Global Geometry of Multichannel Sparse Blind Deconvolution on the Sphere

Yanjun Li Yoram Bresler

CSL and Department of ECE, UIUC

Neural Information Processing Systems Foundation

ILLINOIS
Coordinated Science Lab
Electrical & Computer Engineering
COLLEGE OF ENGINEERING

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Multichannel Sparse Blind Deconvolution (MSBD)

Model:

- Given circular convolution: $y_i = x_i \circledast f$, for $i = 1, 2, \ldots, N$
- Solve for x_i and f

Assumptions:

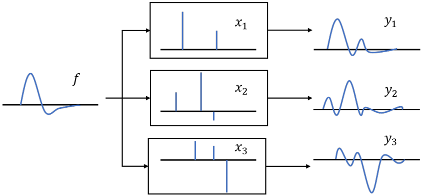
- $f \in \mathbb{R}^n$: invertible signal
- $x_i \in \mathbb{R}^n$: sparse filters

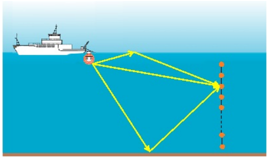
Applications:

- opportunistic underwater acoustics
- reflection seismology
- functional MRI
- super-resolution fluorescence microscopy

Open problem:

- Guaranteed algorithm for unconstrained f





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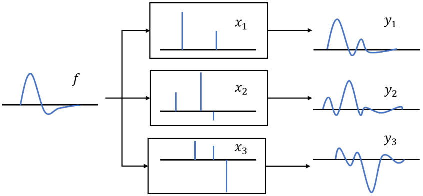

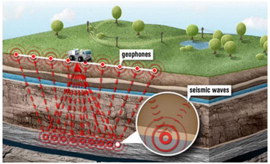

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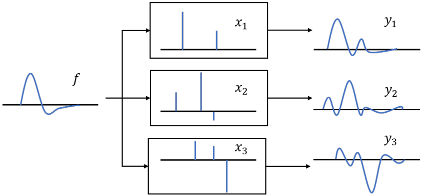

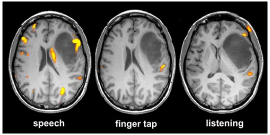

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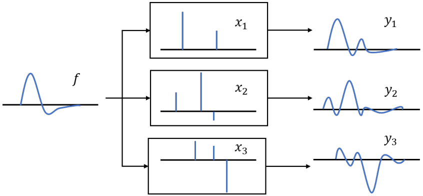
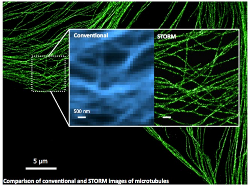


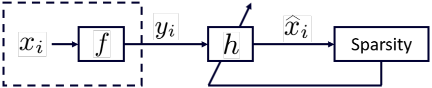


Formulation

Solving for inverse filter

- Find the inverse h of f

$$\text{(P0)} \quad \min_{h \in \mathbb{R}^n} \; \frac{1}{N} \sum_{i=1}^{N} \|C_{y_i} h\|_0, \quad \text{s.t.} \; h \neq 0.$$



- Solution: scaled & shifted

Smooth formulation

- min. “sparsity” ℓ_1 norm $\approx$ max. “spiky” ℓ_4 norm

$$\text{(P1)} \quad \min_{h \in \mathbb{R}^n} \; -\frac{1}{4N} \sum_{i=1}^{N} \|C_{y_i} R h\|_4^4, \quad \text{s.t.} \; \|h\| = 1.$$

Preconditioner $R := \left(\frac{1}{\theta n N} \sum_{i=1}^{N} C_{y_i}^\top C_{y_i}\right)^{-1/2}$

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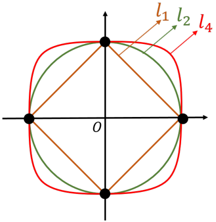

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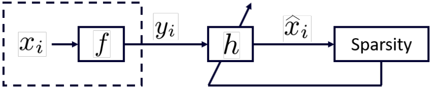

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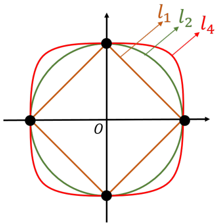

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Main Result

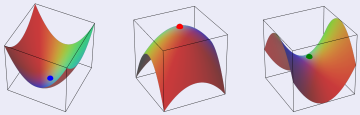
Theorem (Geometric Analysis [L. and Bresler, 2018])

If

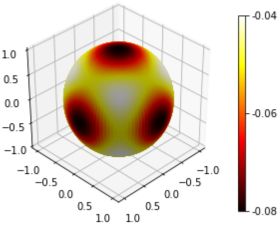
- $\{x_i\}_{i=1}^N \subset \mathbb{R}^n$: *Bernoulli-Rademacher*
- $N \gtrsim \mathrm{polylog}(n)$

Then w.h.p.,

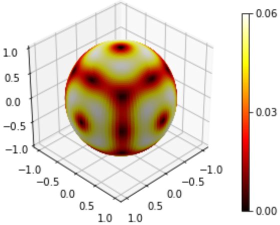
- *local minima* $\Longleftrightarrow$ *signed & shifted ground truth*
- *objective function:*
  - *near local minima: strongly convex*
  - *near local maxima & saddle points: negative curvature (strict saddle points)*



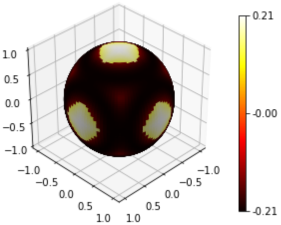
Geometric Structure



objective



norm of gradient



smallest curvature

First-Order Algorithm

Optimize the sparsity promoting objective over the unit sphere

- Manifold gradient descent: $h^{(t+1)} = P_{S^{n-1}}\big(h^{(t)} - \gamma \widehat{\nabla}_L(h^{(t)})\big)$
  - gradient descent along the tangent space
  - retraction (projection onto the sphere)
- Time complexity (per iteration): $O(Nn \log n)$

Theorem

If

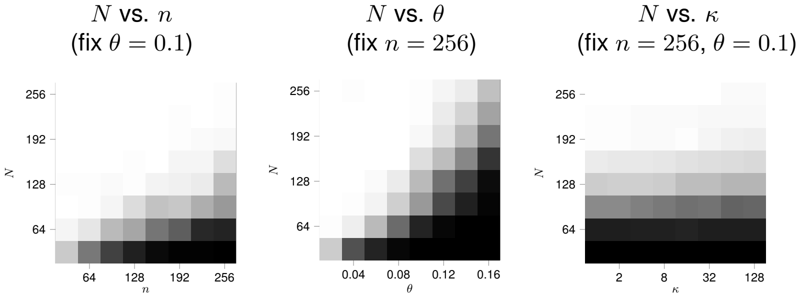
- *geometric properties*
- *random initialization* $h^{(0)} \sim \mathrm{Uniform}(S^{n-1})$
- *fixed step size*

Then manifold gradient descent

- *converges to a local minimum (*$\approx$ *signed & shifted ground truth) a.s.*
- *achieves accuracy* ρ *after* $T \gtrsim \mathrm{poly}(n/\rho)$ *steps*

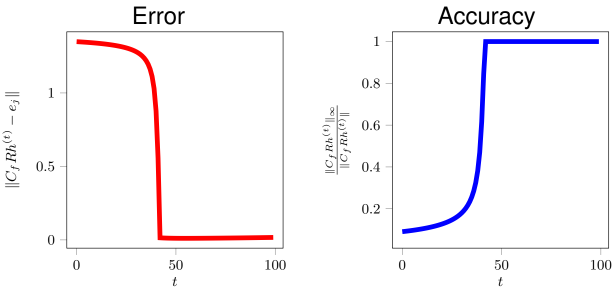
Empirical Phase Transition

- Random $f \in \mathbb{R}^n$, Bernoulli-Rademacher $x_i \in \mathbb{R}^n$
- Noise level: $20\,$dB
- Iteration number $T = 100$, step size $\gamma = 0.1$



- Empirical success:
  - $N \gtrsim n\theta$
  - weak dependence on κ

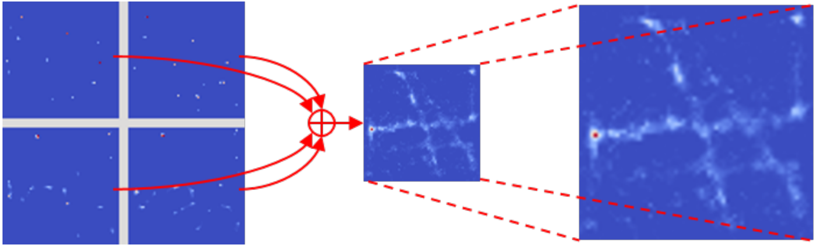
Empirical Convergence



Application: SR Fluorescence Microscopy

Time resolved images

- fluorophores $\Longrightarrow$ sparse
- random activation $\Longrightarrow$ random



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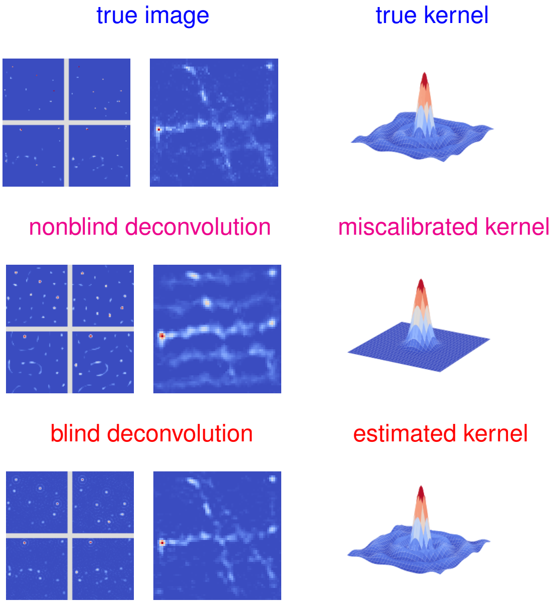

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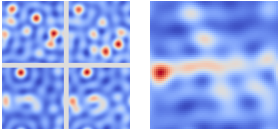
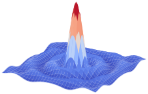
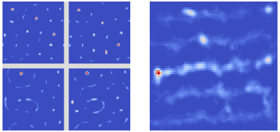
blurry image



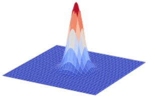
true kernel



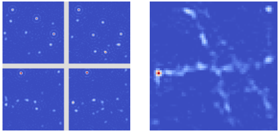
nonblind deconvolution



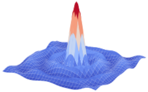
miscalibrated kernel



blind deconvolution



estimated kernel



Thank you!