ALPS: Improved Optimization for Highly Sparse One-Shot Pruning for Large Language Models

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NEURAL INFORMATION PROCESSING SYSTEMS

Large Language Models Compression via Unstructured Pruning

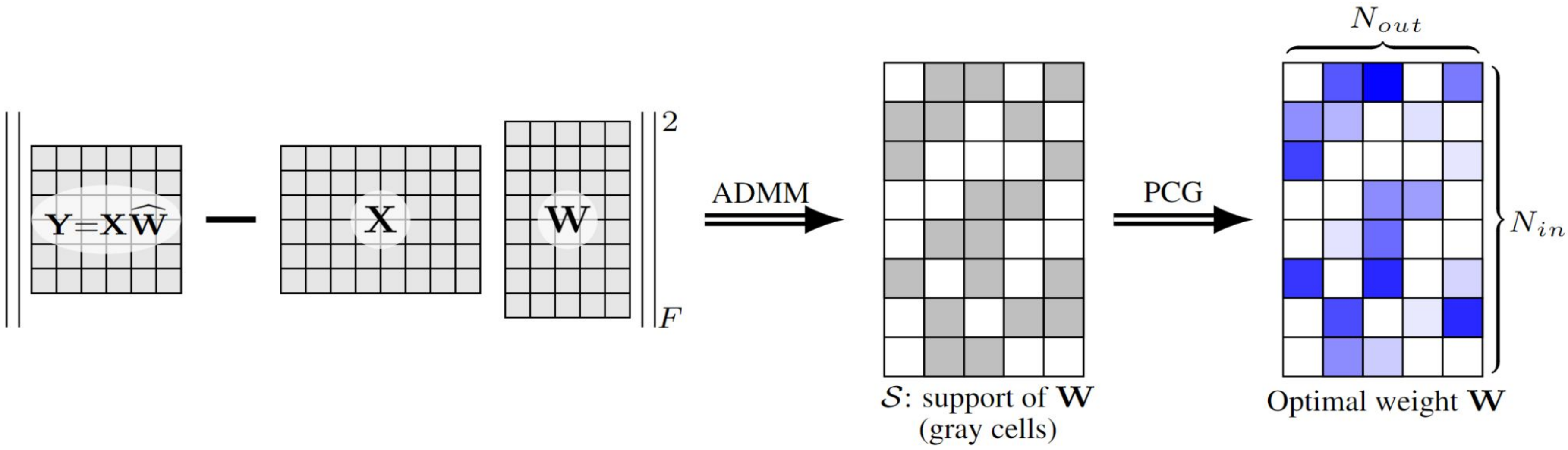
- ❖ Large language models like ChatGPT revolutionize our daily life with massive scale. However, with up to hundreds of billions of parameters, they pose significant challenges in storage and inference

- ❖ **Network Pruning**: reduces model size by identifying redundant weights and setting them to zero

- ❖ Focus on **one-shot unstructured pruning**: compress a pre-trained model once using limited data, without fine-tuning

- ❖ Challenges:
  - ➢ CNN pruning methods don't scale to LLMs;
  - ➢ current LLM pruning relies on heuristics, potentially leading to suboptimal compression-accuracy trade-offs

Overview of ALPS — ADMM-based LLM Pruning in one-Shot



- Layerwise reconstruction objective with an ℓ_0 constraint on the weights.
- ADMM is employed to determine high-quality support for the weight matrix $\mathbf{W}$.
- Restrict the optimization problem to support obtained by ADMM, apply a modified PCG to compute optimal weights within the support.

ADMM for Layer-wise Unstructured Pruning

- Layer-wise pruning problem

$$\min_{\mathbf{W}} \; Q(\mathbf{W}) := \|\mathbf{X}\widehat{\mathbf{W}} - \mathbf{X}\mathbf{W}\|_F^2 + \lambda_2\|\widehat{\mathbf{W}} - \mathbf{W}\|_F^2 \quad \text{s.t.} \; \|\mathbf{W}\|_0 \le k$$

- Reformulation via operater splitting

$$\min_{\mathbf{W},\mathbf{D}} \; Q(\mathbf{W}) + \infty \cdot \mathbf{1}_{\|\mathbf{D}\|_0 > k} \quad \text{s.t.} \quad \mathbf{W} = \mathbf{D}$$

- Augmented Lagrangian

$$L_\rho(\mathbf{W}, \mathbf{D}, \mathbf{V}) = Q(\mathbf{W}) + \infty \cdot \mathbf{1}_{\|\mathbf{D}\|_0 > k} + \langle \mathbf{V}, \mathbf{W} - \mathbf{D} \rangle + \frac{\rho}{2}\|\mathbf{W} - \mathbf{D}\|_F^2$$

- ADMM updates

$$\begin{aligned}
\mathbf{W}^{(t+1)} &= \arg\min_{\mathbf{W}} L_\rho(\mathbf{W}, \mathbf{D}^{(t)}, \mathbf{V}^{(t)}) = \left(\mathbf{X}^\top\mathbf{X} + (\lambda_2 + \rho)\mathbf{I}\right)^{-1}\left((\mathbf{X}^\top\mathbf{X} + \lambda_2\mathbf{I})\widehat{\mathbf{W}} - \mathbf{V}^{(t)} + \rho\mathbf{D}^{(t)}\right) \\
\mathbf{D}^{(t+1)} &= \arg\min_{\mathbf{D}} L_\rho(\mathbf{W}^{(t+1)}, \mathbf{D}, \mathbf{V}^{(t)}) = P_k\left(\mathbf{W}^{(t+1)} + \mathbf{V}^{(t)}/\rho\right), \\
\mathbf{V}^{(t+1)} &= \mathbf{V}^{(t)} + \rho(\mathbf{W}^{(t+1)} - \mathbf{D}^{(t+1)}),
\end{aligned}$$

ADMM for Layer-wise Unstructured Pruning

$$\mathbf{W}^{(t+1)} = \arg\min_{\mathbf{W}} L_\rho(\mathbf{W}, \mathbf{D}^{(t)}, \mathbf{V}^{(t)}) = \left(\mathbf{X}^\top\mathbf{X} + (\lambda_2 + \rho)\mathbf{I}\right)^{-1}\left((\mathbf{X}^\top\mathbf{X} + \lambda_2\mathbf{I})\widehat{\mathbf{W}} - \mathbf{V}^{(t)} + \rho\mathbf{D}^{(t)}\right)$$
$$\mathbf{D}^{(t+1)} = \arg\min_{\mathbf{D}} L_\rho(\mathbf{W}^{(t+1)}, \mathbf{D}, \mathbf{V}^{(t)}) = P_k\left(\mathbf{W}^{(t+1)} + \mathbf{V}^{(t)}/\rho\right),$$
$$\mathbf{V}^{(t+1)} = \mathbf{V}^{(t)} + \rho(\mathbf{W}^{(t+1)} - \mathbf{D}^{(t+1)}),$$

All updates can be efficiently computed via torch (on GPU)

Increase ρ along iterations; ensures convergence & finding high quality solutions

The rate of change of the support between consecutive iterations, comparing ALPS with ADMM using a fixed ρ

Supp change / Iter	5	10	20	30	50	100
ALPS	20.2%	17.0%	2.8%	0.0%	0.0%	0.0%
ADMM($\rho = 0.3$)	6.4%	7.0%	7.0%	7.0%	6.9%	6.9%
ADMM($\rho = 3$)	0.2%	<0.1%	<0.1%	<0.1%	<0.1%	<0.1%

Efficiently Refining Weights after Support Stabilization

Post-processing: refining the solution within support given by ADMM

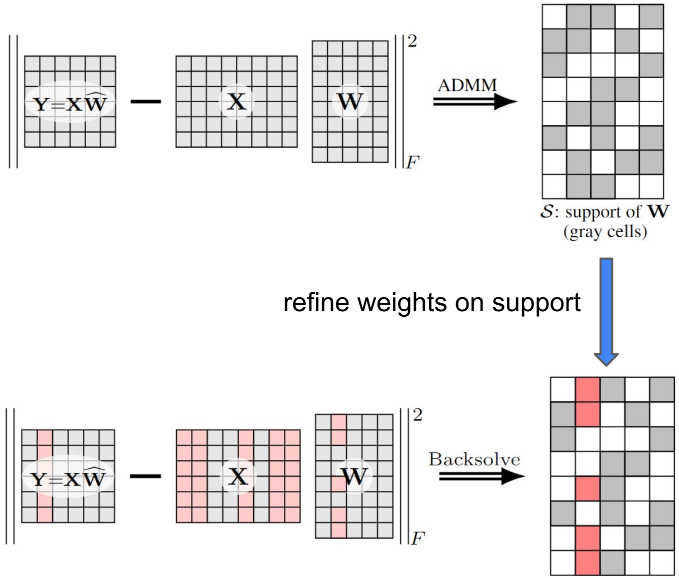
$$\min_{\mathbf{W}} \ Q(\mathbf{W}) \quad \text{s.t.} \ \ \mathrm{Supp}(\mathbf{W}) \subset \mathcal{S}.$$

Decomposes into separate least squares problems across the columns of W.

Direct backsolve — solving >10k different linear systems (very costly)

Proposed: modified conjugate gradient method solving **all columns simultaneously**

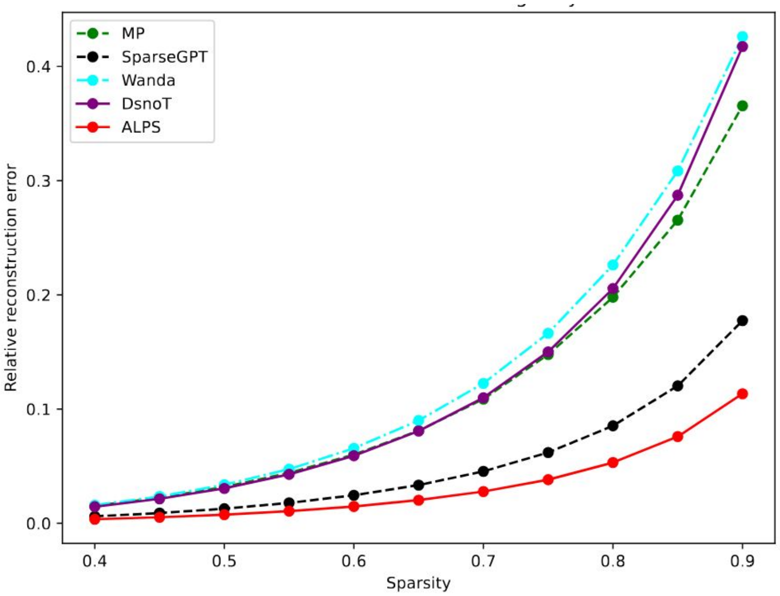
Exploit GPU **parallelism**, >100x acceleration (table)

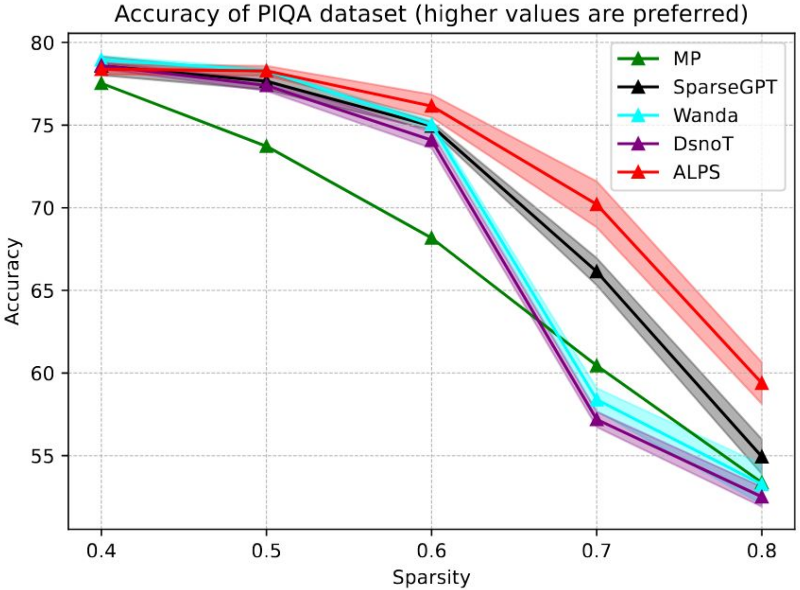


Sparsity	*ALPS*		Backsolve	
	Time(s)	Error	Time(s)	Error
0.5	0.77	1.24e-2	131	1.10e-2
0.6	0.79	2.43e-2	95.0	2.25e-2
0.7	0.78	4.56e-2	64.1	4.38e-2

Experimental Results

Comparing with (i)(MP, [Han et al., 2015]), (ii) SparseGPT [Frantar and Alistarh, 2023], (iii) Wanda [Sun et al. 2023], and (iv) DSnoT [Zhang et al., 2023]





Pruning results on LLaMA-7B. ALPS achieves much lower reconstruction objective (left), which translate to higher performance in various tasks (right).