Learning to Handle Complex Constraints for Vehicle Routing Problems

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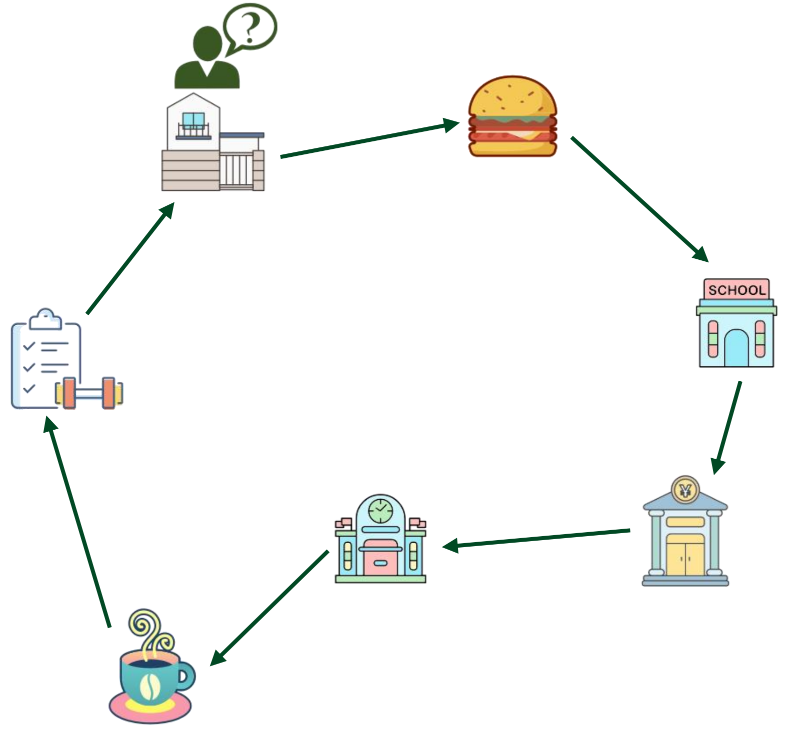
01 Problem Definition

Vehicle Routing Problems (VRP) is a typical combinatorial optimization problem.

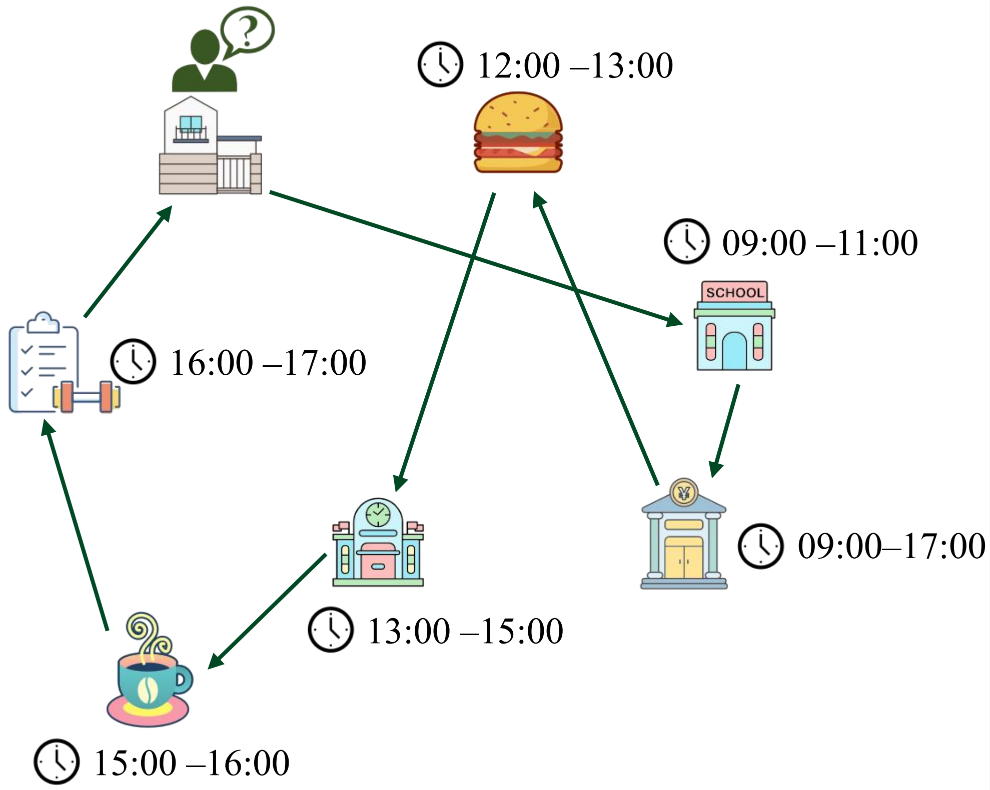
- Objective: to minimize the total travelling cost (e.g., the tour length)
- Constraint: each node should be visited once and only once + constraints in other VRP variants



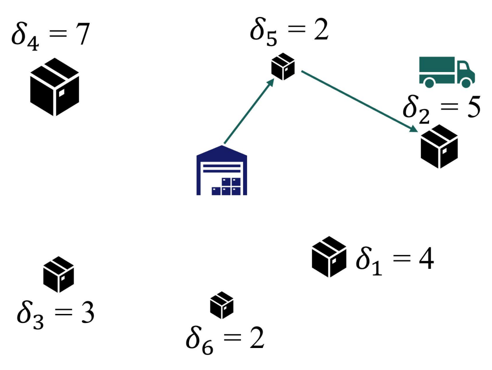
Travelling Salesman Problem (TSP)

+ Problem-specific Constraints

e.g., time window



01 Constructive solvers for VRPs



Route

0 → 5 → 2 → ?

Capacity: 10

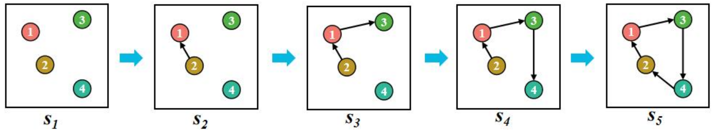
Remaining Load: 3

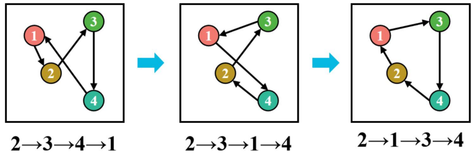
ID	Demand	$P_{original}$	Feasible?	P_{final}
0	/	0.33	☑	0.38
1	4	0.24	☒ (exceed capacity)	0
2	5	0.05	☒ (visited)	0
3	3	0.12	☑	0.31
4	7	0.04	☒ (exceed capacity)	0
5	2	0.09	☒ (visited)	0
6	2	0.13	☑	0.31

Feasibility masking

How do they handle constraints?

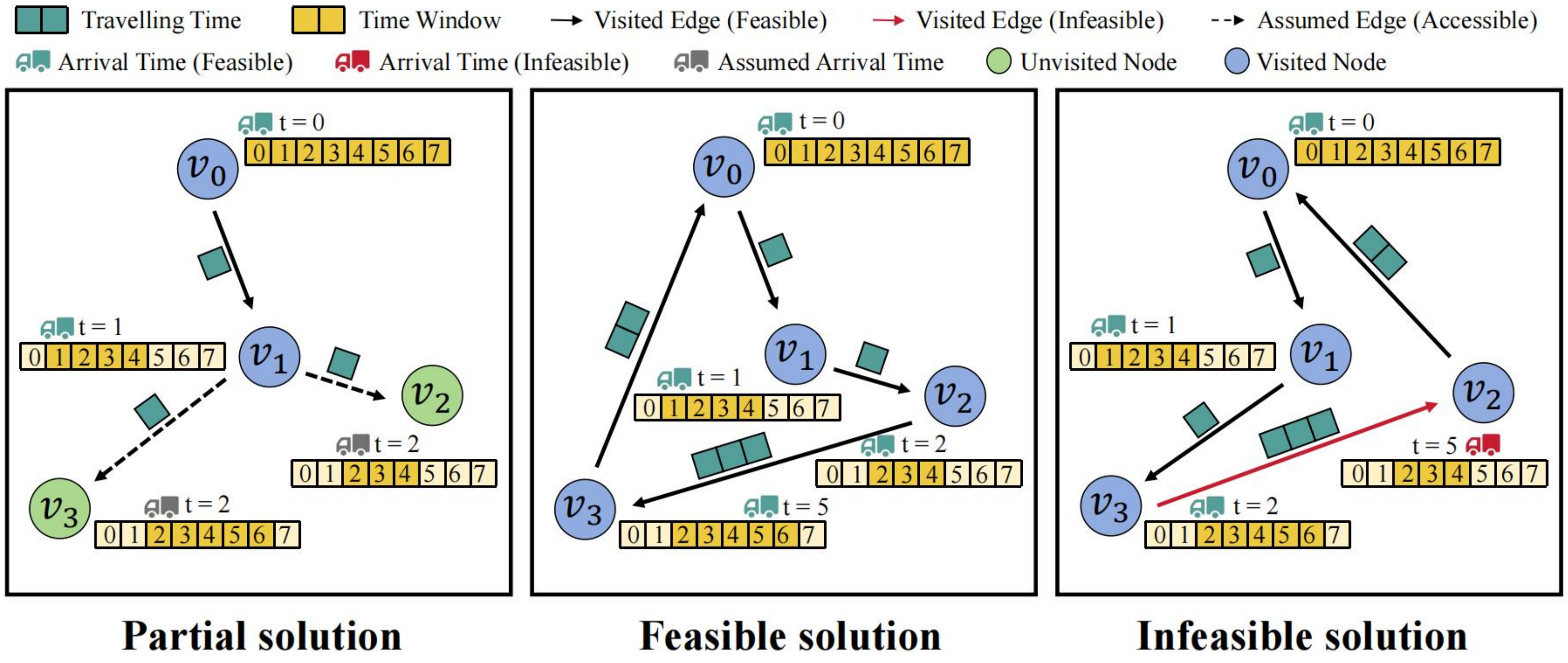
Neural solvers for VRPs
- **Constructive solvers**
  - **Autoregressive solvers**
  - **Non-autoregressive solvers**
- **Iterative solvers**

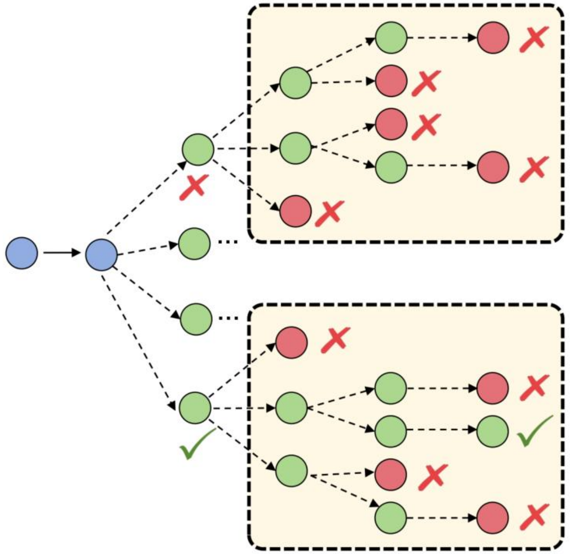




01

Dilemma of feasibility masking – NP-Hard





02 Methodology - Lagrangian-assisted constraint awareness

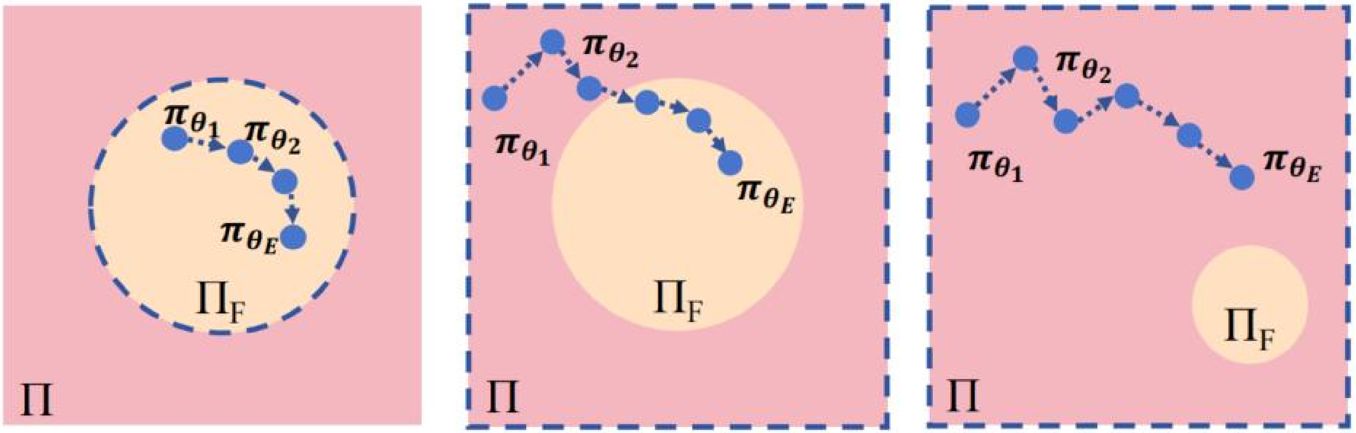
- Acquisition of the feasibility masking is also a NP-Hard problem.

$$\max_{\theta} \mathcal{J}(\pi_\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{e(v_i,v_j)\in\tau} \mathcal{R}\left(e\left(v_i, v_j\right)\right)\right],$$

$$\boxed{\text{s.t. } \pi_\theta \in \Pi_F,\ \Pi_F = \{\pi \in \Pi \,|\, \mathcal{J}_{\mathcal{C}_m}(\pi) \le \kappa_m,\ \forall m \in [1, M]\}.}$$

$$\Rightarrow$$

$$\min_{\lambda\ge 0}\max_{\theta} \mathcal{L}(\lambda,\theta) = \min_{\lambda\ge 0}\max_{\theta} -\mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{e(v_i,v_j)\in\tau} ||v_i - v_j||_2 + \sum_{m=1}^{M} \lambda_m \mathcal{J}_{C_m}(\tau) + \mathcal{J}_{\text{IN}}\right]$$



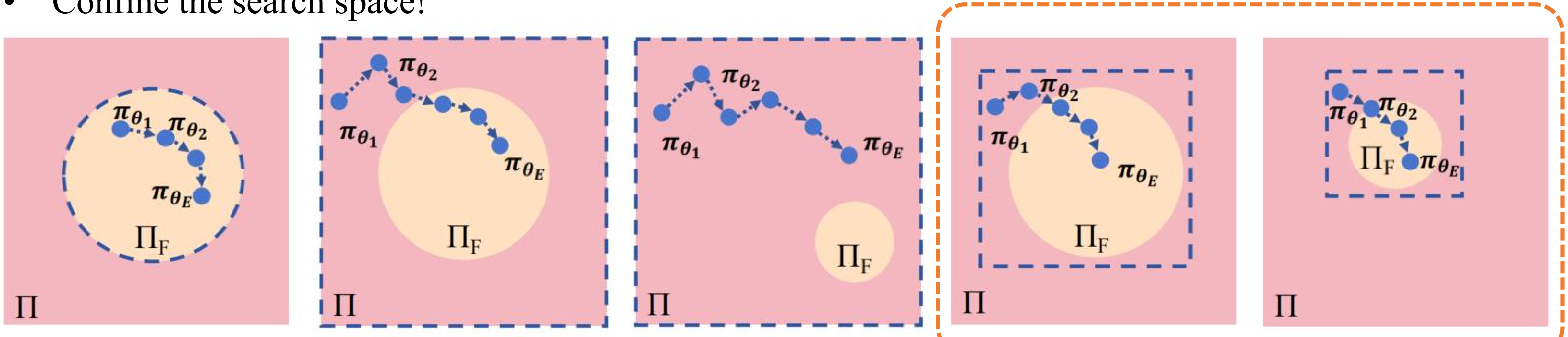
Lagrange on Easy

Lagrange on Hard

- For more complex cases, neural solvers with Lagrangian Relaxation still struggle to navigate the large search space.

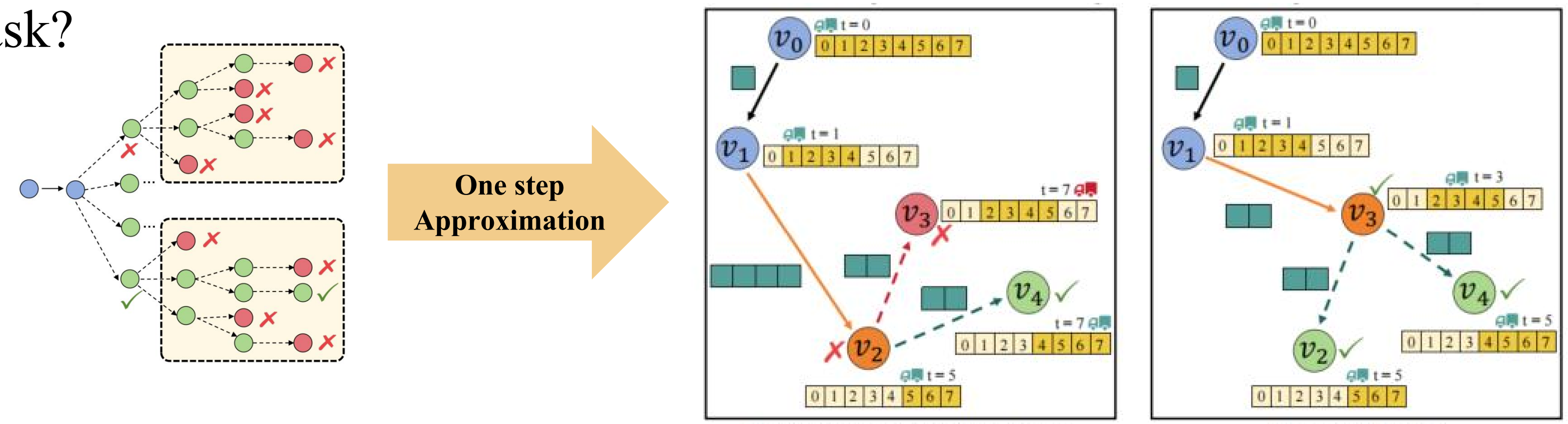
02 Methodology - Preventative infeasibility Prevention (PIP)

- Confine the search space!



Lagrange on Easy   Lagrange on Hard

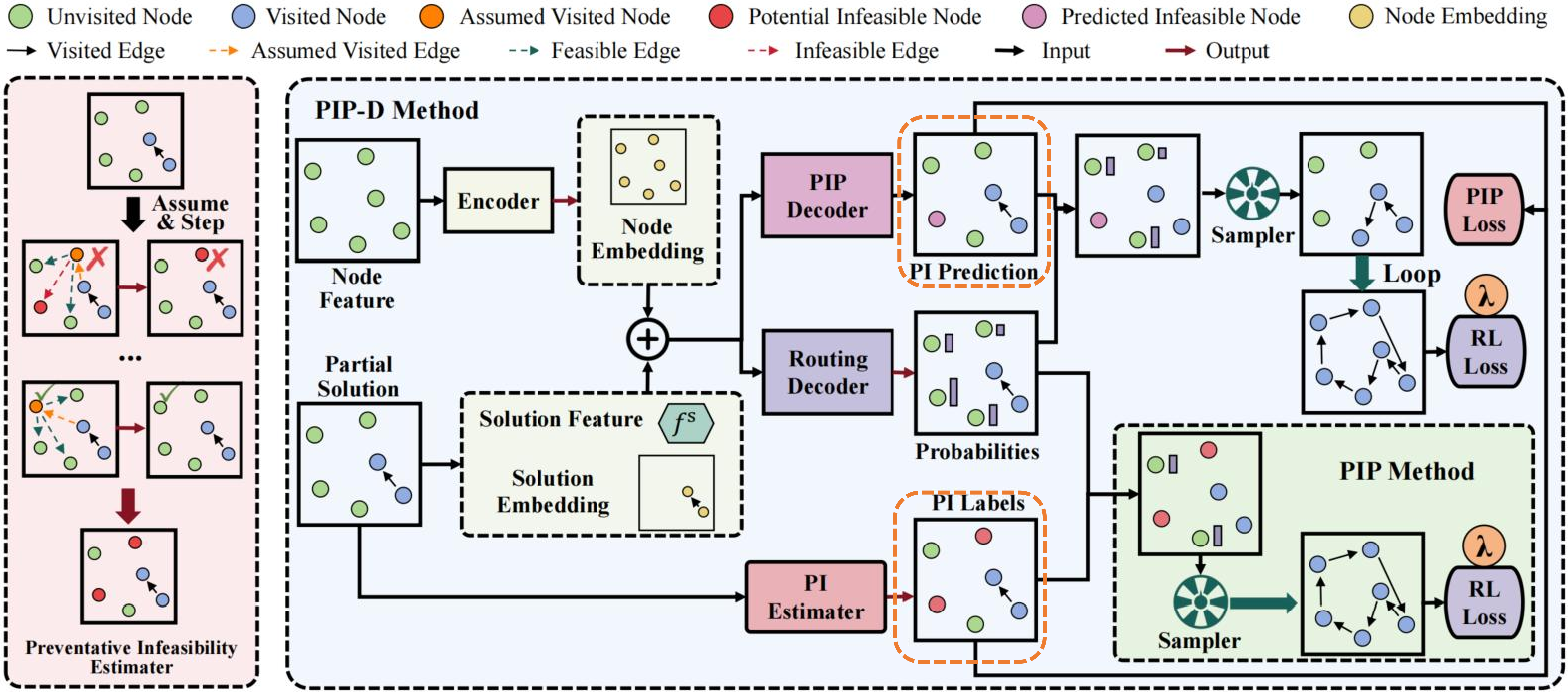
- How to obtain the PI mask?



Preventative Infeasible Nodes   Feasible Nodes

02

Methodology –PIP-D



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Experiment

Constructive solvers

- **Autoregressive solvers: POMO, AM**
- **Non-autoregressive solvers: GFACS**

TSPTW

Easy | **Medium** | **Hard**

Travelling Salesman Problem with Time Window

TSPDL

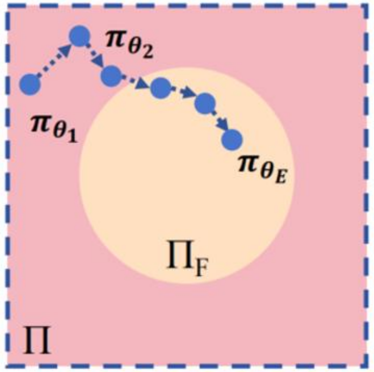
Medium | **Hard**

Travelling Salesman Problem with Draft Limit

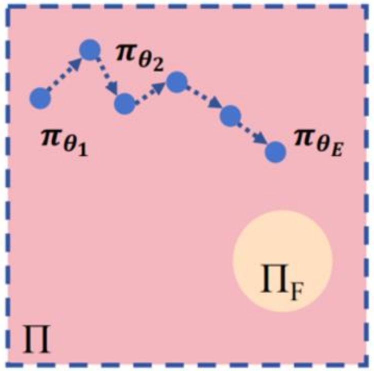
03 Experiments on TSPTW

Easy/ Medium/ Hard

Lagrange can handle easy constraints well, but fail on more complex constraints with larger problem scale.



Lagrange on Easy



Lagrange on Hard

	Method	$n = 50$ Infeasible% Sol.↓	Inst.↓	Obj.↓	Gap↓	Time↓	$n = 100$ Infeasible% Sol.↓	Inst.↓	Obj.↓	Gap↓	Time↓
Easy	LKH3	0.00%	0.00%	7.31	0.00%	4.6h	0.00%	0.00%	10.21	0.00%	8.5h
	ORTools	0.00%	0.00%	7.34	0.96%	7h	0.00%	0.00%	10.41	1.97%	14h
	Greedy-L	100.00%	100.00%	/	/	13.8s	100.00%	100.00%	/	/	1.3m
	Greedy-C	0.00%	0.00%	26.08	257.27%	4.5s	0.00%	0.00%	52.14	411.13%	12s
	JAMPR #	/	0.00%	/	249.03%	1.2m	/	100.00%	/	/	1.6m
	OSLA #	/	11.80%	/	8.15%	15.6s	/	/	/	/	/
	MUSLA #	/	8.20%	/	7.32%	1.3m	/	18.60%	/	14.6%	9.8m
	MUSLA adapt #	/	0.10%	/	5.63%	7.7m	/	0.60%	/	12.01%	1.1h
	AM	100.00%	100.00%	/	/	5m	100.00%	100.00%	/	/	21m
	AM*	3.46%	0.22%	8.02	9.82%	5.2m	7.87%	1.49%	11.84	16.07%	21m
	AM*+PIP	0.55%	0.00%	7.87	7.67%	10.7m	0.45%	0.00%	11.42	11.86%	1h
	AM*+PIP-D	0.51%	0.00%	7.91	8.19%	11m	0.25%	0.00%	11.53	13.02%	1h
	POMO	100.00%	100.00%	/	/	13s	100.00%	100.00%	/	/	21s
	POMO*	1.75%	0.00%	7.54	3.08%	13s	2.11%	0.00%	10.83	6.07%	21s
	POMO* + PIP	0.32%	0.00%	7.50	2.65%	15s	0.15%	0.00%	10.57	3.53%	48s
	POMO* + PIP-D	0.28%	0.00%	7.49	2.51%	15s	0.06%	0.00%	10.66	4.39%	48s
Medium	LKH3	0.00%	0.00%	13.02	0.00%	7h	0.00%	0.00%	18.74	0.00%	10.8h
	ORTools	15.77%	15.77%	13.02	0.30%	5.9h	0.52%	0.52%	19.34	3.23%	13.8h
	Greedy-L	100.00%	100.00%	/	/	15s	100.00%	100.00%	/	/	1m
	Greedy-C	47.52%	47.52%	25.33	96.43%	4.2s	20.34%	20.34%	51.62	176.07%	11.4s
	AM	100.00%	100.00%	/	/	5m	100.00%	100.00%	/	/	21m
	AM*	24.84%	0.27%	13.81	6.11%	5m	50.19%	0.09%	21.42	14.34%	21m
	AM*+PIP	7.62%	0.35%	13.68	5.06%	11m	12.73%	0.04%	20.57	9.82%	1h
	AM*+PIP-D	11.96%	0.33%	13.65	4.87%	11m	8.80%	0.02%	20.80	11.03%	1h
	POMO	100.00%	100.00%	/	/	13s	100.00%	100.00%	/	/	21s
	POMO*	14.92%	3.77%	13.68	5.23%	13s	18.77%	0.12%	20.78	10.93%	21s
	POMO* + PIP	4.53%	0.90%	13.40	2.91%	15s	3.88%	0.19%	19.61	4.65%	48s
	POMO* + PIP-D	3.83%	0.65%	13.45	3.32%	15s	3.34%	0.03%	19.79	5.64%	48s
Hard	LKH3	0.12%	0.12%	25.61	0.00%	7h	0.07%	0.07%	51.24	0.00%	1.4d
	ORTools	65.72%	65.72%	25.76	-0.00%	2.4h	89.07%	89.07%	51.61	0.00%	1.6h
	Greedy-L	100.00%	100.00%	/	/	21.8s	100.00%	100.00%	/	/	1.3m
	Greedy-C	72.55%	72.55%	26.39	1.53%	4.5s	93.38%	93.38%	52.95	1.43%	11.1s
	AM	100.00%	100.00%	/	/	5m	100.00%	100.00%	/	/	21m
	AM*	39.87%	18.88%	26.08	1.425%	5m	100.00%	100.00%	/	/	21m
	AM*+PIP	18.07%	1.98%	25.71	0.38%	11m	41.92%	16.46%	51.49	0.47%	1h
	AM*+PIP-D	30.39%	4.40%	25.80	0.67%	11m	53.09%	5.33%	51.55	0.57%	1h
	POMO	100.00%	100.00%	/	/	13s	100.00%	100.00%	/	/	21s
	POMO*	39.26%	35.25%	26.22	1.61%	13s	100.00%	100.00%	/	/	21s
	POMO* + PIP	5.54%	2.67%	25.66	0.18%	15s	31.49%	16.27%	51.42	0.37%	48s
	POMO* + PIP-D	6.76%	3.07%	25.69	0.28%	15s	13.18%	6.48%	51.39	0.31%	48s

Experiments on TSPDL

Medium/ Hard

	Method	n = 50 Infeasible% Sol.↓	Inst.↓	Obj.↓	Gap↓	Time↓	n = 100 Infeasible% Sol.↓	Inst.↓	Obj.↓	Gap↓	Time↓
Medium	LKH3	0.00%	0.00%	10.87	0.00%	5.1h	0.00%	0.00%	16.39	0.00%	14h
	ORTools	100.00%	100.00%	/	/	10.9s	100.00%	100.00%	/	/	56.9s
	Greedy-L	100.00%	100.00%	/	/	2.4m	100.00%	100.00%	/	/	9.5m
	Greedy-C	0.00%	0.00%	26.09	144.24%	9.1s	0.00%	0.00%	52.16	222.71%	27s
	POMO*	17.72%	12.52%	10.98	3.80%	6.9s	49.39%	32.19%	17.11	9.15%	18s
	POMO* + PIP	2.21%	0.43%	11.22	3.41%	8.5s	2.88%	0.38%	17.71	8.08%	31s
	POMO* + PIP-D	2.64%	0.37%	11.26	3.78%	8.4s	2.14%	0.23%	17.84	8.86%	31s
Hard	LKH3	0.00%	0.00%	13.30	0.00%	6.8h	0.00%	0.00%	20.70	0.00%	1.2d
	ORTools	100.00%	100.00%	/	/	10.6s	100.00%	100.00%	/	/	56.8s
	Greedy-L	100.00%	100.00%	/	/	2.4m	100.00%	100.00%	/	/	9.4m
	Greedy-C	0.00%	0.00%	26.07	99.73%	10.9s	0.00%	0.00%	52.17	156.37%	25s
	POMO*	37.01%	29.25%	13.03	4.11%	6.8s	99.98%	99.85%	20.95	15.87%	18s
	POMO* + PIP	4.53%	2.10%	13.66	3.13%	8.5s	28.55%	20.66%	22.30	12.67%	31s
	POMO* + PIP-D	3.89%	0.82%	13.80	3.95%	8.5s	12.84%	7.91%	22.84	12.32%	31s

03

Experiments on GFACS

Constructive solvers
- Autoregressive solvers: POMO; AM
- Non-autoregressive solvers: GFACS

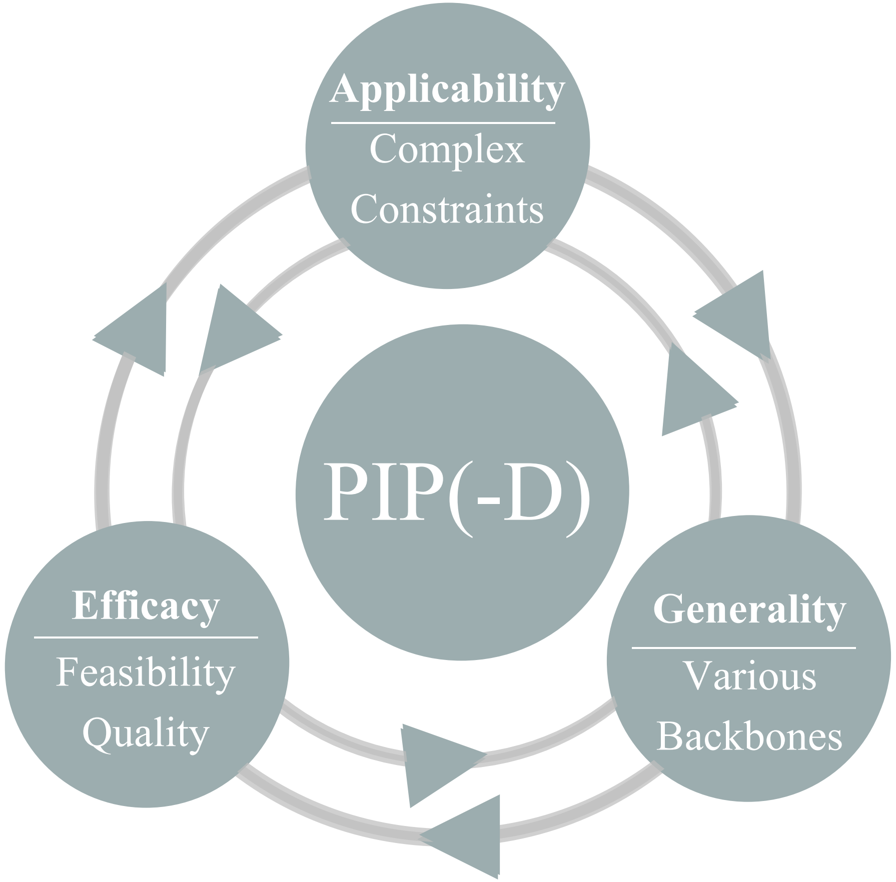
Method	Infeasible% Sol.↓	Infeasible% Inst.↓	Gap↓	Time↓
LKH3	0.00%	0.00%	0.00%	26m
Greedy-L	100.00%	100.00%	/	3.2m
Greedy-C	100.00%	100.00%	/	4.1s
GFACS*	58.20%	57.81%	21.32%	6.4m
GFACS* + PIP	4.72%	1.56%	15.04%	6.5m
GFACS* + PIP-D	0.03%	0.00%	11.95%	6.5m

Results on Medium TSPTW-500

04

Conclusion

Lagrangian multiplier method+ Preventative infeasibility masking + Auxiliary decoder



Applicability

- Challenges of NCO: Complex constraints and Scalability
- Complex VRPs with various constraints hardness levels

Efficacy

- Significant (up to 93.52%) reduction in infeasible rate
- Improvement in solution quality

Generality

Across various backbone models (i.e., AM, POMO and GFACS)

Thanks

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