# Tri-Level Navigator: LLM-Empowered Tri-Level Learning for Time Series OOD Generalization

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# Outline

# Introduction

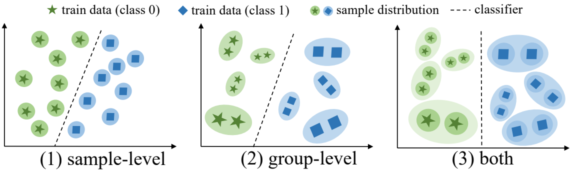
- **Background**: Out-of-Distribution (OOD) generalization is a key challenge in machine learning when training and test distributions differ significantly.
- **Problem**: Time series data present unique challenges for OOD generalization due to temporal dependencies and dynamic changes.
- **Goal**: We propose a novel tri-level learning framework (TTSO) based on Large Language Models (LLMs) for time series OOD generalization.

# Motivation

- **Current State**: Extensive research has focused on OOD generalization in vision and text domains, but limited work exists for time series.
- **Pre-trained Models**: LLMs like GPT demonstrate strong generalization and representation learning capabilities across modalities, including time series.
- **Solution**: Our TTSO framework leverages LLMs and tri-level optimization to address time series OOD generalization challenges.

# Related Work

- **OOD Generalization**: Previous work focuses on sample-level or group-level uncertainties.
- **LLMs in Time Series**: An emerging field where LLMs show promise in time series analysis.
- **Contribution**: TTSO uniquely integrates sample-level and group-level uncertainties into a tri-level learning problem with LLMs to enhance OOD generalization.



# Tri-Level Learning Framework

- **Overview**: TTSO addresses sample and group uncertainties through tri-level optimization:

$$
\begin{aligned}
\min_{\boldsymbol{\theta},\boldsymbol{q},\boldsymbol{\delta}} \quad & \textstyle\sum_{i=1}^{K} q_i \ell_{\text{con}}\left(\boldsymbol{\theta}, \boldsymbol{\delta}; \mathcal{D}_{S_i}\right) \\
\text{s.t.} \quad & \boldsymbol{q} = \arg\max_{\boldsymbol{q}' \in \Delta^K} \textstyle\sum_{i=1}^{K} q_i' \ell_{\text{con}}\left(\boldsymbol{\theta}, \boldsymbol{\delta}; \mathcal{D}_{S_i}\right) \\
& \text{s.t. } d(\boldsymbol{p}, \boldsymbol{q}') \leq \tau \\
& \qquad \boldsymbol{\delta} = \arg\max_{\boldsymbol{\delta}' \sim p(\boldsymbol{\delta}'; \boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma})} \textstyle\sum_{i=1}^{K} q_i' \ell_{\text{align}}(\boldsymbol{\theta}, \boldsymbol{\delta}'; \mathcal{D}_{S_i}) \\
& \qquad \text{s.t. } \|\boldsymbol{\mu}\| \leq C_1, \|\boldsymbol{\sigma}\| \leq C_2, \textstyle\sum_{m=1}^{M} \pi_m = 1, \pi_m \geq 0,
\end{aligned} \tag{1}
$$

- **Advantages**: Tackles both sample and group uncertainties, providing strong theoretical and practical generalization benefits.

# Stratified Localization Algorithm

- **Step 1**: Use gradient descent and Taylor approximation to transform the tri-level problem into a single-layer optimization problem.
- **Step 2**: Generate cutting planes to approximate the non-convex feasible region.
- **Step 3**: Iteratively refine the solution using the cutting planes.
- **Step 4**: Update variables and add new cutting planes every k iterations to ensure tighter approximation.
- **Step 5**: Return the optimized parameters after satisfying convergence criteria.

# Theoretical Analysis

- **Convergence Analysis**: The Stratified Localization Algorithm (SLA) is proven to converge to an $\epsilon$-stationary point.
- **Iteration Complexity**: The iteration complexity is given by:

$$T(\epsilon) \sim \mathcal{O}\left(t_1 + \frac{L^2(m(\alpha_1^2+\alpha_2^2+\alpha_3^2)+\sigma_1^2+\sigma_2^2+\sigma_3^2)^2}{4m^2(\epsilon - F(\boldsymbol{\theta}^{T_1}, \boldsymbol{q}^{T_1}, \boldsymbol{\delta}^{T_1}) + F^*)^2}\right), \tag{2}$$

- **Generalization Bound**: Based on VC (Vapnik-Chervonenkis) dimension theory, the generalization properties is given by:

$$\begin{aligned} \epsilon_T(\hat{h}) \leq 3\epsilon_T(h_T^*) + \lambda + d_{\mathcal{H}\Delta\mathcal{H}}(\mathbb{P}_\mathsf{C}, \mathbb{P}_\mathsf{T}) \\ + \max_{i,j} d_{\mathcal{H}\Delta\mathcal{H}}(\mathbb{P}_{S_i}, \mathbb{P}_{S_j}) + C(\delta, m, d), \end{aligned} \tag{3}$$

where $\lambda$ and $C(\delta, m, d)$ is a statistical term. $d_{\mathcal{H}\Delta\mathcal{H}}(\cdot,\cdot)$ is a metric function which measures differences in distribution. $\epsilon_{S_i}(h)$ and $\epsilon_T(h)$ is the source error and the target error.

# Experiments

- **Datasets**: Experiments conducted on six real-world time series datasets (HHAR, PAMAP, WESAD, etc.).
- **Baselines**: Compared against traditional OOD methods (ERM, IRM, GroupDRO) and time-series-specific approaches (AdaRNN, GILE, DIVERSIFY, DFDDG, CCDG).
- **Results**: TTSO achieves an maximum 4.9% improvement in performance on time series classification in OOD scenarios.

# Conclusion

- The TTSO framework integrates sample-level and group-level uncertainties with LLMs for time series OOD generalization.
- TTSO demonstrates significant improvements in real-world datasets, outperforming existing methods.
- Future Work: Explore the application of TTSO on other modalities beyond time series, such as image, audio.