A scalable generative model for dynamical system reconstruction from neuroimaging data

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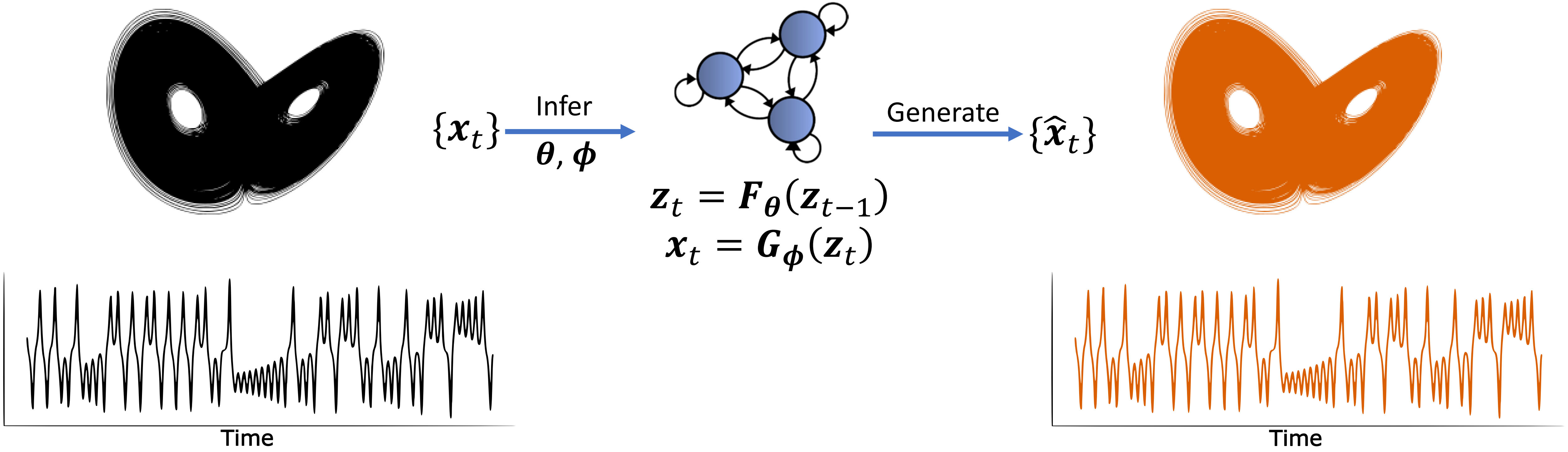
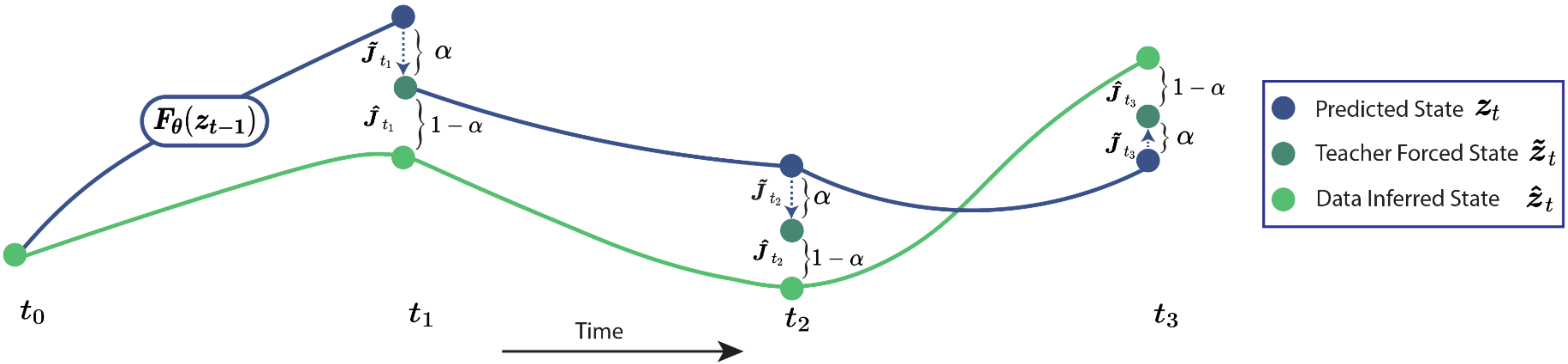
Dynamical Systems Reconstruction (DSR)



Generalized Teacher Forcing (GTF)[1]

[1]Hess et al, ICML 2023, Generalized teacher forcing for learning chaotic dynamics



Interpolate between forward-iterated states z_t and data-inferred states $\hat{z}_t = G_\phi^{-1}(x_t)$:

$$\tilde{z}_t = F_\theta\big((1-\alpha)z_{t-1} + \alpha \hat{z}_{t-1}\big), \qquad \alpha \in (0,1)$$

→ optimal choice of α prevents EGP

DSR for convolved time series

Important empirical setting: observations are a convolution of signal of interest f and a impulse response filter h

$$(f * h)[n] \coloneqq \sum_{m=-\infty}^{\infty} f[n-m]h[m]$$

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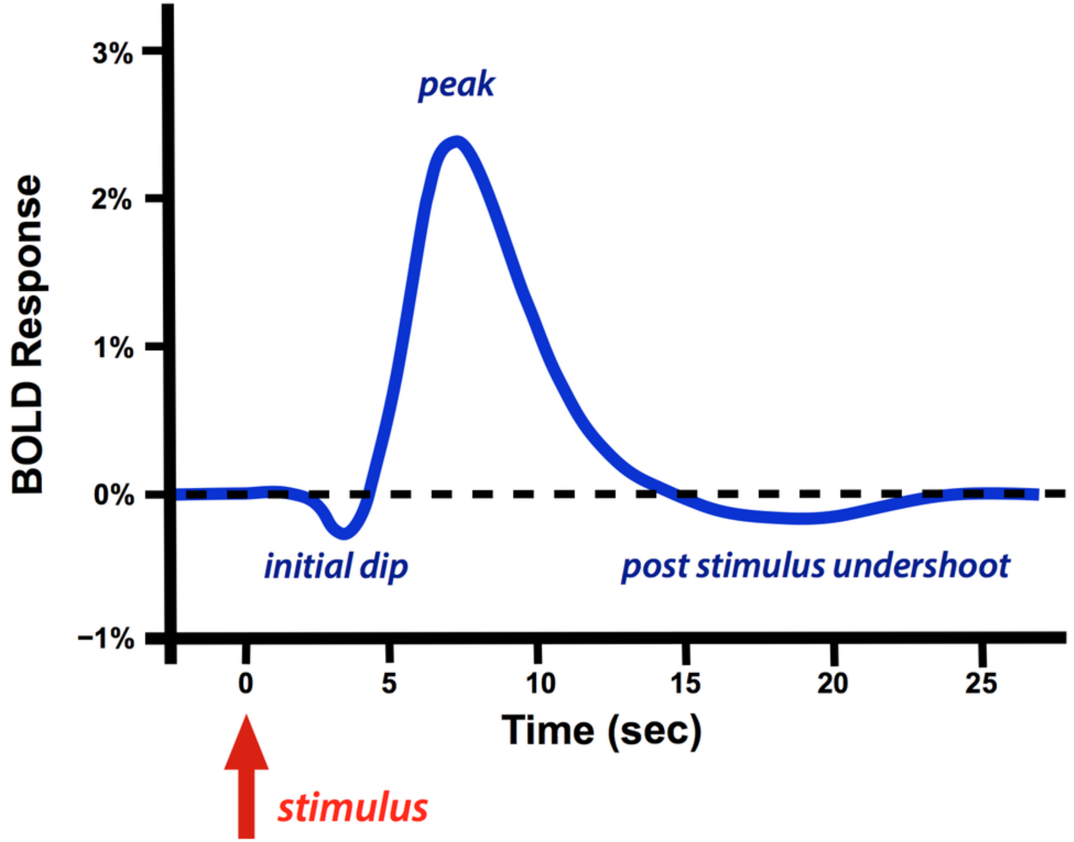
$$(f * h)[n] := \sum_{m=-\infty}^{\infty} f[n-m]h[m]$$

Example: fMRI

- Signal of interest: neuronal activity
- Impulse response: hemodynamic response function (hrf)



ConvSSM

- Modified observation equation

$$\hat{x}_t = \boldsymbol{G_\phi}(\ h * z_{\{t_0:t\}})$$

- Latent states are convolved over time with filter function h

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- **Problem**: non-invertible observation model

  → How to calculate forcing targets at t for GTF?

ConvSSM

Deconvolution approach for fMRI time series motivated by Wu et al. (2021)

Wiener deconvolution filter $G(k) = \frac{\tilde{Y}^*(k)H(k)}{|\tilde{Y}(k)|^2 H(k) + \tilde{N}(k)}$

Wu et al, NeuroImage 2021, A toolbox for resting-state HRF estimation and deconvolution

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4. Use estimate $\tilde{x}$ in Teacher Forcing Algorithm

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1. **State space divergence** D_{stsp}
   Geometrical overlap of orbits in state space

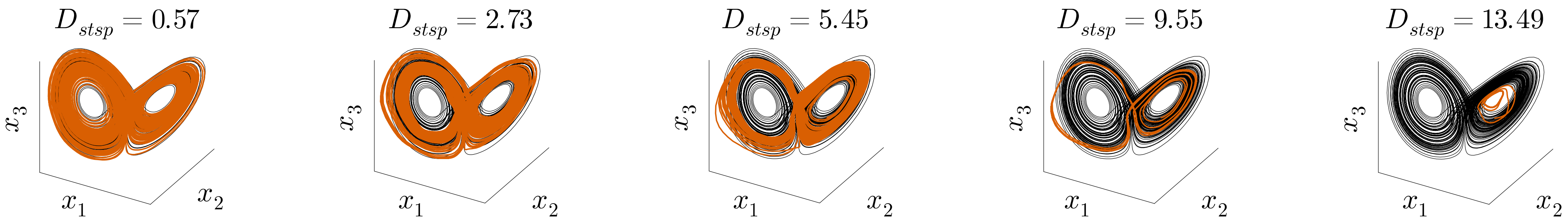
$$D_{stsp} = \int_{\mathbb{R}^N} p(\mathbf{x}) \frac{p(\mathbf{x})}{q(\mathbf{x})}\, d\mathbf{x}$$

2. **Power spectrum error** D_{PSE}
   Average Hellinger distance between power spectra

$$D_{PSE} = \frac{1}{N} \sum_{i=1}^{N} \left(1 - \int_{\mathbb{R}} \sqrt{f_i(\omega) g_i(\omega)}\, dx \right)^{\frac{1}{2}}$$

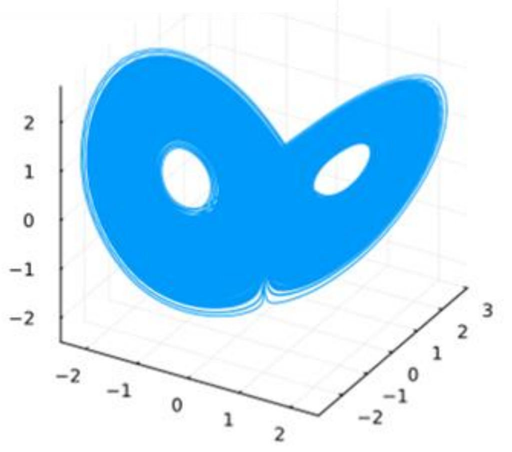
3. **Prediction error** PE
   n-step prediction error

$$PE(n) = \frac{1}{N(T-n)} \sum_{t=1}^{T-n} ||x_{t+n} - \hat{x}_{t+n}||_2^2$$



Validation on Lorenz63 system

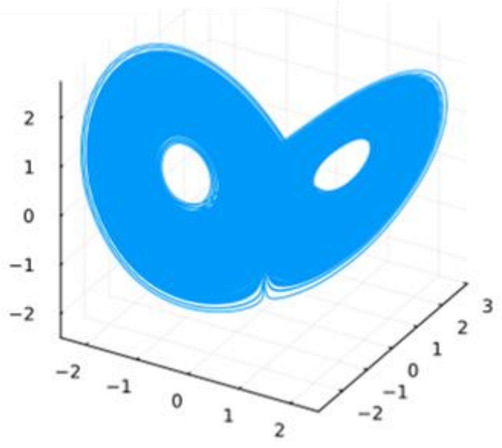
Unobserved latent trajectory



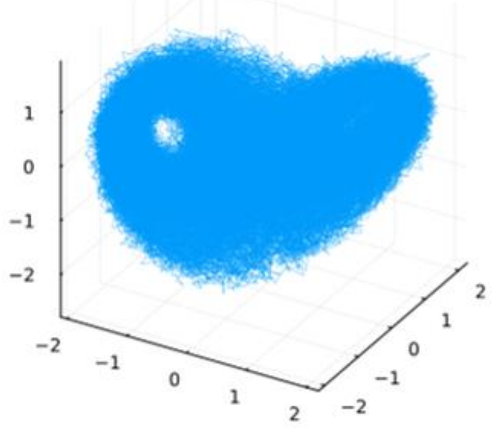
Validation on Lorenz63 system

BOLD signal

Unobserved latent trajectory



Observed trajectory



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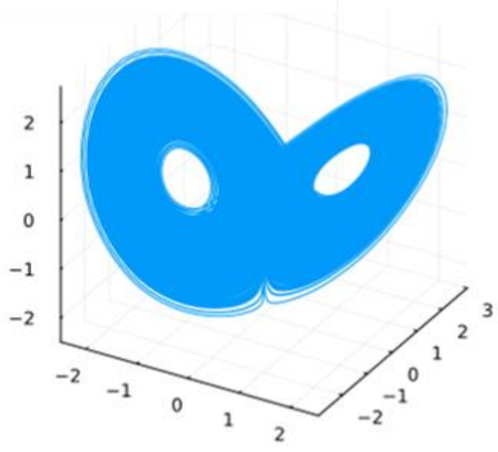

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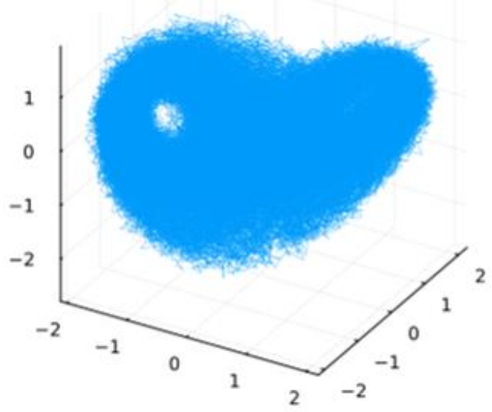
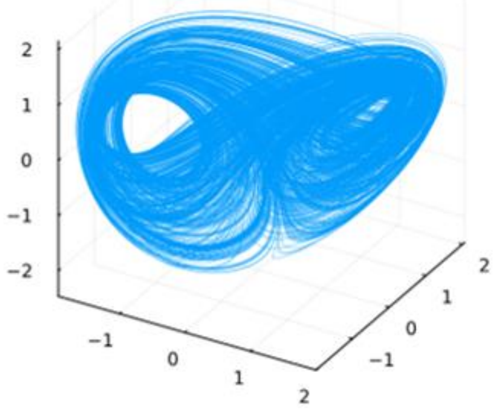
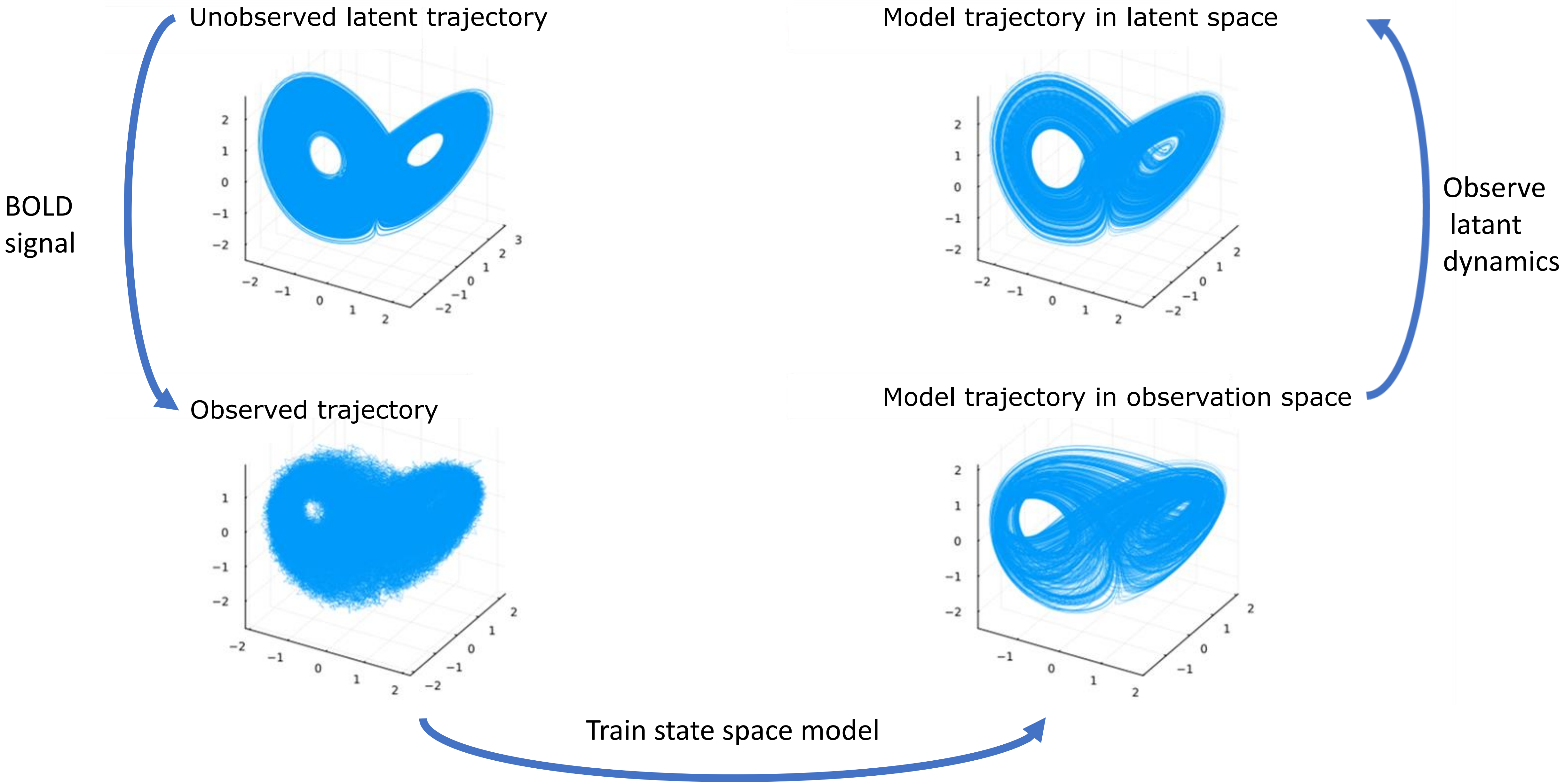


Model trajectory in observation space



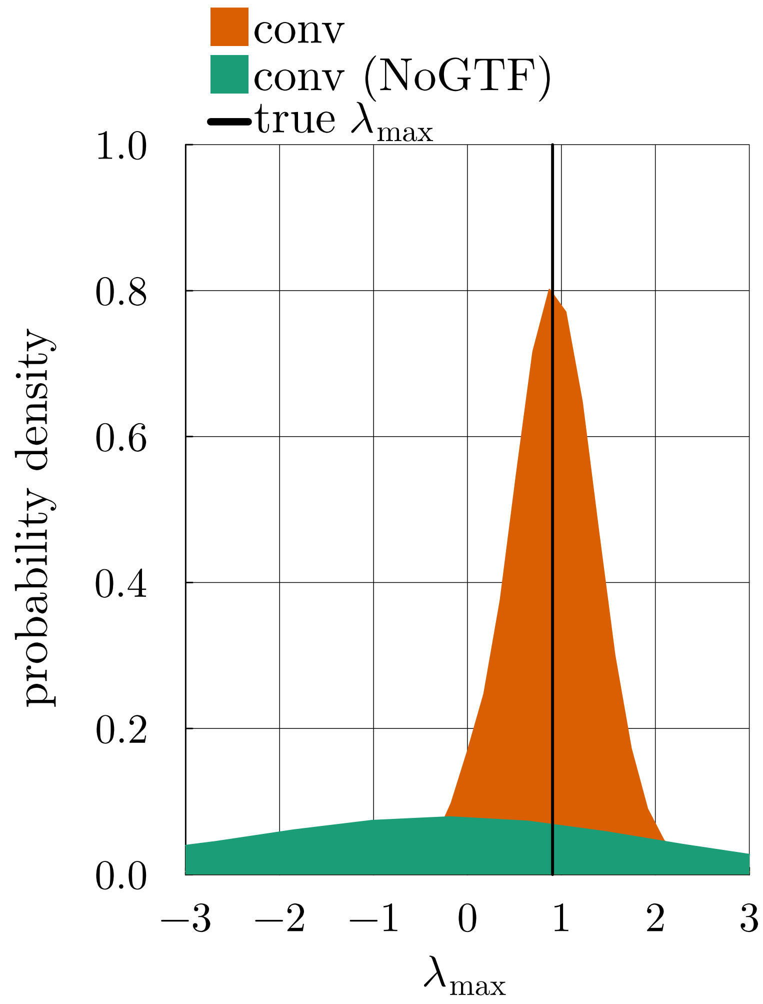
Train state space model

Validation on Lorenz63 system



Validation on Lorenz63 system

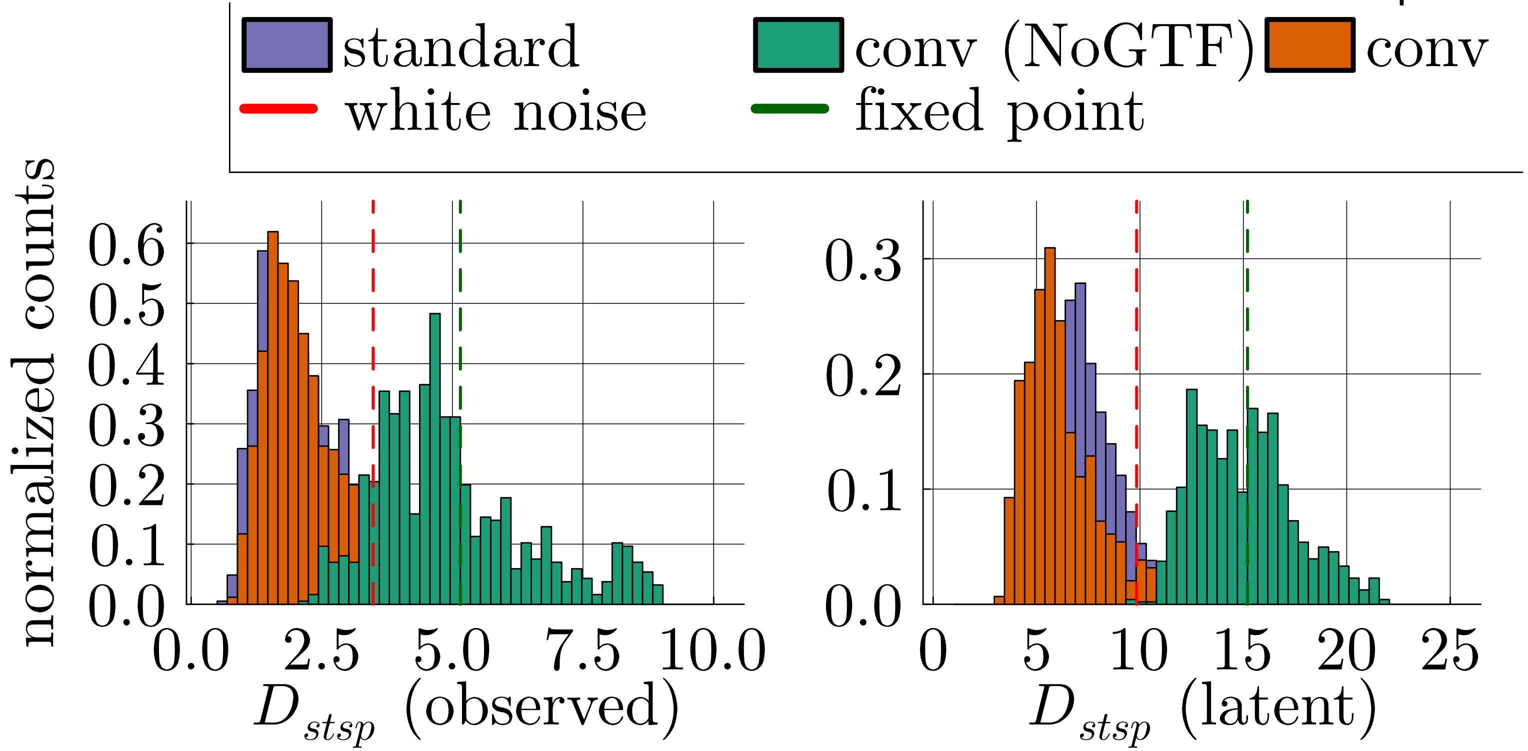
Distribution of learned λ_{max} values with/without GTF



ALN: Simulated fMRI data

Key finding: ConvSSM improves latent space reconstruction

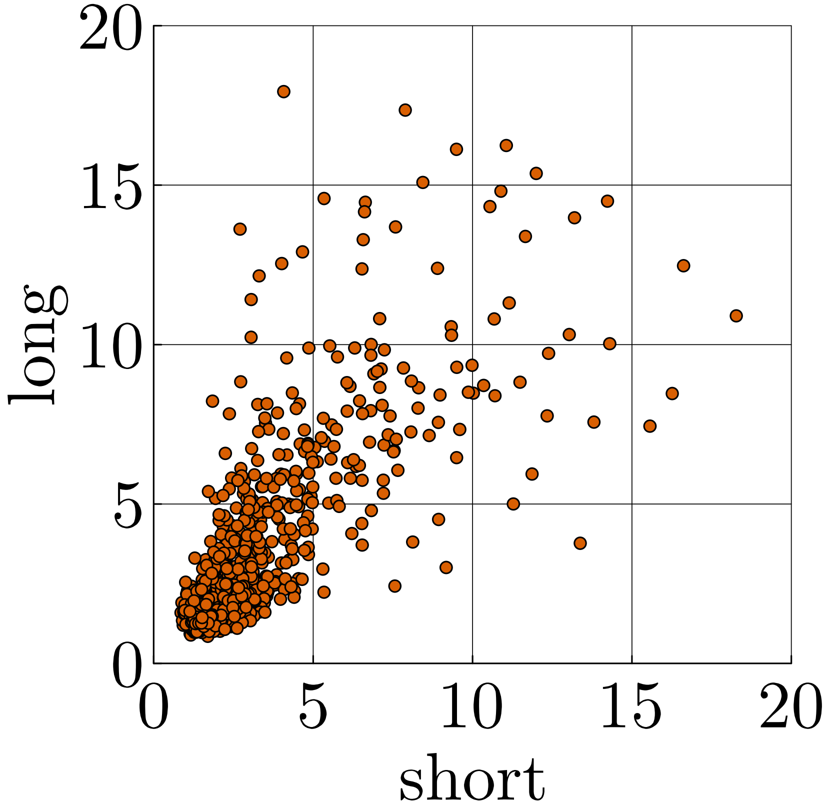
Distribution of reconstruction measure in obervation and in latent space



ALN: Simulated fMRI data

Key finding: We can identify a successful DSR model using our measures on short time series

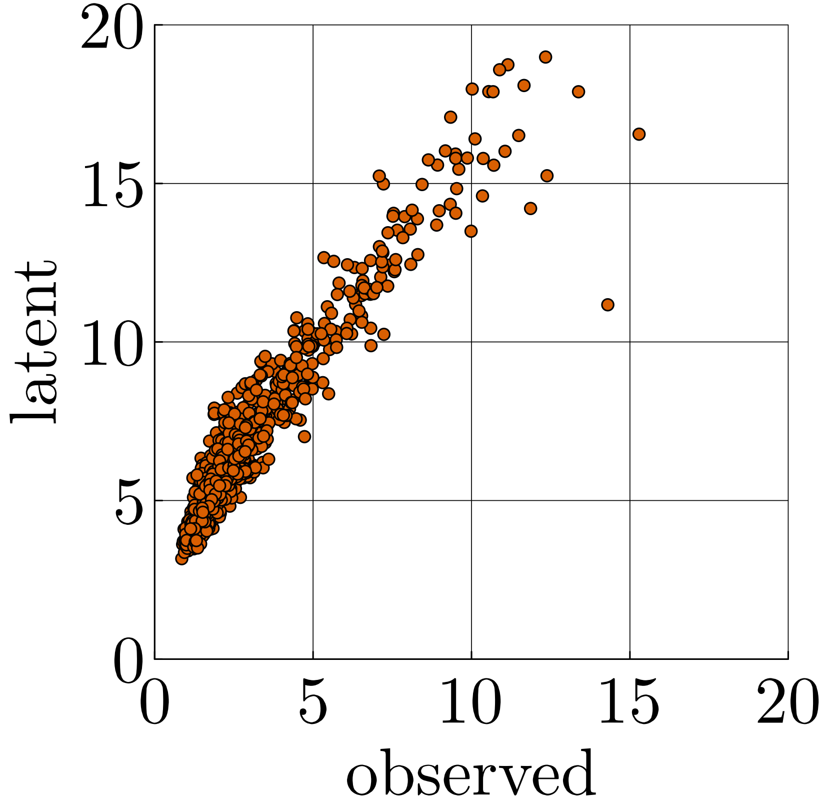
Correlation of DSR measures evaluted on short vs long testing time series



ALN: Simulated fMRI data

Key finding: Consistent DSR measures in observation and latent space for ConvSSM

Correlation of DSR measures evaluted in oberserved vs in latent space for ConvSSM



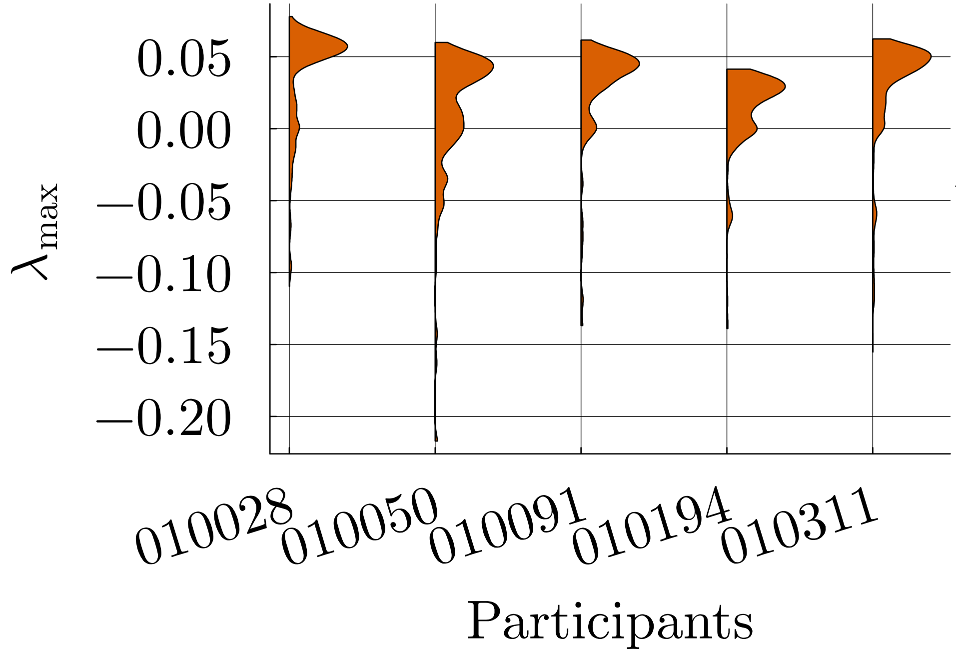
Application to experimental fMRI data

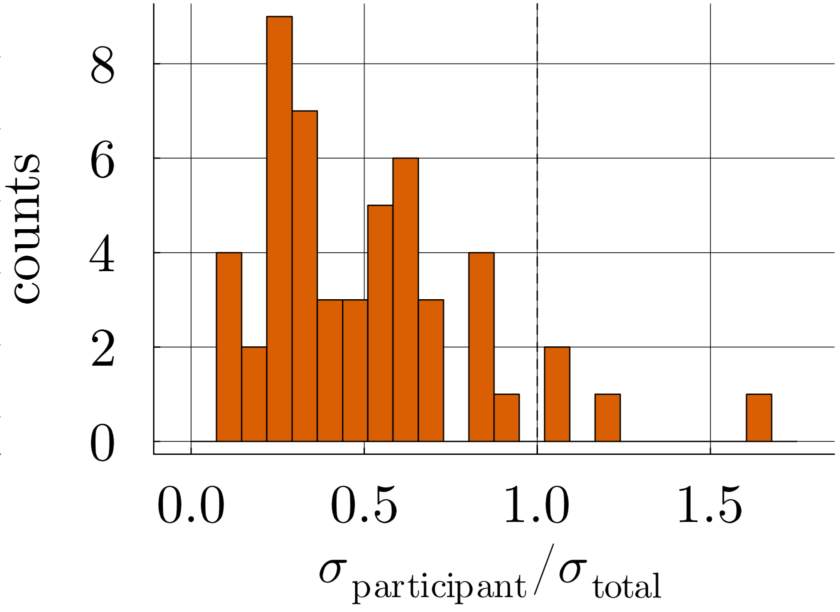
Key findings: ConvSSM outperforms other methods in DSR performance

Application to experimental fMRI data

Key findings:

- positive λ_{max}, indicating chaotic attractors
- Reliably Inferred λ_{max} differentiate between subjects





Outlook: Classification and Regression using DS features identified by ConvSSM

Thank you for your attention

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