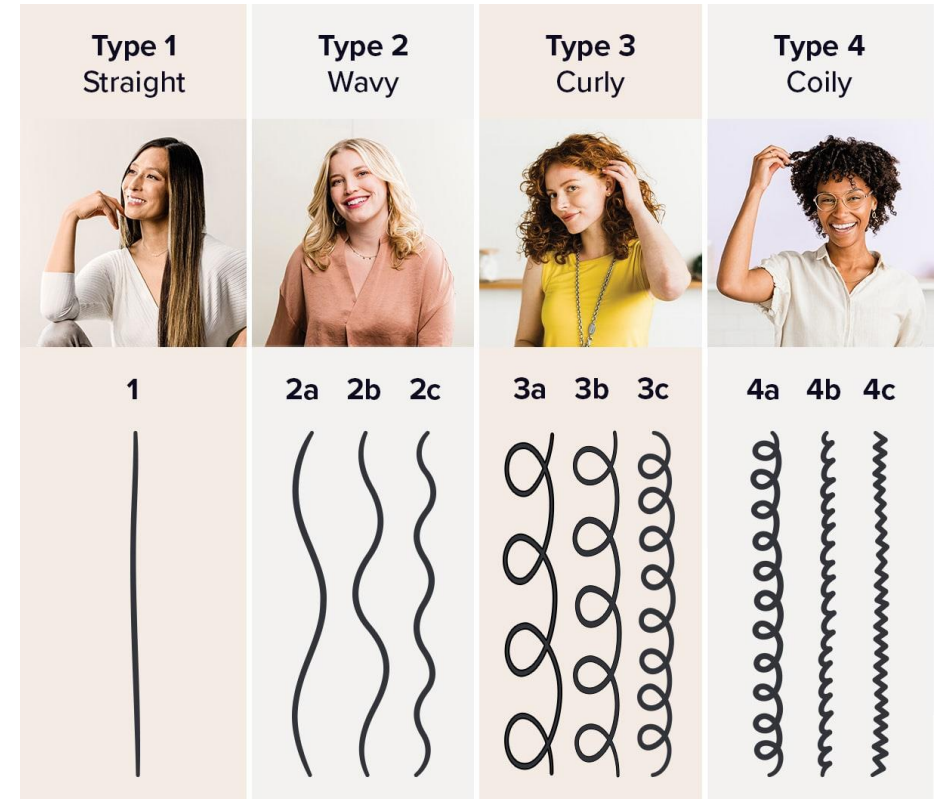


# Doubly Hierarchical Geometric Representations for Strand-based Human Hair Generation

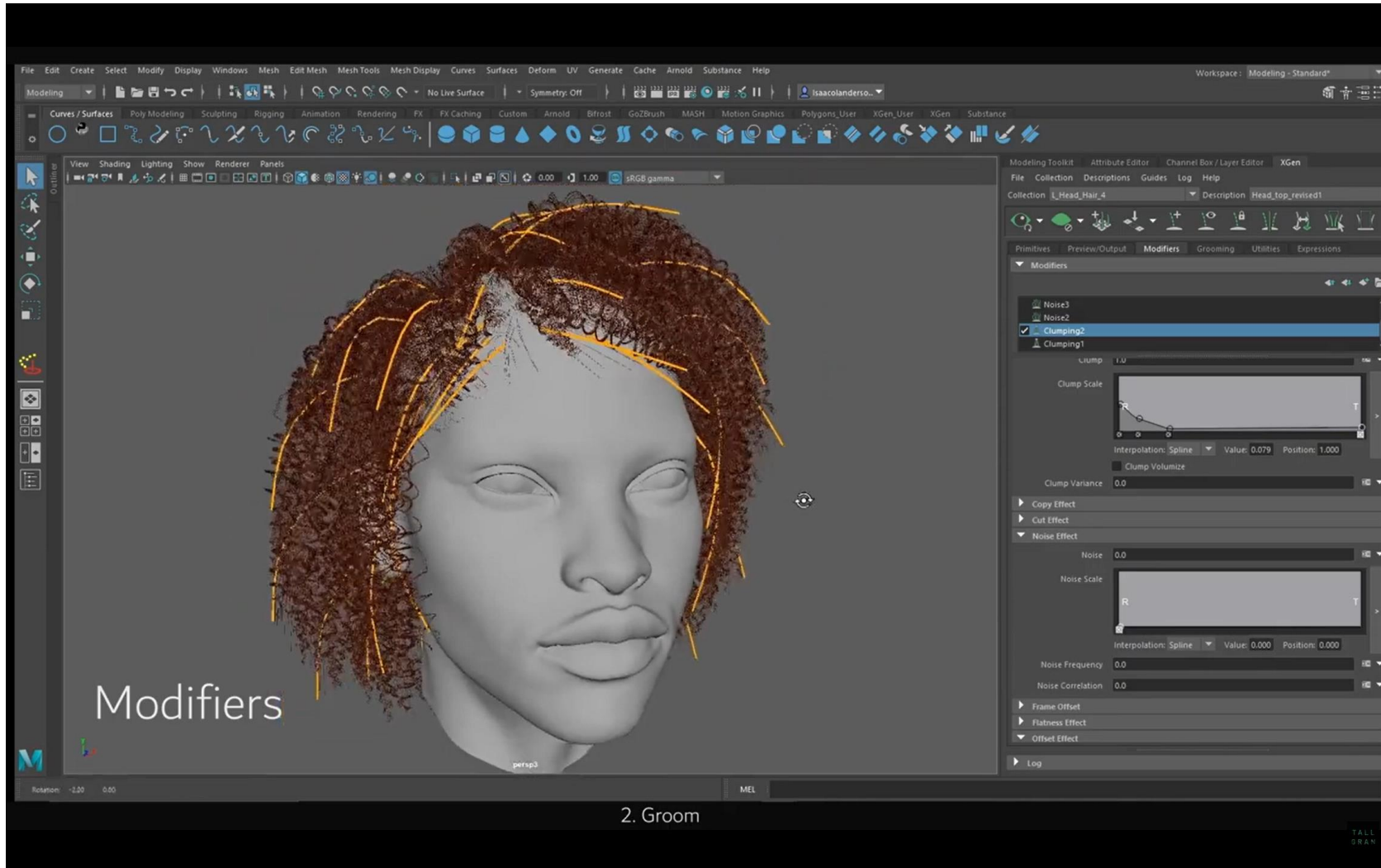
Yunlu Chen, Francisco Vicente Carrasco, Christian Häne,  
Giljoo Nam, Jean-Charles Bazin, Fernando De la Torre

# Strand-based hair geometry is complex and high-dimensional

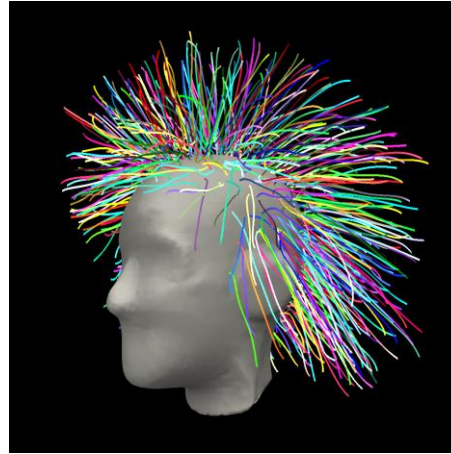
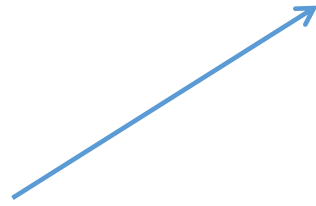
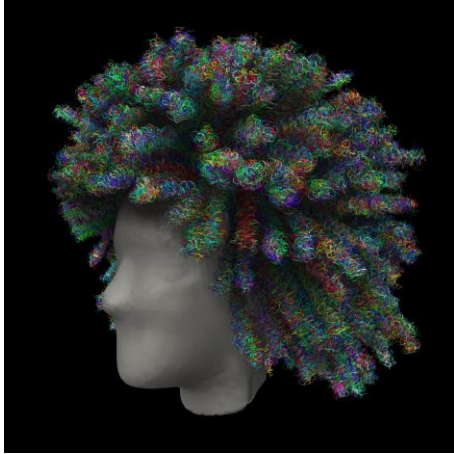
- Each person has ~100K strands
  - Flexible density and amount
- Different hair types / curliness
  - Patterns in frequency space
- A set of 1D curves in 3D space, with roots on the 2D head scalp manifold
- Due to high dimensionality, a **hierarchical, coarse-to-fine** representation is needed for generative learning



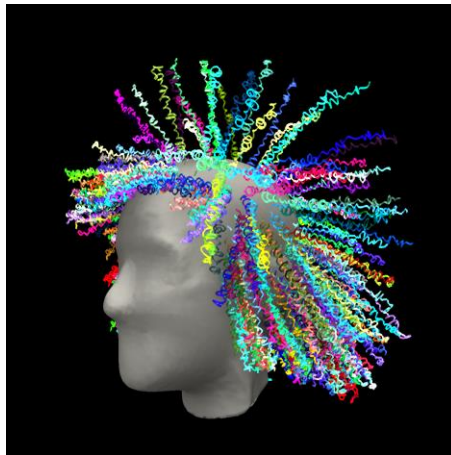
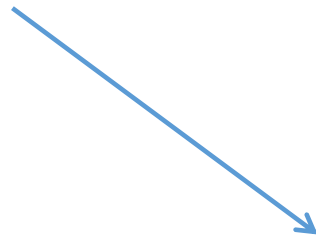
- Guide hair modelling in modern graphics tools (e.g. Maya XGen, Houdini, ...)
  - The artist sculpts **sparse** and **smooth** curves as guide hair, to guide the coarse growing direction;
  - Dense hair with high-frequency details of curliness and noise generated from the algorithm.



# Extract pseudo guide hair from data



Ideal guide hair

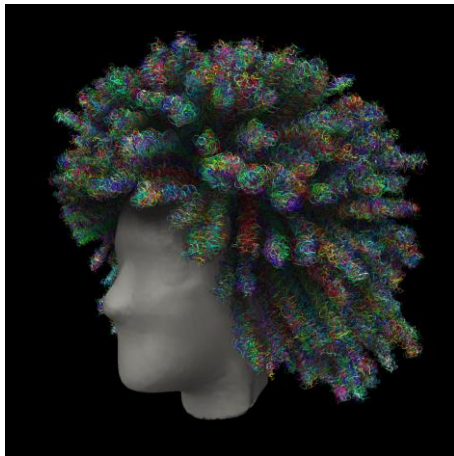


Guide hair extracted  
by existing methods

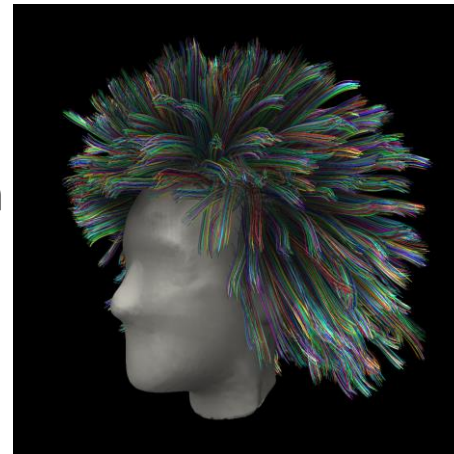
**with high-freq local curliness and noise details  
undesired for coarse-scale features**

# Doubly hierarchical guide hair modelling

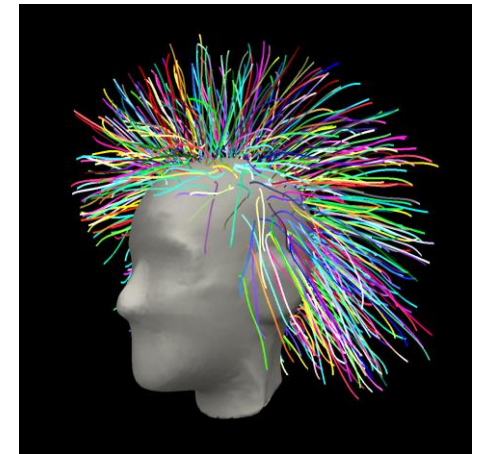
- Per-strand frequency decomposition
- Optimal sampling of important strands



DCT  
freq decomposition

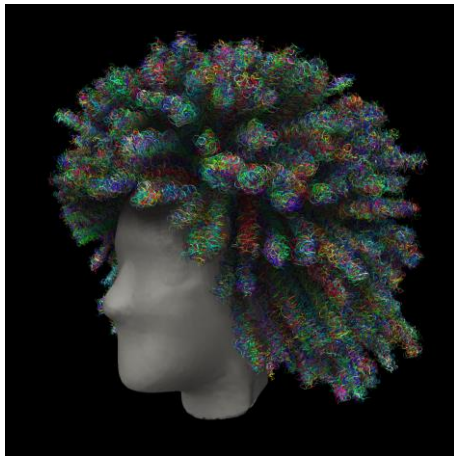


K-medoids  
Guide hair sampling

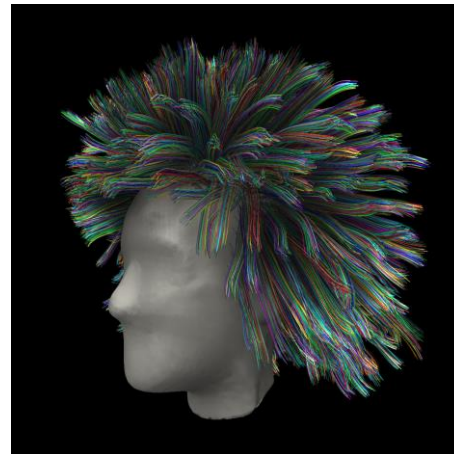


# Doubly hierarchical guide hair modelling

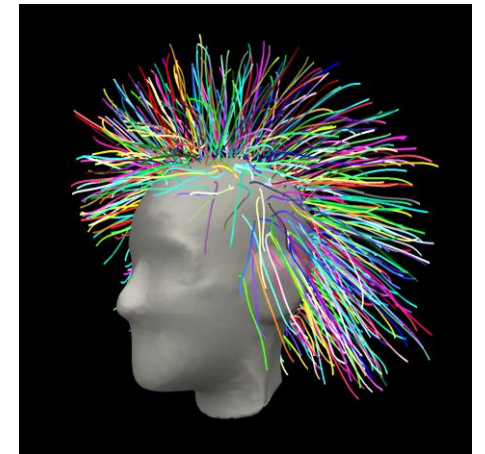
- Per-strand frequency decomposition
- Optimal sampling of important strands



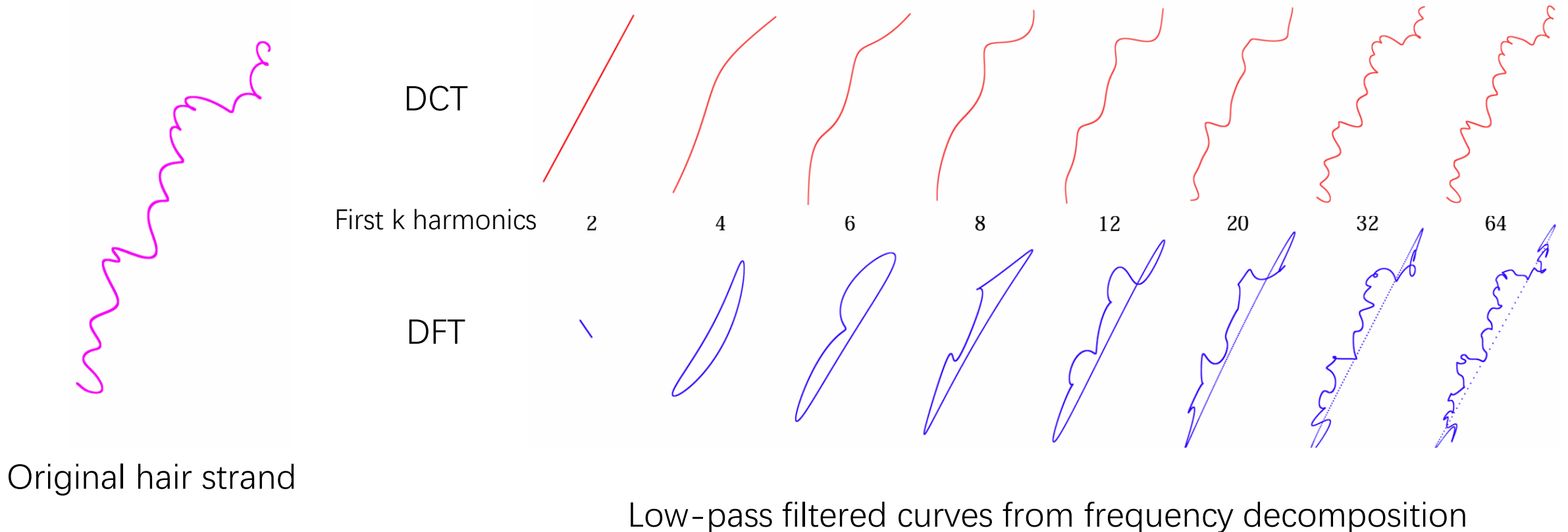
DCT  
freq decomposition



K-medoids  
Guide hair sampling

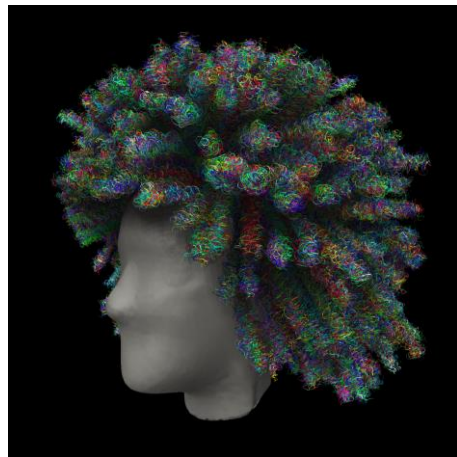


- Per-strand frequency decomposition for low-pass filtered curves
  - **DFT (Discrete Fourier transform)** bias to closed curves (Gibbs' oscillation)
  - **DCT (Discrete Cosine Transform)** better solution for open curves.

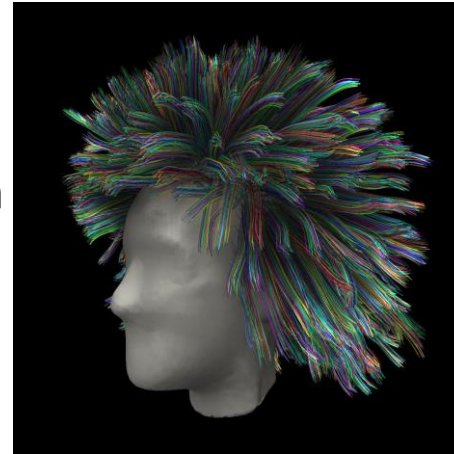


# Doubly hierarchical guide hair modelling

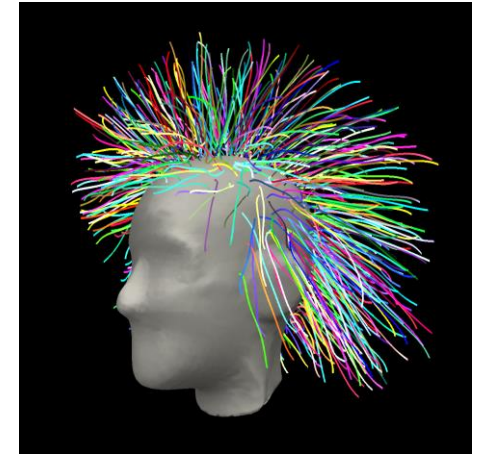
- Per-strand frequency decomposition
- Optimal sampling of important strands



DCT  
freq decomposition

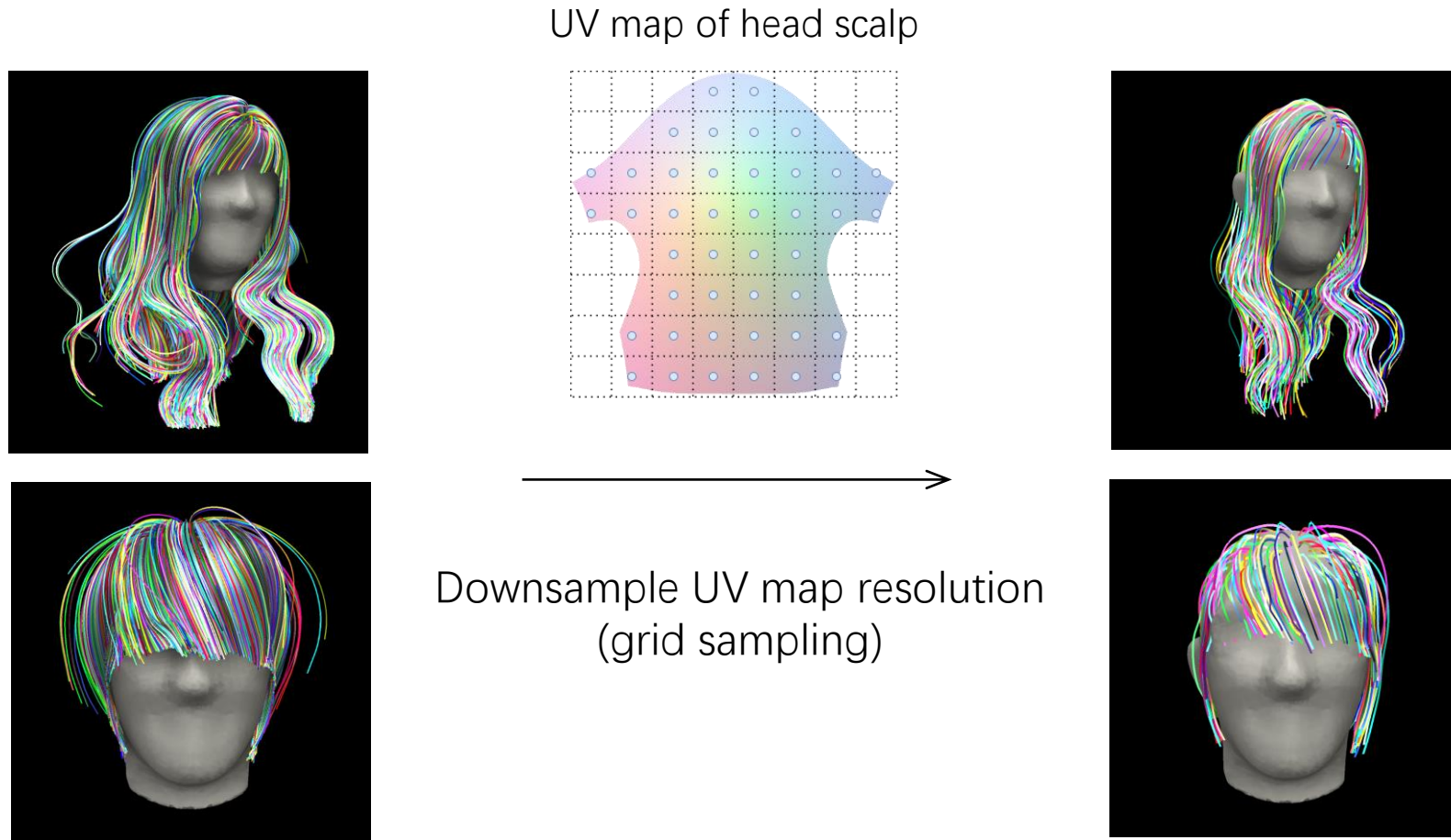


K-medoids  
Guide hair sampling

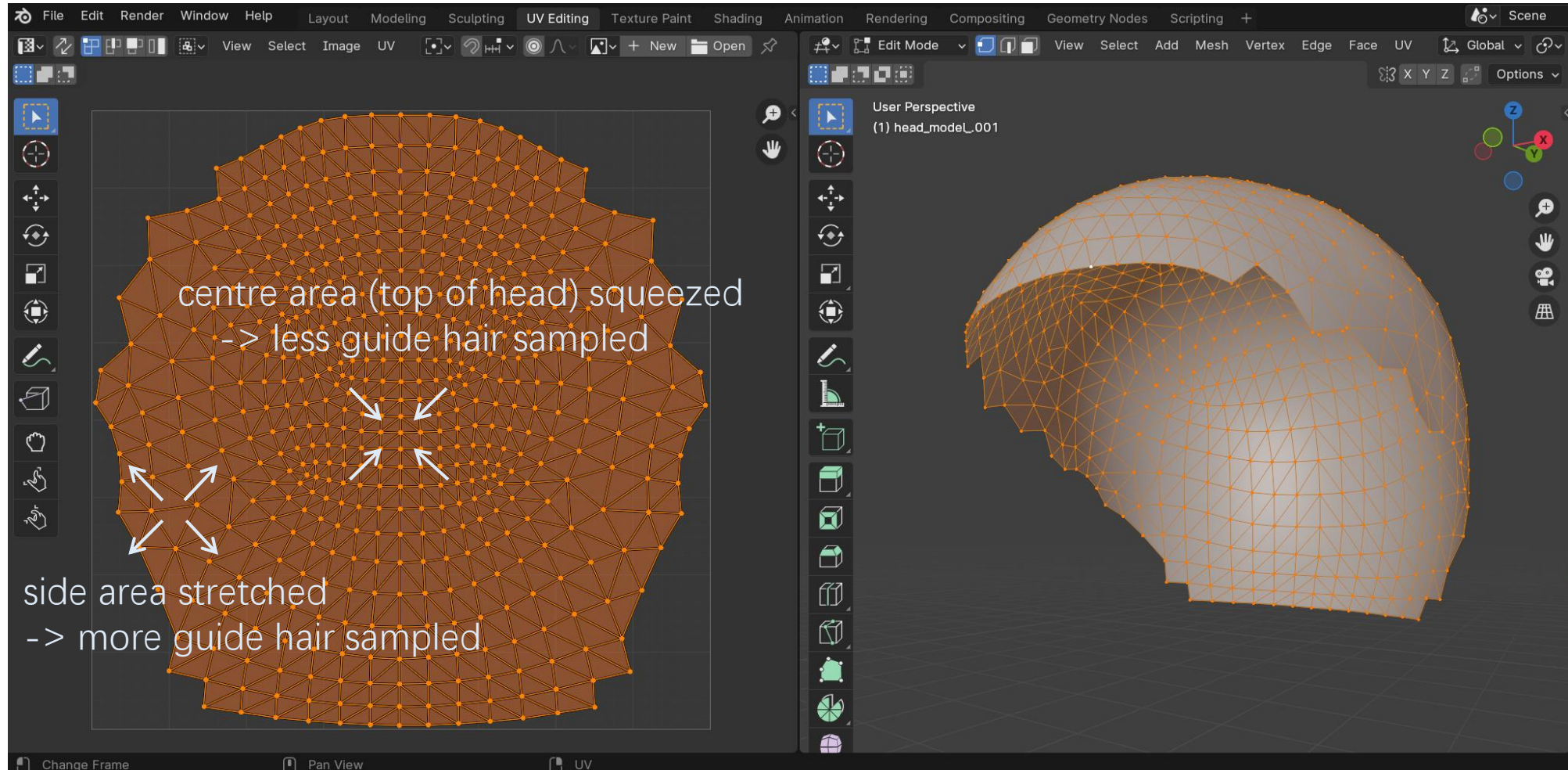




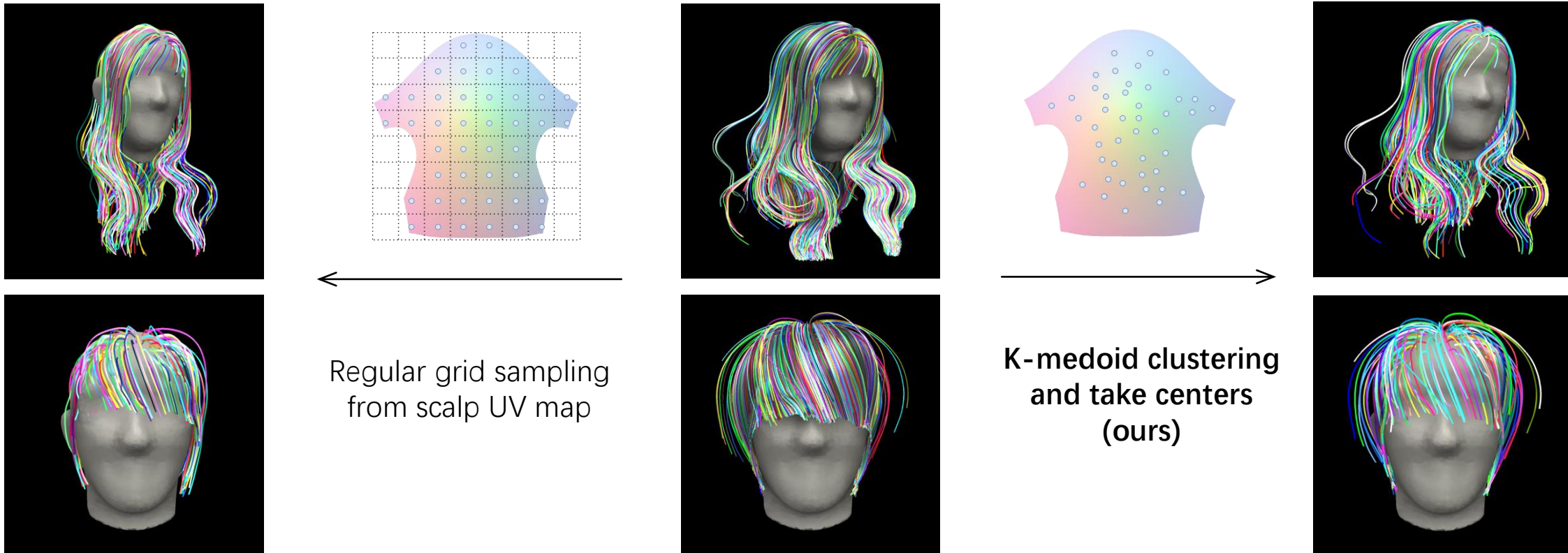
- Existing methods simply down-sample the scalp UV map for guide hair
- Problems with the regular grid sampling on the UV map (1)
  - Miss capture important details



- Problems with the regular grid sampling on the UV map (2)
  - UV mapping not area-preserving -> Less guide hair sampled from important area



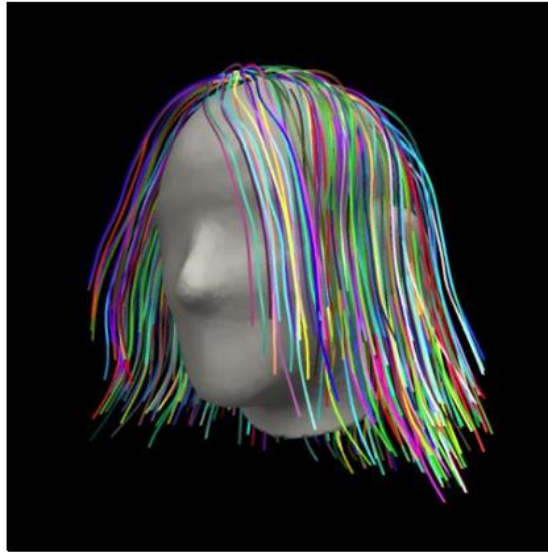
- Optimal sampling of guide strands subset (ours)
  - Extract k-medoids clustering centres as guide strands
  - Optimal: the guide curve set is the closest to the original strand set (in terms of chamfer measurement).



We show that our way of sampling guide hair is theoretically optimal

**Theorem 1.** The medoid set  $\mathcal{U} = \{u_1, \dots, u_k\}$  from the  $k$ -medoids clustering of  $\mathcal{H}$  is the optimal sampled hair curve set  $\mathcal{G}_k^*$ , if aggregated squared Euclidean distance  $d(l, l') = \frac{1}{n} \|l - l'\|_2^2 := \frac{1}{n} \sum_{t=0}^{n-1} \|l(t) - l'(t)\|_2^2$  for two individual curves is used as the divergence function for  $k$ -medoids.

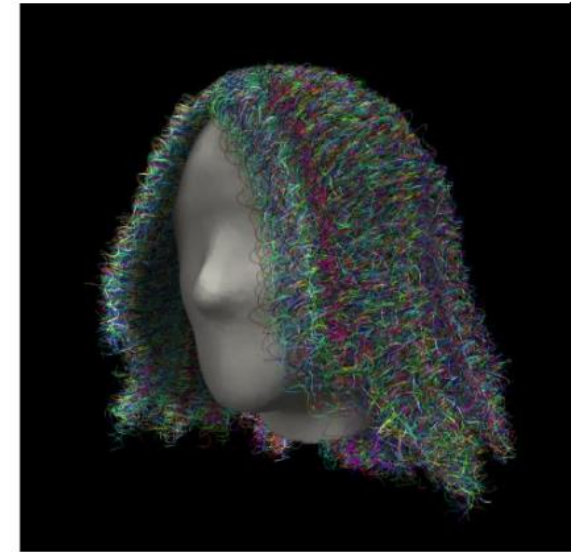
# Generation pipeline and network architecture



Guide hair



Dense hair

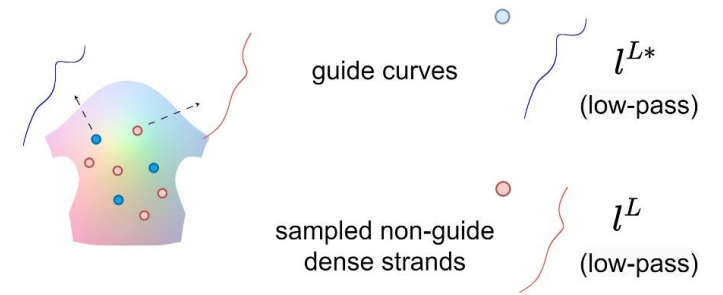


high-freq curliness and noise

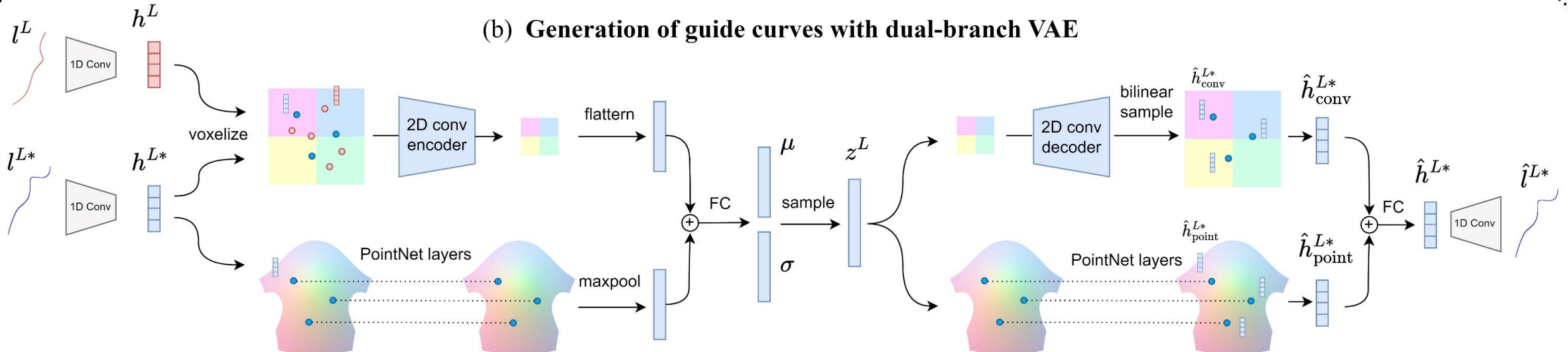
# Network architecture (1): Guide hair generation

- Dual branch VAE,
  - Conv + PointNet branches, inspired by PVCNN
  - to handle flexible root positions.

(a) Extract guide curves  
and sample non-guide dense strands



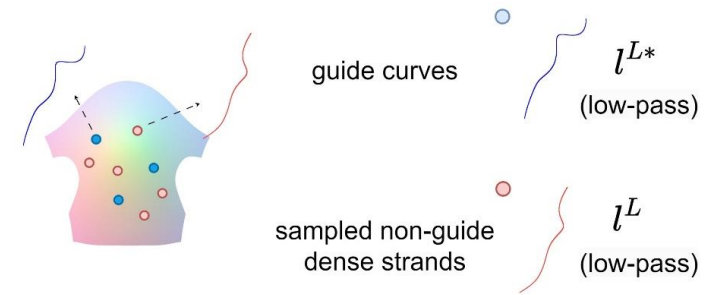
(b) Generation of guide curves with dual-branch VAE



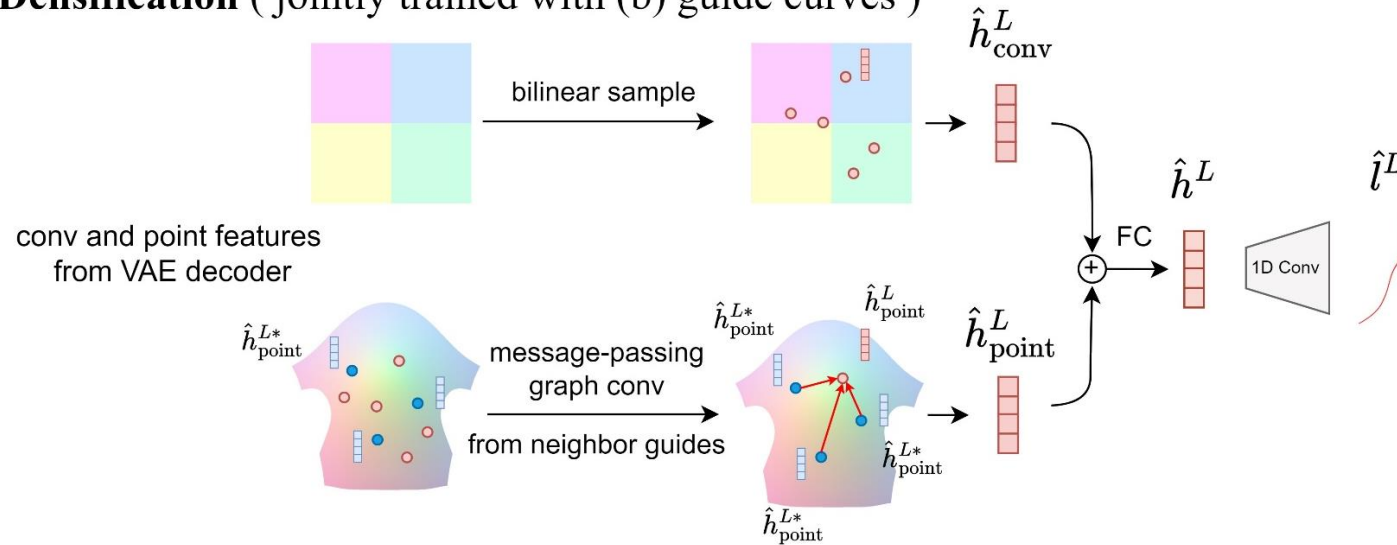
# Network architecture (2): Upsample to dense hair

- Hybrid neural fields + graph message passing
- Can handle any amount and density
- Can be jointly trained with guide hair generation

## (a) Extract guide curves and sample non-guide dense strands



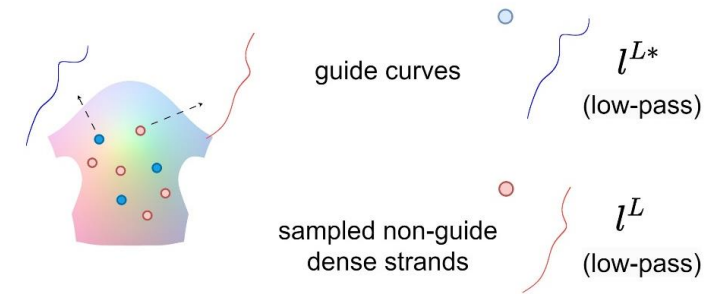
## (c) **Densification** ( jointly trained with (b) guide curves )



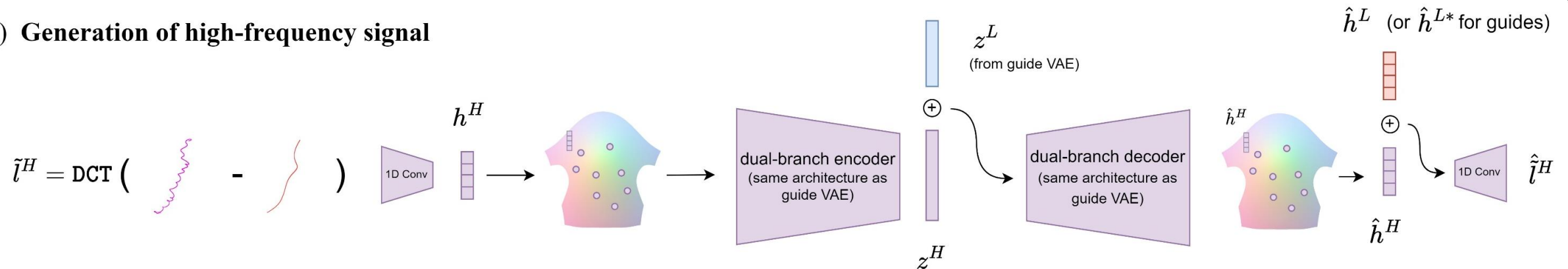
# Network architecture (3): Add high-frequency details

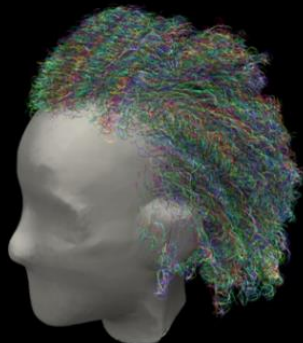
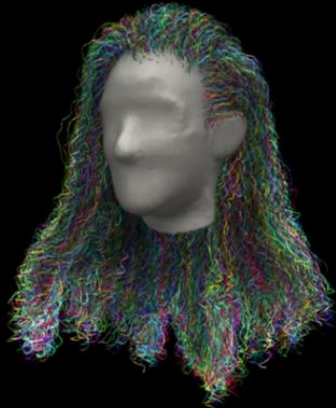
- Another dual-branch network.
- Conditioned on low-freq. latent features

## (a) Extract guide curves and sample non-guide dense strands



## (d) Generation of high-frequency signal







# Recap: doubly hierarchical representations for generating strand-based hair geometry

- A novel hierarchical representation with mathematical insights
  - DCT to filter out high-frequency local noise and curliness
  - Optimal sampling of important strands from k-medoids clustering
- Generative network design for our representation
  - PVCNN-inspired architecture for guide hair generation with flexible roots
  - Upsample to dense strands: hybrid neural fields + graph message-passing
    - Handles any amount and any density
  - High-frequency details conditioned on latent features from low-freq model