Compressing Large Language Models using Low Rank and Low Precision Decomposition

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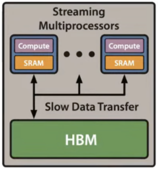
Compressing Large Language Models

[Credits: GPT-4 + DALL.E 3]



[Credits: FlashAttention, Dao et. al.]

- LLMs are memory hungry and often cannot be loaded on consumer GPUs: Eg: LLaMa 70B in BF16 takes up 140 GiB. Consumer GPUs (eg. NVIDIA A10G) have only 24 GiB of HBM.
- High inference latency (fewer tokens per second): Inference with low batch sizes is typically memory bound, i.e., back-and-forth communication between GPU HBM and SRAM is the bottleneck.
- Out-of-memory (OOM) issues while finetuning: Fine-tuning LLMs requires storing weights, activations, and optimizer states.
- Communication bandwidth becomes a bottleneck in distributed inference using multi-GPU (eg. NVLink) or multi-node (eg. InfiniBand).
- Increased model sharing latency (HuggingFace upload/download)
- **Goal of our work: Compress an LLM while preserving its accuracy.**

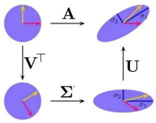
Low Rankness of LLM weights

- LLM weights (Query, Key, ...) are represented as matrices. Matrices are linear transforms on input activations. While compressing a weight matrix, we should preserve this functionality.

- **Singular value decomposition**: Any matrix $\mathbf{A} \in \mathbb{R}^{n \times d}$ can written as:

$$\mathbf{A} = \sum_{i=1}^{\mathrm{rank}(\mathbf{A})} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top,$$

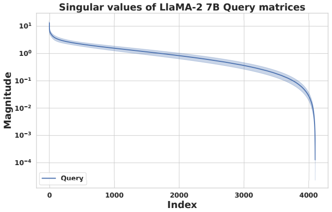
where $\{\sigma_i\}$ are the singular values, and $\mathbf{u}_i \in \mathbb{R}^n$, $\mathbf{v}_i \in \mathbb{R}^d$ are singular vectors.



$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$

- Higher singular value components majorly capture how input activations are transformed into output activations for each layer in a forward pass.

We leverage the top singular components to compress weight matrices by obtaining an approximate low-rank structure!



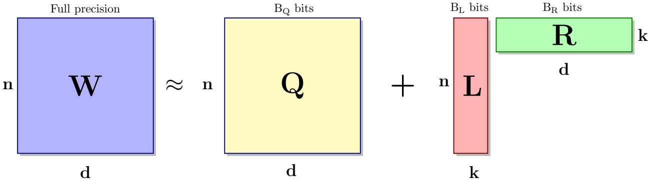
Low-precision representations

[Credits: GPT-4 + DALL.E 3]

- Low-precision formats also reduce memory footprint by using fewer bits to represent real numbers.
  Eg. INT4, FP4, MXFP4, ...
- Low-precision compute is faster.
  Eg. NVIDIA H100 specs: 1979 teraFLOPS with BFLOAT16 vs. 3958 teraFLOPS with FP8.
- Low-precision operations also require fewer Watts, i.e., more energy efficient.

Low-precision and Low-Rank Decomposition

Problem: **How to jointly obtain a low rank as well as low precision approximation of a matrix?**



Calibration Aware Low-Precision and Low-Rank Decomposition

$$\min_{\mathbf{Q},\mathbf{L},\mathbf{R}} \|(\mathbf{Q}+\mathbf{L}\mathbf{R}-\mathbf{W})\mathbf{X}^{\top}\|_{\mathrm{F}}^2$$ subject to $\mathbf{Q}, \mathbf{L}, \mathbf{R}$ using $\mathrm{B_Q}$, $\mathrm{B_L}$, and $\mathrm{B_R}$ bits respectively.

- Calibration data $\mathbf{X}$: Sampled from RedPajama dataset.
- Low-rank factors $\mathbf{L}$ and $\mathbf{R}$ capture the large singular components of $\mathbf{W}$ with fewer parameters but high fidelity ($\mathrm{B_L} = \mathrm{B_R} = 4$ bits).
- Full-rank backbone $\mathbf{Q}$ is quantized aggressively ($\mathrm{B_Q} = 2$ bits), coarsely capturing the essence of the moderately decaying and low singular components of $\mathbf{W}$.
- Choose quantizers such that $\mathrm{B_Q} = 2$ bits, $\mathrm{B_L} = \mathrm{B_R} = 4$ bits. For an LLM weight matrix with $n = d = 4096$, choosing rank $k = 64$ implies 2.125 bits per entry.

Our Algorithm: CALDERA

Calibration Aware Low-Precision and Low-Rank Decomposition

$$\min_{\mathbf{Q},\mathbf{L},\mathbf{R}} \|(\mathbf{Q}+\mathbf{LR}-\mathbf{W})\mathbf{X}^\top\|_{\mathrm{F}}^2 \quad \text{subject to } \mathbf{Q},\mathbf{L},\mathbf{R} \text{ using } \mathrm{B_Q}, \mathrm{B_L}, \text{and } \mathrm{B_R} \text{ bits respectively.}$$

- CALDERA: **C**alibration **A**ware **L**ow-Precision **DE**composition with Low-**R**ank **A**daptation.
- Our algorithm: Alternately update $\mathbf{Q}$ and $(\mathbf{L},\mathbf{R})$.
  - Initialize $t \leftarrow 0$, $\mathbf{L}_0 \leftarrow \mathbf{0}$, $\mathbf{R}_0 \leftarrow \mathbf{0}$.
  - Step 1: $\mathbf{Q}_{t+1} \leftarrow \textsc{Quantize}(\mathbf{W}-\mathbf{L}_t\mathbf{R}_t)$ using $\mathrm{B_Q}$ bits.
    Solve $\min_{\mathbf{Q}}\|(\mathbf{Q}-\mathbf{L}_t\mathbf{R}_t-\mathbf{W})\mathbf{X}^\top\|_{\mathrm{F}}^2$ using LDLQ quantizer [Chee et al., NeurIPS '23].
  - Step 2: $\mathbf{L}_{t+1},\mathbf{R}_{t+1} \leftarrow \textsc{LPLRFactorize}(\mathbf{W}-\mathbf{Q}_{t+1},k)$, where $(\mathbf{L},\mathbf{R})$ use $(\mathrm{B_L},\mathrm{B_R})$ bits.
    Solve $\min_{\mathbf{L},\mathbf{R}}\|(\mathbf{Q}_t-\mathbf{LR}-\mathbf{W})\mathbf{X}^\top\|_{\mathrm{F}}^2$ (submodule described in next slide).
  - Iterate between Step 1 and Step 2 for a maximum number of iterations.

Low-Precision Low-Rank (LPLR) Factorize submodule

- Rank-constrained regression (RCR): $\min_{\text{rank}(\mathbf{Z}) \le k} \|\mathbf{XZ} - \mathbf{Y}\|_F^2$ is a non-convex problem that can be solved to global optimality in closed form [Xiang et al., KDD '12].
- LPLRFactorize solves RCR subject to quantization constraints, i.e., $\min_{\mathbf{L},\mathbf{R}} \|(\mathbf{LR} - \mathbf{A})\mathbf{X}^\top\|_F^2$, where $(\mathbf{L}, \mathbf{R})$ are constrained to $(\mathrm{B_L}, \mathrm{B_R})$ bits.
- For fixed $\mathbf{A}$, run an inner loop alternately update $\mathbf{L}$ and $\mathbf{R}$.
    - Initialize $(\mathbf{L}_0, \mathbf{R}_0)$ from the RCR solution.
    - Step 1: $\mathbf{L}_i = \textsc{Quantize}\left(\arg\min_{\mathbf{Z} \in \mathbb{R}^{n \times k}} \|(\mathbf{Z}\mathbf{R}_i - \mathbf{A})\mathbf{X}^\top\|_F^2\right)$.
    - Step 2: $\mathbf{R}_i = \textsc{Quantize}\left(\arg\min_{\mathbf{Z} \in \mathbb{R}^{k \times d}} \|(\mathbf{L}_i\mathbf{Z} - \mathbf{A})\mathbf{X}^\top\|_F^2\right)$.
    - Iterate between Step 1 and Step 2 for a maximum number of inner iterations.
- *Note*: The solutions of minimization problems in steps 1 and 2 above are available in closed form.

Compressing LLaMa family of LLMs

- Results on different sizes of LLaMa models (without finetuning):[†]

Method	Rank	Avg Bits	Wiki2 ↓	C4 ↓	Wino ↑	RTE ↑	PiQA ↑	ArcE ↑	ArcC ↑
CALDERA (7B)	256	2.4	6.19	8.14	66.0	60.6	75.6	63.6	34.0
QuIP# (7B, No FT)	0	2	8.23	10.8	61.7	57.8	69.6	61.2	29.9
CALDERA (13B)	256	2.32	5.41	7.21	66.9	62.1	76.2	70.3	40.4
QuIP# (13B, No FT)	0	2	6.06	8.07	63.6	54.5	74.2	68.7	36.2
CALDERA (70B)	256	2.2	3.98	5.76	77.6	71.5	79.8	79.5	47.4
QuIP# (70B, No FT)	0	2	4.16	6.01	74.2	70.0	78.8	77.9	48.6

- Can finetune randomized Hadamard transform (RHT) parameters for improved results:

Method	Rank	Avg Bits	Wiki2 ↓	C4 ↓	Wino ↑	RTE ↑	PiQA ↑	ArcE ↑	ArcC ↑
CALDERA (7B)	256	2.4	5.84	7.75	65.7	60.6	76.5	64.6	35.9
QuIP#*	0	2	6.58	8.62	64.4	53.4	75.0	64.8	34.0

- Low-rank factors can be (optionally) fine-tuned via LoRA to boost performance on specific tasks.

Method	Rank	RHT FT	Avg Bits**	Wiki2 ↓	RTE ↑	Wino ↑
CALDERA (7B)	128	Yes	2.5	5.77	84.12	85.00
CALDERA (7B)	256	Yes	2.7	5.55	86.28	84.93

[†] E8 lattice quantization with indices packed as INT64 data type.
[*] Only end-to-end RHT finetuning, and not layer-by-layer finetuning. [**] The top 64 components of $\mathbf{L}$ and $\mathbf{R}$ are in BF16.

Summary

- We propose CALDERA for compressing an LLM in the regime of 2 to 2.5 bits per parameter, with the goal of reducing the accuracy gap to uncompressed models.
- CALDERA provides a unified framework that jointly optimizes the backbone $\mathbf{Q}$ and the low-rank factors $\mathbf{LR}$ – providing the flexibility to represent them in different precisions.
- We provide rigorous theoretical guarantees on the approximation error of CALDERA, provably showing that it is better compared to rank-agnostic compression algorithms.
- Our CALDERA decomposition can be used with other strategies like randomized Hadamard transform fine-tuning [QuIP#], Low-Rank adaptation, etc.
- Auto-regressive generation throughput for the 2 to 2.5 bit-quantized model is higher than unquantized.

Thank you!

Reach out for questions or discussions

Poster Session:
The 12 Dec 11 a.m. PST — 2 p.m. PST
https://nips.cc/virtual/2024/poster/93805

Paper: https://arxiv.org/abs/2405.18886
GitHub: https://github.com/pilancilab/caldera