

# Multi-scale Decomposition of Sea Surface Height Snapshots using Machine Learning

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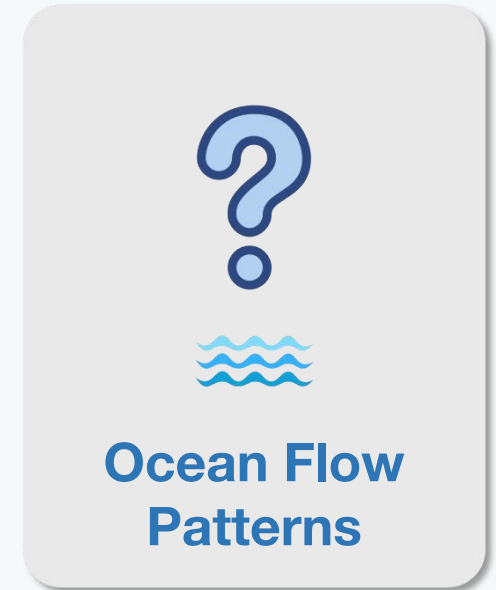
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\*Equal contribution

# Ocean's Role in Climate Change

## Climate Impact:

- 93% of Earth's excess heat storage
- 30% of CO<sub>2</sub> absorption
- Major climate regulator



## Our Task:

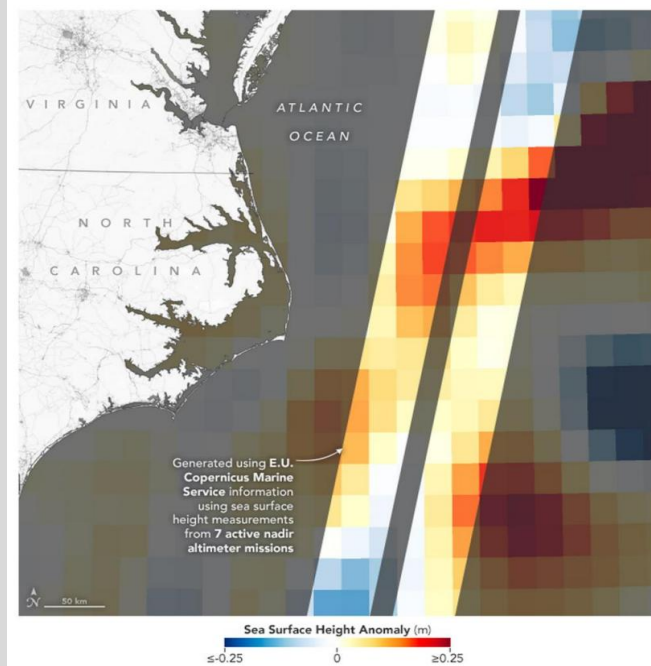
Accurately **decompose SSH snapshots** to understand ocean circulation patterns, enabling:

- **Better** climate predictions
- **Improved** carbon uptake estimates
- **Enhanced** climate model validation

# Satellite Revolution

From Traditional Satellite to SWOT

## Traditional Altimeter



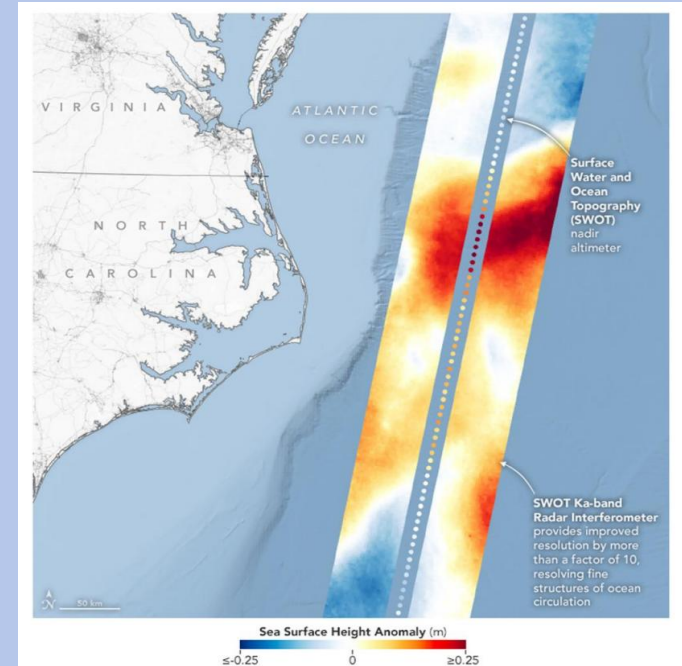
## Difference

 **Spatial Resolution**  
~100km → **5-10km**

 **Coverage**  
1D Tracks → **2D Swath**

 **Temporal Resolution**  
Daily → **21 Days**

## SWOT Satellite

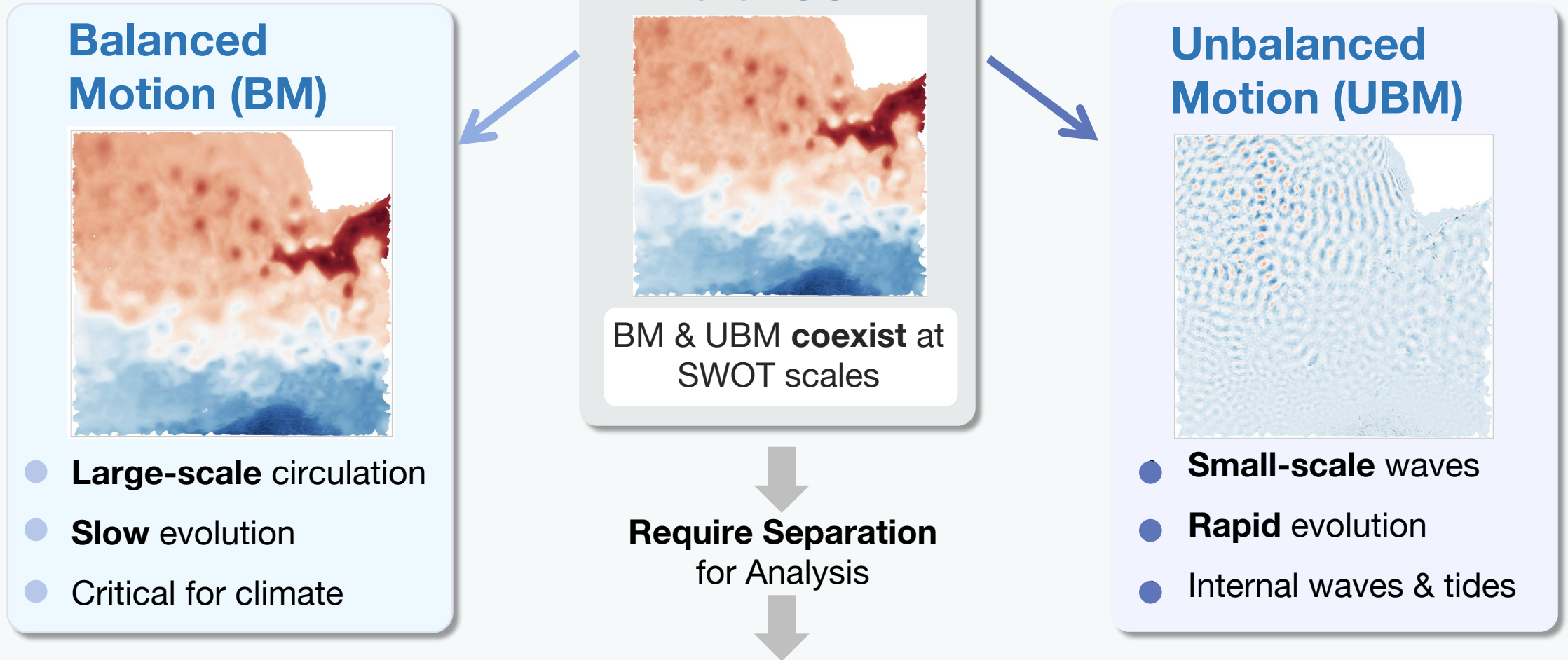


## New Challenge:

Longer repeat time (21 days) requires new methods for analyzing single snapshots

# Decomposing SSH

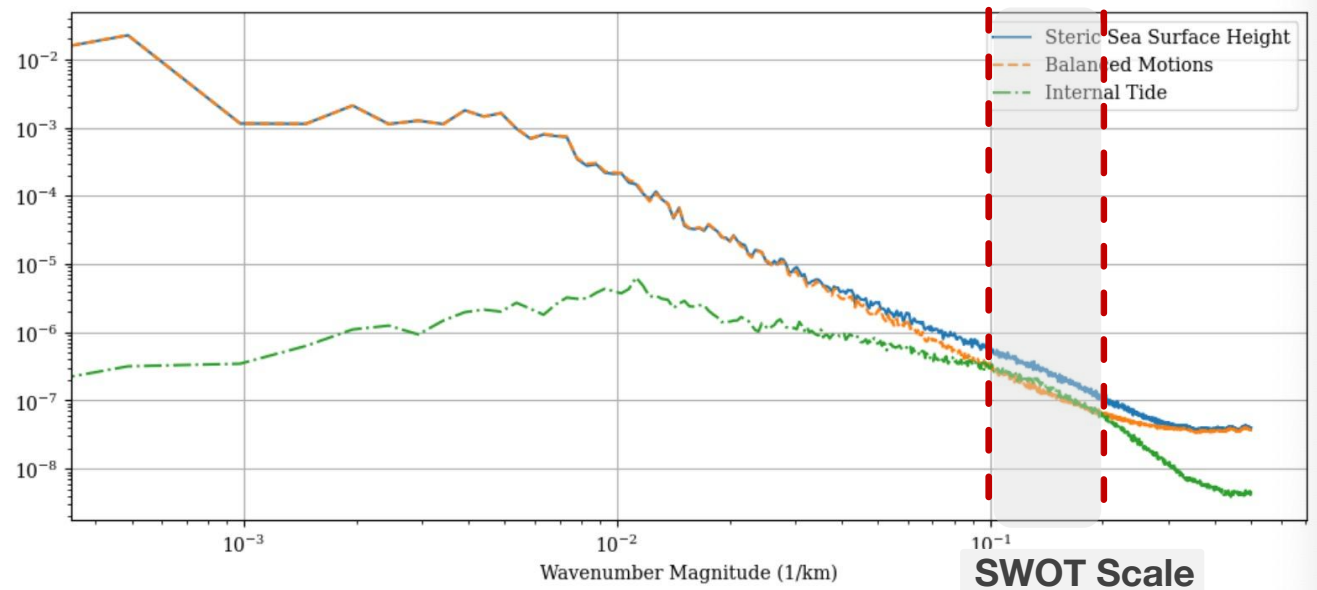
$$\text{SSH} = \text{BM} + \text{UBM}$$



**Challenge:** How to separate BM & UBM from a **single** SSH snapshot?

# Challenge: A Multi-Scale Problem

## Power Spectral Density Analysis



## Key Challenges

### Signal overlap

BM/UBM coexist at SWOT scales

### Power law decay

PSD decays exponentially with wavenumber

# Current Solution: Gradient-Enhanced U-Net

A Step Forward, But Not Enough

## State-of-the-Art Approach

### U-Net + Gradient Loss

$$\mathcal{L}(\eta_{\text{UB}}, \tilde{\eta}_{\text{UB}}) = \|\eta_{\text{UB}} - \tilde{\eta}_{\text{UB}}\|^2 + \alpha \|\nabla \eta_{\text{UB}} - \nabla \tilde{\eta}_{\text{UB}}\|^2$$

$\alpha$ : gradient loss weight (requires tuning)

## Key Limitations

- Heavy computational overhead
- Sensitive to  $\alpha$  hyperparameter tuning
- High-frequency noise in prediction

## Required: A More Fundamental Solution

- 1 Self-adaptive framework that eliminates gradient regularization tuning
- 2 Enhanced spectral fidelity at high wavenumbers without noise artifacts

# ZCA Whitening: A Fundamental Solution

## ZCA Core Process

- 1 Feature Decorrelation**  
Separate features across different spatial scales
- 2 Power Normalization**  
Normalize power across all frequencies
- 3 Spatial Recovery**  
Return to original coordinate space

## Key Advantages

- ➔ **Enhanced Feature Detection**  
Reveals previously ignored small-scale features
- ➔ **Improved Separation**  
Clear BM/UBM distinction at all scales
- ➔ **Simplified Training**  
Standard MSE loss becomes effective

**Innovation:** ZCA transforms multi-scale learning problem into standard learning task

# ZCA Whitening: Mathematical Formula

## 1. Data Preprocessing

### 1 Input Matrix

$$X_{\eta_{UB}} \in \mathbb{R}^{n \times d}$$

n: number of samples

d:  $108 \times 108$  (spatial dimensions)

### 2 Center Data

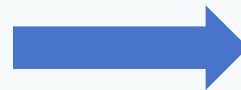
$$X_{\eta_{UB}}^c = X_{\eta_{UB}} - \mu_{\eta_{UB}}$$

$\mu$ : mean across UBM samples

### 3 Compute Covariance

$$\Sigma = \frac{1}{n-1} (X_{\eta_{UB}}^c)^T X_{\eta_{UB}}^c$$

Captures feature relationships



## 2. ZCA Transform

### 1 Eigendecomposition

$$\Sigma = U \Lambda U^T$$

U: eigenvectors

$\Lambda$ : diagonal eigenvalue matrix

### 2 Whitening Matrix

$$W_{ZCA} = U(\Lambda + \epsilon I)^{-1/2} U^T$$

$\epsilon$ : small constant for numerical stability

### 3 Apply Transform

$$X_{ZCA} = X_{\eta_{UB}}^c W_{ZCA}$$

Final whitened representation

## Key Properties

### Spectral Equalization

Flat power distribution  
across all wavenumbers

### Decorrelation

$$\text{Cov}(X_{ZCA}) \approx I$$

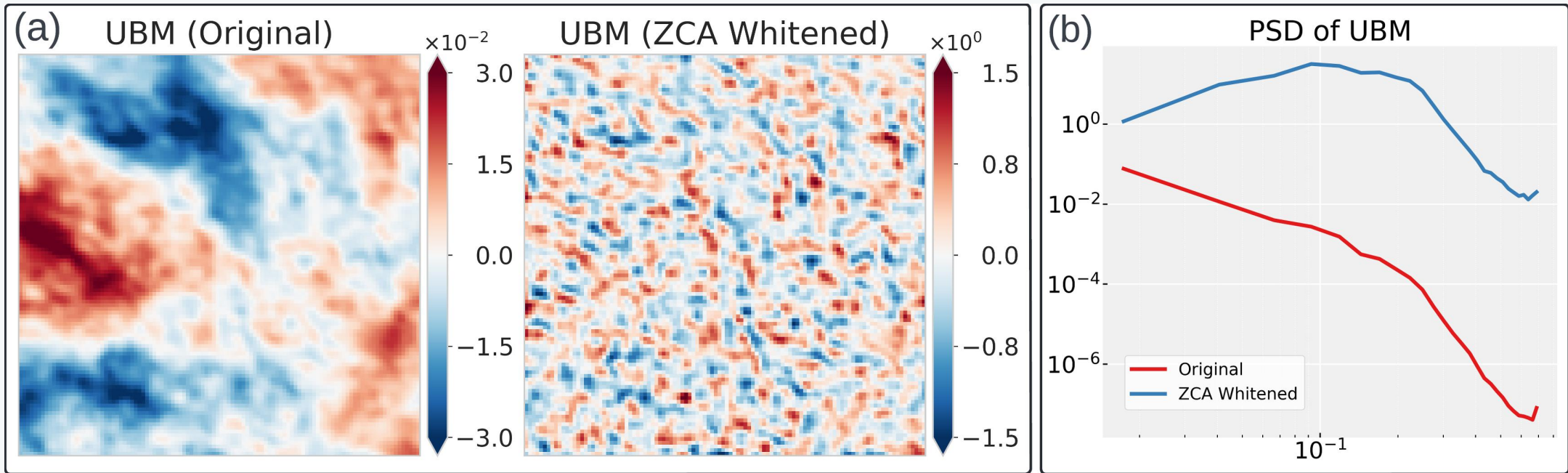
Features become statistically independent

### Invertibility

Exists analytical inverse  
transformation



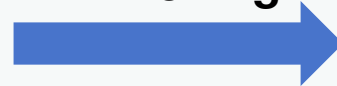
# ZCA Whitening: Effect Visualization



## Original UBM

- **Large-scale** features dominate
- **PSD**: Steep power decay
- **Amplitude scale**:  $O(10^{-2})$

After ZCA  
Whitening



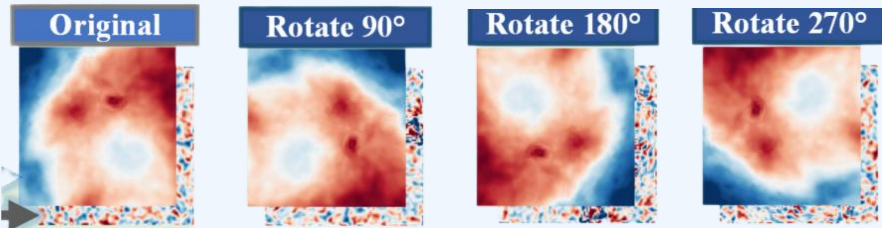
## UBM (ZCA Whitened)

- **Small-scale** features visible
- **PSD**: Flatter power distribution
- **Amplitude scale**:  $O(10^0)$

# Data Augmentation: Solution for Data-Efficiency

Maximizing Learning from Limited Ocean Data

## ↻ Rotational Augmentation



- **4 orientations** per sample
- Preserves physical relationships
- Rotation-invariant ocean dynamics

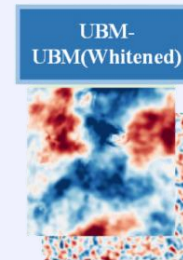
## 🗄 Synthetic Sample Generation



### Pure BM Samples

$(\eta_{\text{BM}}, 0)$

Large-scale circulation patterns



### Pure UBM Samples

$(\eta_{\text{UBM}}, \eta_{\text{UBM\_whitened}})$

Small-scale wave patterns

## ↗ Training Enhancement

Original Dataset

**24,000**

training samples

After Augmentation

**144,000**

training samples

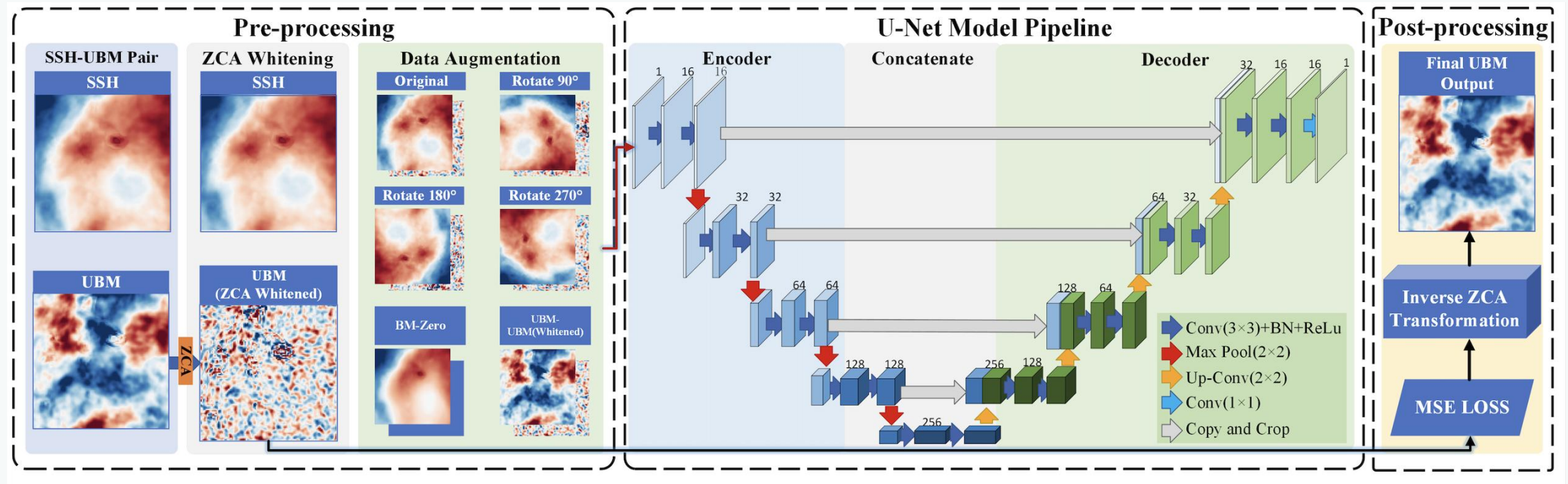
**Improvement**

**6X**

### Key Benefits:

- ↑ Model robustness
- ↑ Feature recognition
- ↑ Generalization

# Our Approach Overview



## Key Innovations

### 1 ZCA Whitening

Enhances high frequency information capture

### 2 Data Augmentation

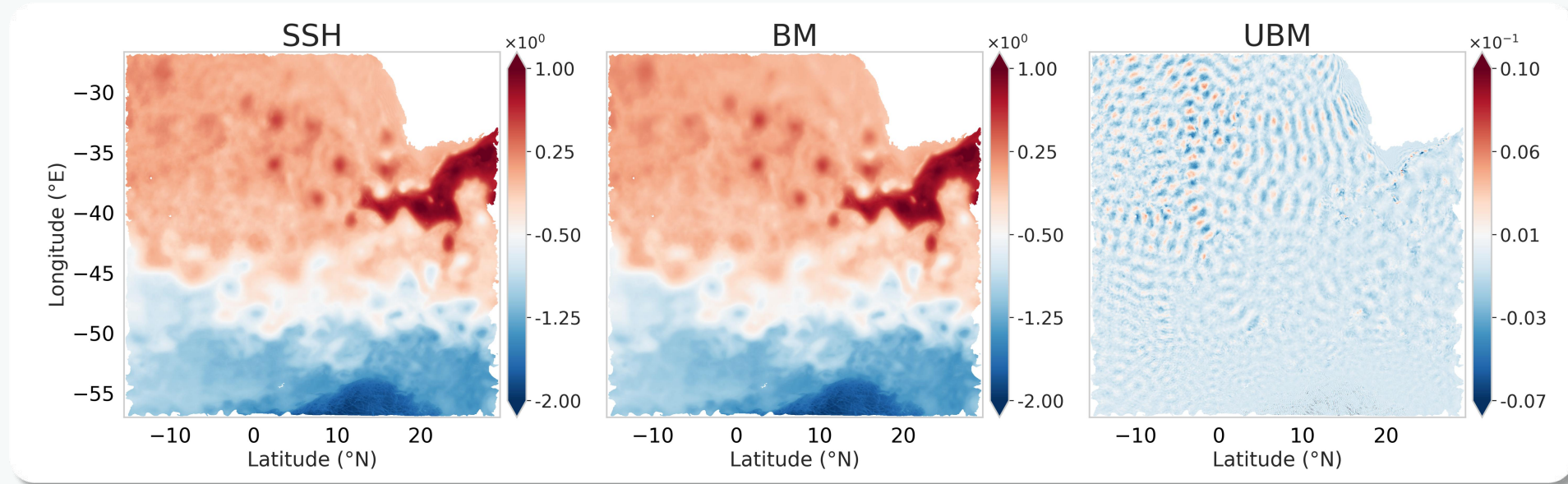
Mitigates limited training samples

### 3 U-Net

Multi-scale feature processing

# Dataset: LLC4320 Ocean Simulation

High-Resolution Global Ocean Model for SSH Decomposition



## Dataset Overview

### Temporal Coverage

70 daily snapshots  
(Sept-Nov 2011)

### Spatial Coverage

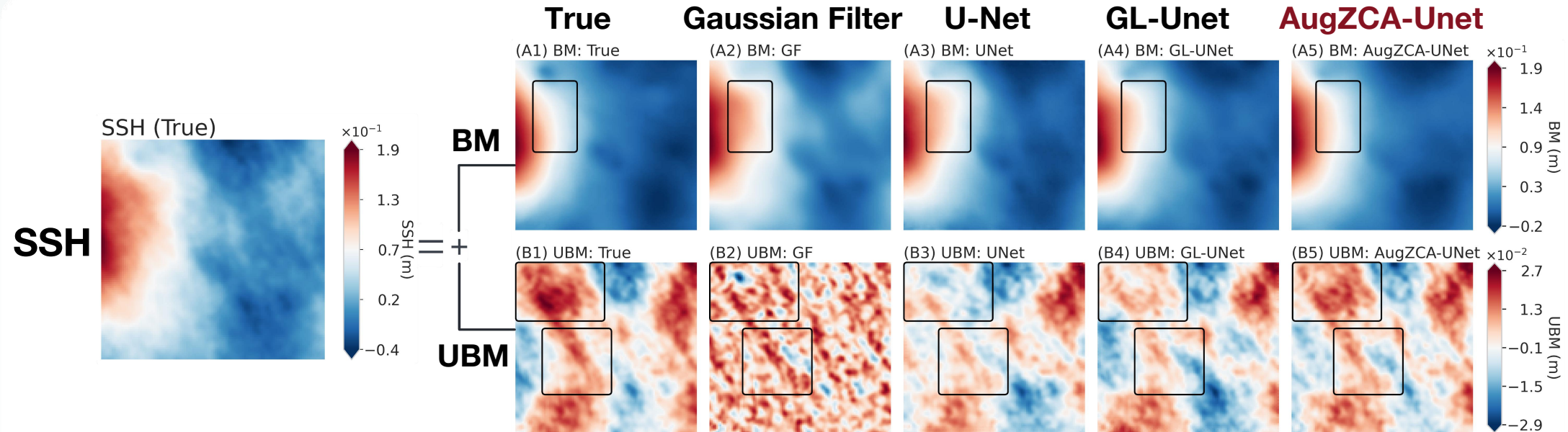
Agulhas retroreflection region  
( $15^\circ$  W- $29^\circ$  E,  $27^\circ$  S- $57^\circ$  S)

### Resolution

Temporal: 1 Day  
Spatial:  $\sim 2$ km

# Results: Visual Comparison

Model Performance Across Different Methods



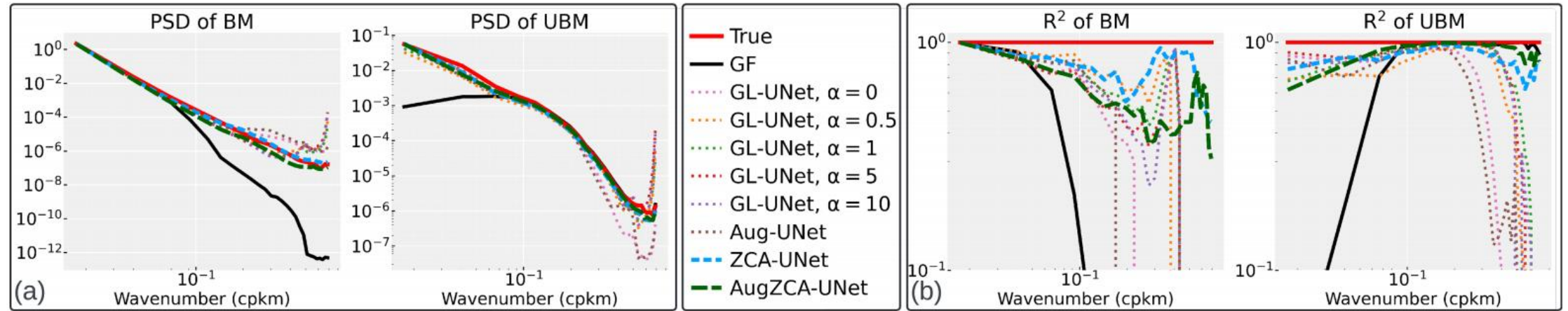
## Balanced Motion (BM)

- Preserves **large-scale** features without over-smoothing

## Unbalanced Motion (UBM)

- Better recovery of **fine-scale** structures

# Results: Spectral Analysis



## Low Wavenumbers

- All methods perform well for large-scale features

## High Wavenumbers

- AugZCA-UNet maintains accuracy at small scales

# Results: Performance Metrics

Quantitative Analysis of Model Performance

Table 1: Pixel-wise Absolute Error Distribution Measures for  $\hat{\eta}_B$  ( $\times 10^{-2}$ ) and  $|\nabla \hat{\eta}_B|$  ( $\times 10^{-3}$ )

Measures	GF	GL-UNet					Aug-UNet	ZCA-UNet	AugZCA-UNet	
		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 5$	$\alpha = 10$				
$\hat{\eta}_B$	Median	0.406	0.404	0.420	0.397	0.398	0.406	0.384	0.393	<b>0.372</b>
	P95 (95%)	1.68	1.52	1.49	1.48	1.51	1.53	1.50	1.52	<b>1.40</b>
$ \nabla \hat{\eta}_B $	Median	0.505	0.464	0.447	0.450	0.450	0.453	0.460	0.450	<b>0.427</b>
	P95 (95%)	2.20	1.80	1.70	1.72	1.74	1.75	1.78	1.71	<b>1.64</b>

## Error Reduction

↓ **8.4%** BM Error Reduction  
vs GL-UNet

↓ **15.4%** Gradient Error Reduction  
vs GF

## Key Advantages

- **No** gradient loss tuning needed
- **Consistent performance** across scales
- **Lower error** in both BM and gradients

# Conclusion

## Key Contributions

- AugZCA-UNet: Superior multi-scale decomposition without gradient tuning
- ZCA whitening enables consistent cross-scale performance
- Effective processing of SWOT satellite data

## Future Work

- ❑ Address ZCA memory scaling for larger images
- ❑ Investigate transfer learning for generalization
- ❑ Incorporate physical constraints in ML frameworks