# Accurate Interpolation for Scattered Data through Hierarchical Residual Refinement

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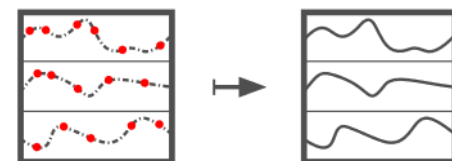
Scattered data interpolation

- Given n scattered observed points $O = \{(x_i, y_i)\}_{i=1}^{n}$

  interpolate O to reconstruct a function f

  It is assumed that f belongs to a latent function distribution, denoted by $f \in \mathcal{F}$

- Wide range of application scenarios
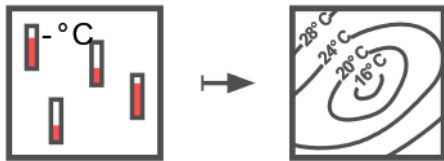
Temperature field reconstruction
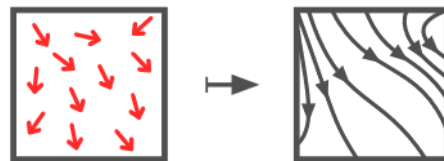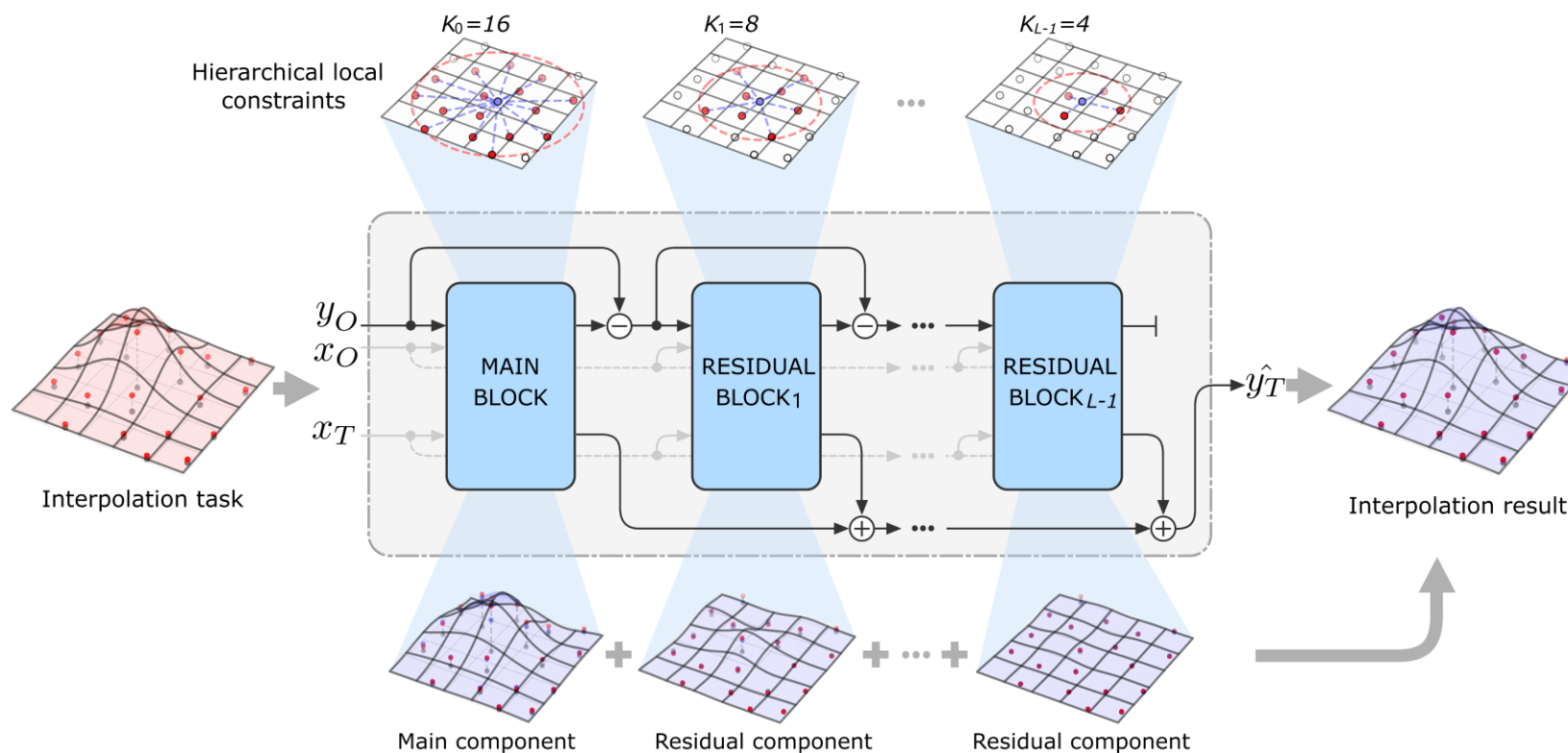
Particle tracking velocimetry

Irregularly-sampled time-series interpolation

Ding et al. arXiv:2209.09078, 2023.

Motivation

- Existing neural interpolators neglect the information of interpolation residuals

- The interpolation residuals can be progressively and hierarchically exploited

Our **H**ierarchical **INT**erpolation framework (HINT)

1. Multiple lightweight interpolation block, dual residual linked, utilizing residual of observed points
2. Hierarchical local constraint for better refining residual predictions

Transformer-based interpolation block

Partial self-attention with
K-nearest neighbor graph constraint
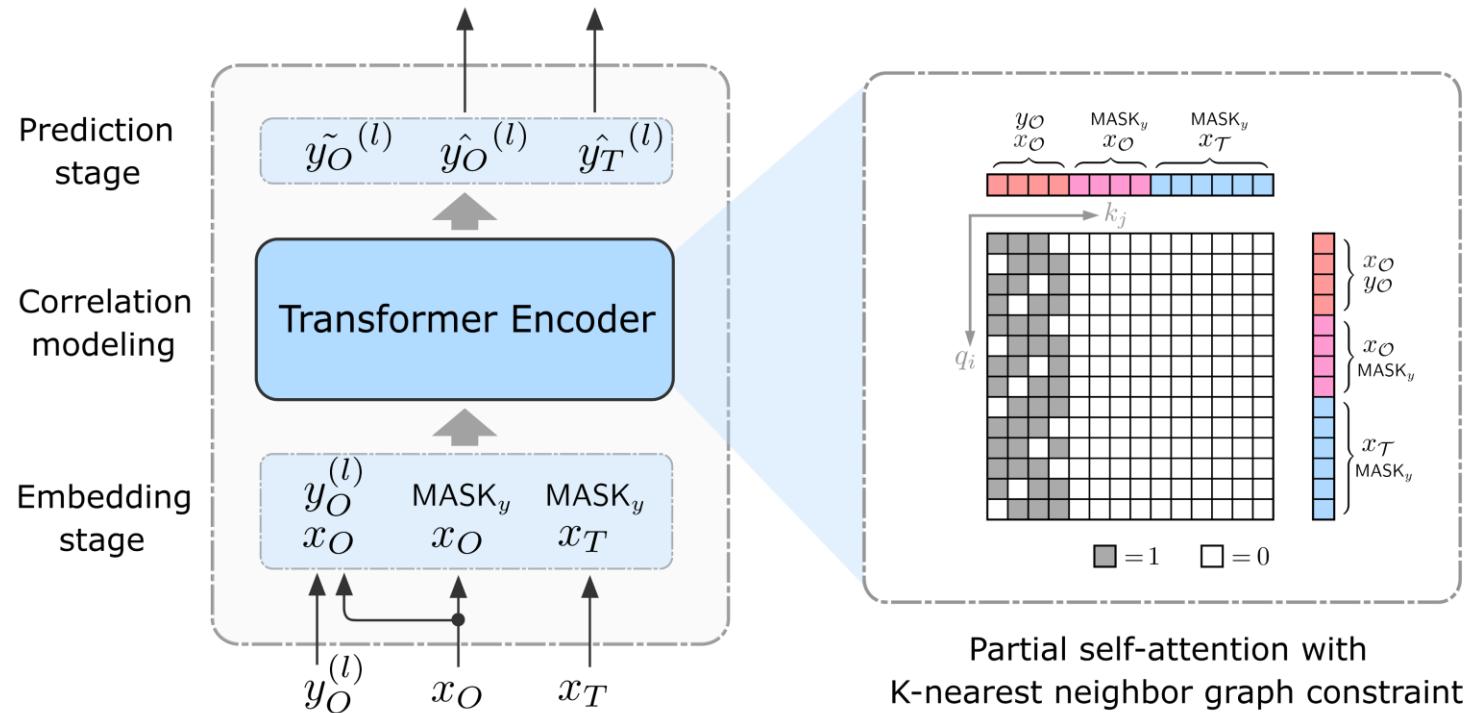
1. Masked Transformer encoder for accurate correlation modeling of scattered points
2. KNN graph mask as local constraint

Results

- SOTA interpolation accuracy on representative datasets

Table 1: Interpolation accuracy on Mathit dataset.

| Interpolation approach	MSE ($\times 10^{-4}$) on Mathit-2D test set
CNP	24.868
ANP	14.001
BANP	8.419
TFR-Transformer	5.857
NIERT	3.167
HINT (ours)	**2.903**

↓ 8.34 %

Table 2: Interpolation accuracy on Perlin dataset.

Interpolation approach	MSE ($\times 10^{-5}$) on Perlin test set
CNP	48.642
ANP	23.731
BANP	20.737
TFR-Transformer	12.101
NIERT	7.185
HINT (ours)	**5.848**

↓ 18.61 %

Table 3: Interpolation accuracy on PTV dataset.

Interpolation approach	MSE ($\times 10^{-3}$) on PTV test set
CNP	137.573
ANP	32.111
BANP	33.585
TFR-Transformer	17.125
NIERT	5.167
HINT (ours)	**3.507**

↓ 32.13 %

Table 4: Interpolation accuracy on TFRD dataset.

Interpolation approach	MAE ($\times 10^{-3}$) on TFRD test set		
	HSink	ADlet	DSine
CNP	204.351	91.782	92.456
ANP	164.491	54.684	58.589
BANP	59.728	28.671	19.107
TFR-Transformer	64.987	27.074	29.961
NIERT	23.519	3.473	8.785
HINT (ours)	**13.758**	**1.761**	**4.912**

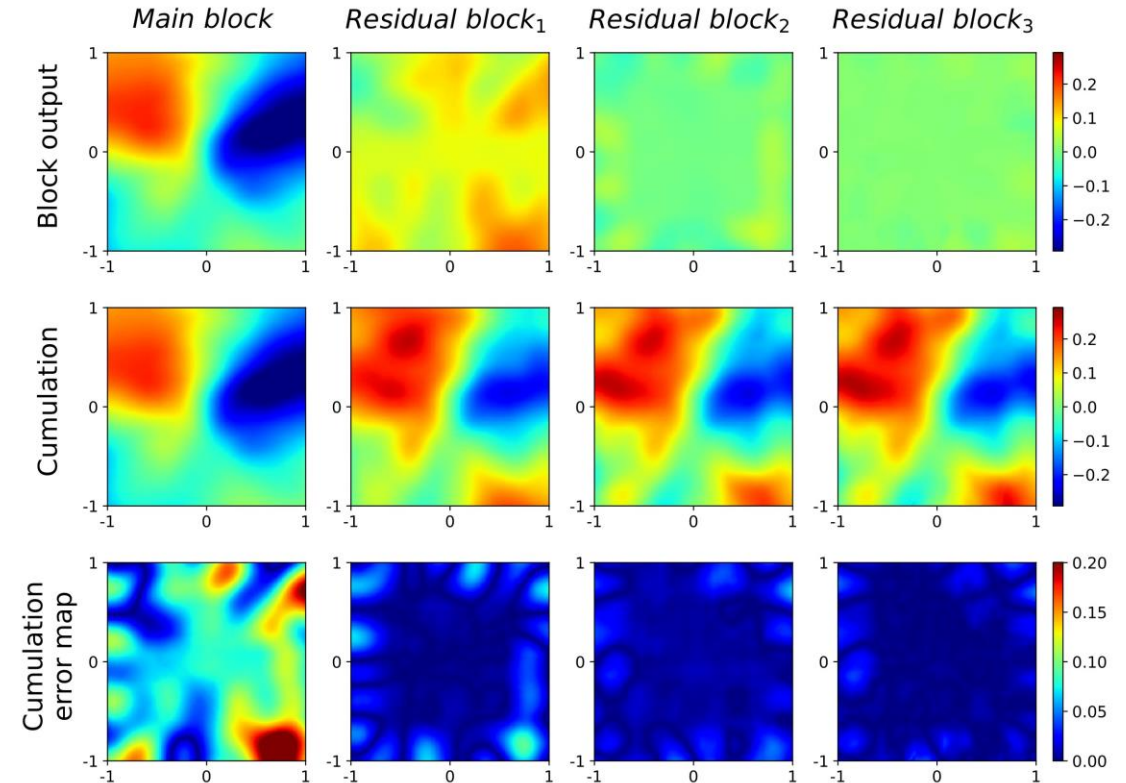↓ 44.96 % on Avg.

Case study and analysis

- More accurate interpolation result

- Finer-scaled error map
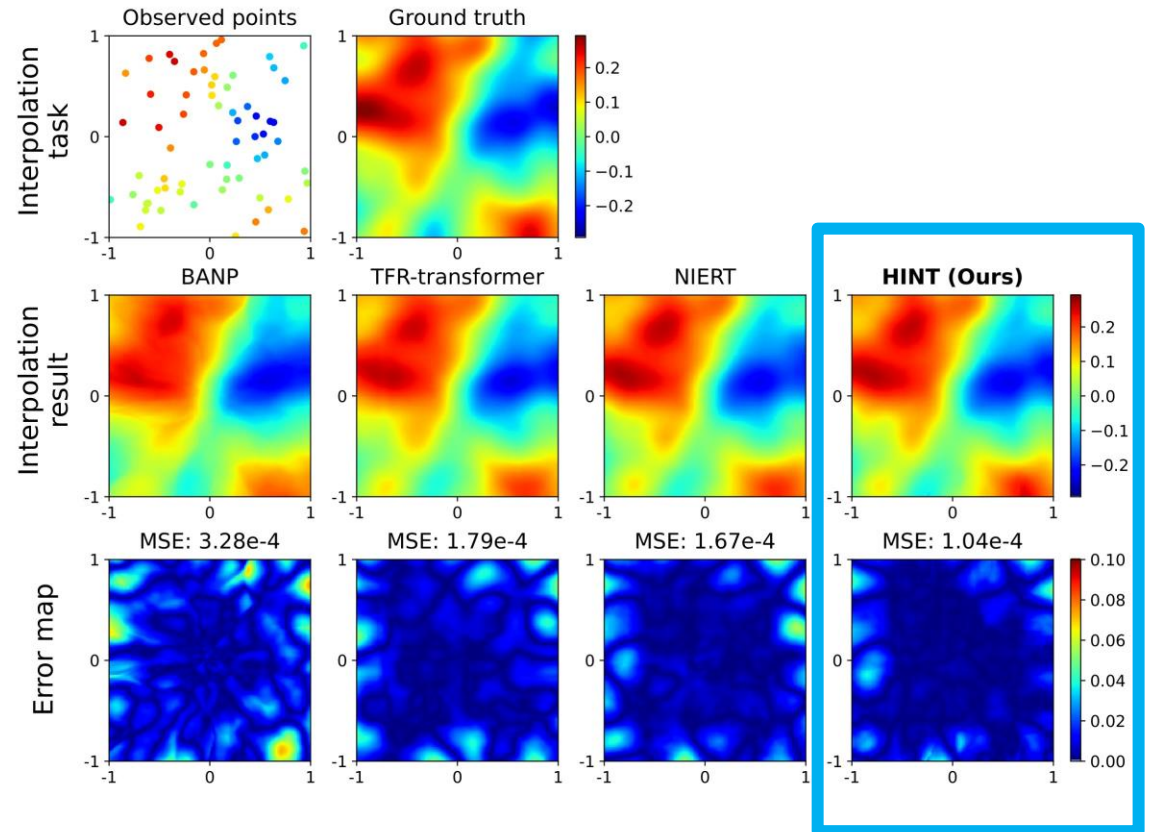
Qualitative comparison on a 2D interpolation
task extracted from Perlin dataset

Case study and analysis

- Progressively predict of the main function components and interpolation residuals

- Decouple the function at different scales

Output from each interpolation block on an interpolation task from Perlin dataset

Conclusion

- We introduce HINT, a novel hierarchical framework for scattered point interpolation

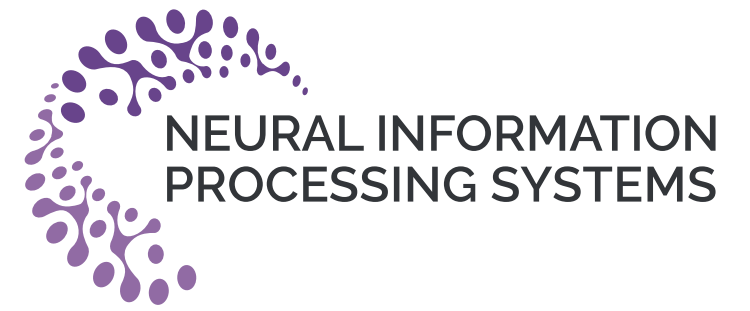- HINT enhances accuracy using coarse-to-fine interpolation blocks and outperforms SOTA methods

- Paper:

  https://nips.cc/virtual/2023/poster/72636

- Source code:

  https://github.com/DingShizhe/HINT

Thanks