Federated Linear Bandits with Finite Adversarial Actions

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Agenda

- Problem Formulation
- Preliminaries
- FedSupLinUCB Framework
- Algorithms
- Experiments
- Conclusion

Problem Formulation

- Star-shape system: M clients communicate with the central server.
- At round t, some clients $I_t \in [M]$ activate and receive K arms associated with contexts $\{x_{t,a}^i\}_{a\in[K]}$.
- Client $i \in I_t$ pulls arm a_t^i and receives reward $r_{t,a_t^i} = \theta^\top x_{t,a_t^i}^i + \epsilon_t$, where $\theta \in \mathbb{R}^d$ is unknown parameter, and ϵ_t is sub-Gaussian noise.
- Two arrival patterns: 1) Synchronous: all M clients are active at each round. 2) Asynchronous: one client is active at each round.
- Total regret: $R_T = \sum_{i=1}^M R_T^i = \sum_{i=1}^M \mathbb{E}\left[\sum_{t\in P_T^i} r_{t,a_t^{i,*}}^i - r_{t,a_t^i}^i\right]$, where P_T^i is the set of time indices at which client i is active.
- Communication cost: total number of communication rounds between clients and the server.

Preliminaries

Algorithm 1 Sync(s, server, client 1, … client n)

for $i = 1, 2, \ldots, n$ **do** ▷ Client-side local information upload
  Client i sends the local new layer s information $(\Delta A_s^i, \Delta b_s^i)$ to the server
end for
Update server's layer s information: ▷ Server-side information aggregation and distribution

$$A_s^{ser} \leftarrow A_s^{ser} + \sum\nolimits_{i=1}^{n} \Delta A_s^i, \quad b_s^{ser} \leftarrow b_s^{ser} + \sum\nolimits_{i=1}^{n} \Delta b_s^i$$

Send server information A_s^{ser}, b_s^{ser} back to all clients
for $i = 1, 2, \ldots, n$ **do**
  $A_s^i \leftarrow A_s^{ser}, b_s^i \leftarrow b_s^{ser}, \Delta A_s^i \leftarrow 0, \Delta b_s^i \leftarrow 0$ ▷ Client i updates the local information
end for

- Information encoding: matrix $A_n = \sum_{t=1}^{n} x_t x_t^\top$ and vector $b_n = \sum_{t=1}^{n} r_t x_t$, encoding function: $A_n, b_n \leftarrow Encode(\{x_t, r_t\}_{t=1}^{n})$.
- Communication criterion: 'doubling trick' introduced in [Abbasi-Yadkori et al., 2011] and the protocol by [Wang et al., 2019, He et al., 2022].
- Synchronization procedure: clients upload the local information, and the server aggregates and distributes updated information.

FedSupLinUCB Framework

Algorithm 2 S-LUCB

Initialization: $S = \lceil \log d \rceil$, $\overline{w}_0 = d^{1.5}/\sqrt{T}$, $\overline{w}_s \leftarrow 2^{-s}\overline{w}_0, \forall s \in [1:S]$.
$\alpha_0 = 1 + \sqrt{d \ln(2M^2T/\delta)}$, $\alpha_s \leftarrow 1 + \sqrt{2\ln(2KMT \ln d/\delta)}, \forall s \in [1:S]$
Input: Client i (with local information $A^i, b^i, \Delta A^i, \Delta b^i$), contexts set $\{x^i_{t,1}, \ldots, x^i_{t,K}\}$
$A^i_{t,s} \leftarrow A^i_s + \Delta A^i_s, b^i_{t,s} \leftarrow b^i_s + \Delta b^i_s$ or $A^i_{t,s} \leftarrow A^i_s, b^i_{t,s} \leftarrow b^i_s$ for `lazy update`
$\hat{\theta}_s \leftarrow (A^i_{t,s})^{-1} b^i_{t,s}$, $\hat{r}^i_{t,s,a} = \hat{\theta}_s^\top x^i_{t,a}$, $w^i_{t,s,a} \leftarrow \alpha_s \|x^i_{t,a}\|_{(A^i_{t,s})^{-1}}, \forall s \in [0:S], \forall a \in [K]$
$s \leftarrow 0$; $\mathcal{A}_0 \leftarrow \{a \in [K] \mid \hat{r}^i_{t,0,a} + w^i_{t,0,a} \geq \max_{a \in [K]}(\hat{r}^i_{t,0,a} - w^i_{t,0,a})\}$ ▷ Initial screening
repeat ▷ Layered successive screening
  if $s = S$ **then**
    Choose action a^i_t arbitrarily from $\mathcal{A}_S$
  else if $w^i_{t,s,a} \leq \overline{w}_s$ for all $a \in \mathcal{A}_s$ **then**
    $\mathcal{A}_{s+1} \leftarrow \{a \in \mathcal{A}_s \mid \hat{r}^i_{t,s,a} \geq \max_{a' \in \mathcal{A}_s}(\hat{r}^i_{t,s,a'}) - 2\overline{w}_s\}$; $s \leftarrow s + 1$
  else
    $a^i_t \leftarrow \arg\max_{\{a \in \mathcal{A}_s, w^i_{t,s,a} > \overline{w}_s\}} w^i_{t,s,a}$
  end if
until action a^i_t is found
Take action a^i_t and and receive reward r^i_{t,a^i_t}
$\Delta A^i_s \leftarrow \Delta A^i_s + x^i_{t,a^i_t} x^{i\top}_{t,a^i_t}$, $\Delta b^i_s \leftarrow \Delta b^i_s + r^i_{t,a^i_t} x^i_{t,a^i_t}$ ▷ Update local information
Return layer index s

- Combination the principles of SupLinUCB [Chu et al., 2011] and OFUL [Abbasi-Yadkori et al., 2011, Ruan et al., 2021].
- Maintain $S = \lceil \log d \rceil$ information layers.
- Accuracy strengthens as the layer index increases.

Asynchronous FedSupLinUCB

Algorithm 3 Async-FedSupLinUCB

Initialization: $T, C, S = \lceil \log d \rceil$

$\{A_s^{ser} \leftarrow I_d, b_s^{ser} \leftarrow 0 \mid s \in [0:S]\}$ ▷ Server initialization

$\{A_s^i \leftarrow I_d, \Delta A_s^i, b_s^i, \Delta b_s^i \leftarrow 0 \mid s \in [0:S], i \in [M]\}$ ▷ Clients initialization

for $t = 1, 2, \cdots, T$ **do**

  Client $i_t = i$ is active, and observes K contexts $\{x_{t,1}^i, x_{t,2}^i, \cdots, x_{t,K}^i\}$

  $s \leftarrow$ S-LUCB $\left(\text{client } i, \{x_{t,1}^i, x_{t,2}^i, \cdots, x_{t,K}^i\}\right)$ with lazy update

  if $\frac{\det\left(A_s^i + \Delta A_s^i\right)}{\det(A_s^i)} > (1 + C)$ **then**

    Sync(s, server, clients i) for each $s \in [0:S]$

  end if

end for

- Only one client is active in each round.
- Global synchronization and coordination are not required

Synchronous FedSupLinUCB

Algorithm 4 `Sync-FedSupLinUCB`

Initialization: T_c, D, $S = \lceil \log d \rceil$, $t^s_{last} \leftarrow 0, \forall s \in [0:S]$, CommLayers $\leftarrow \emptyset$.
$\{A^{ser}_s \leftarrow I_d, b^{ser}_s \leftarrow 0 \mid s \in [0:S]\}$ ▷ Server initialization
$\{A^i_s \leftarrow I_d, \Delta A^i_s, b^i_s, \Delta b^i_s \leftarrow 0 \mid s \in [0:S], i \in [M]\}$ ▷ Clients initialization
for $t = 1, 2, \cdots, T_c$ **do**
  for $i = 1, 2, \cdots, M$ **do**
    Client $i_t = i$ is active, and observes K contexts $\{x^i_{t,1}, x^i_{t,2}, \cdots, x^i_{t,K}\}$
    $s \leftarrow$`S-LUCB` $\left(\text{client } i, \{x^i_{t,1}, x^i_{t,2}, \cdots, x^i_{t,K}\}\right)$
    if $(t - t^s_{last}) \log \frac{\det\left(A^i_s + \Delta A^i_s\right)}{\det(A^i_s)} > D$ **then**
      Add s to CommLayers
    end if
  end for
end for
for $s \in$ CommLayers **do**
  Sync(s, server, clients $[M]$); $t^s_{last} \leftarrow t$, CommLayers $\leftarrow \emptyset$
end for

- All clients are active and make decisions at each round.
- Refined communication criterion to enable time-independent communication cost

Performance

Theorem

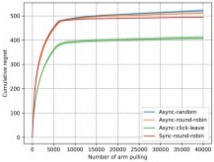
For any $0 < \delta < 1$, if we run Async with $C = 1/M^2$, then with probability at least $1 - \delta$, the regret of the algorithm is bounded as $R_T \leq \tilde{O}\left(\sqrt{d \sum_{i=1}^{M} T_i}\right) = \tilde{O}\left(\sqrt{dT}\right)$. Moreover, the corresponding communication cost is bounded by $O(dM^2 \log d \log T)$.

Theorem

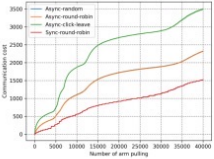
For any $0 < \delta < 1$, if we run Sync with $D = \frac{T_c \log T_c}{d^2 M}$, with probability at least $1 - \delta$, the regret of the algorithm is bounded as $R_T \leq \tilde{O}(\sqrt{dMT_c})$ where T_c is the total per-client arm pulls. Moreover, the corresponding communication cost is bounded by $O(\sqrt{d^3 M^3} \log d)$.

- Both achieve minimax optimal regret with sublinear communication cost.

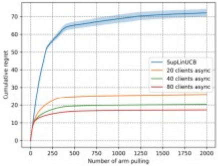
Experiments



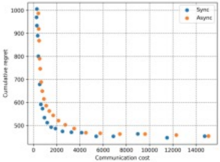
(a) Regret: arrival patterns.



(b) Communication: arrival patterns.



(c) Regret: client numbers.



(d) Regret vs communications.

- Compare with different arrival patterns and amount of clients.
- The trade-off between regrets and communications.

Thank you!

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