PRINCIPLED WEIGHT INITIALISATION FOR INPUT-CONVEX NETWORKS

Pieter-Jan Hoedt @ NeurIPS 2023

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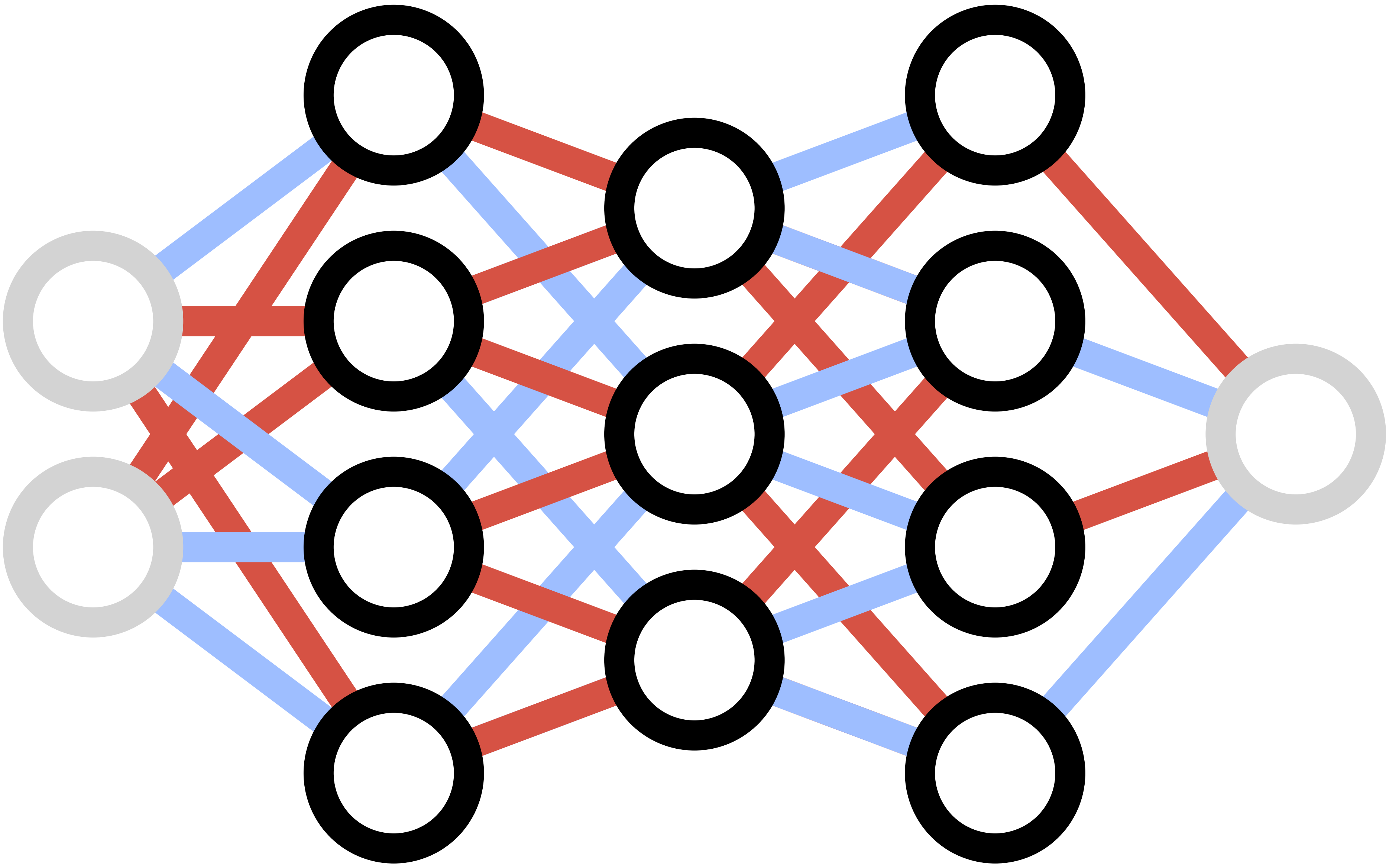
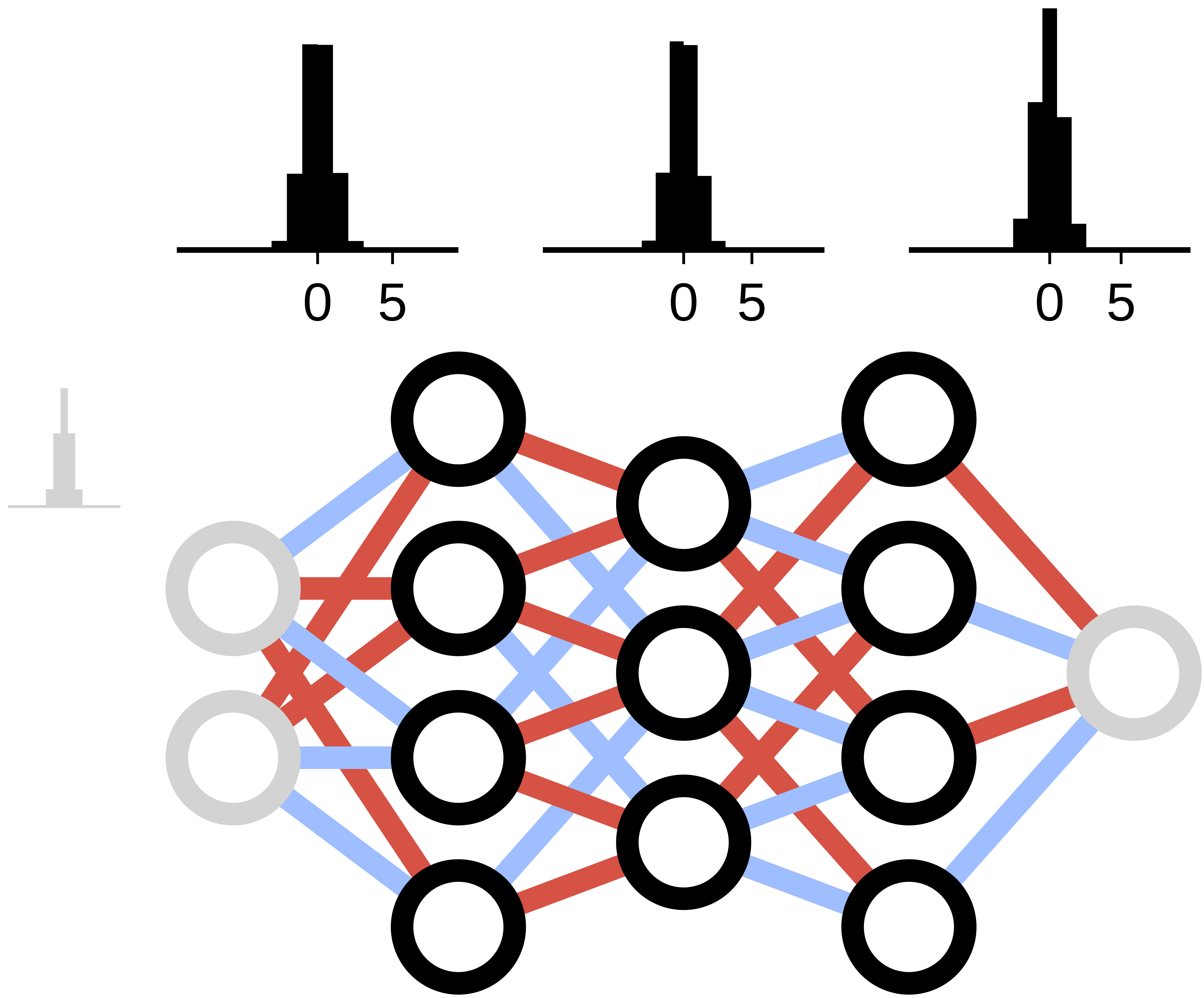
Institute for Machine Learning

JOHANNES KEPLER UNIVERSITY LINZ

INITIALISATION AND SIGNAL PROPAGATION



INITIALISATION AND SIGNAL PROPAGATION



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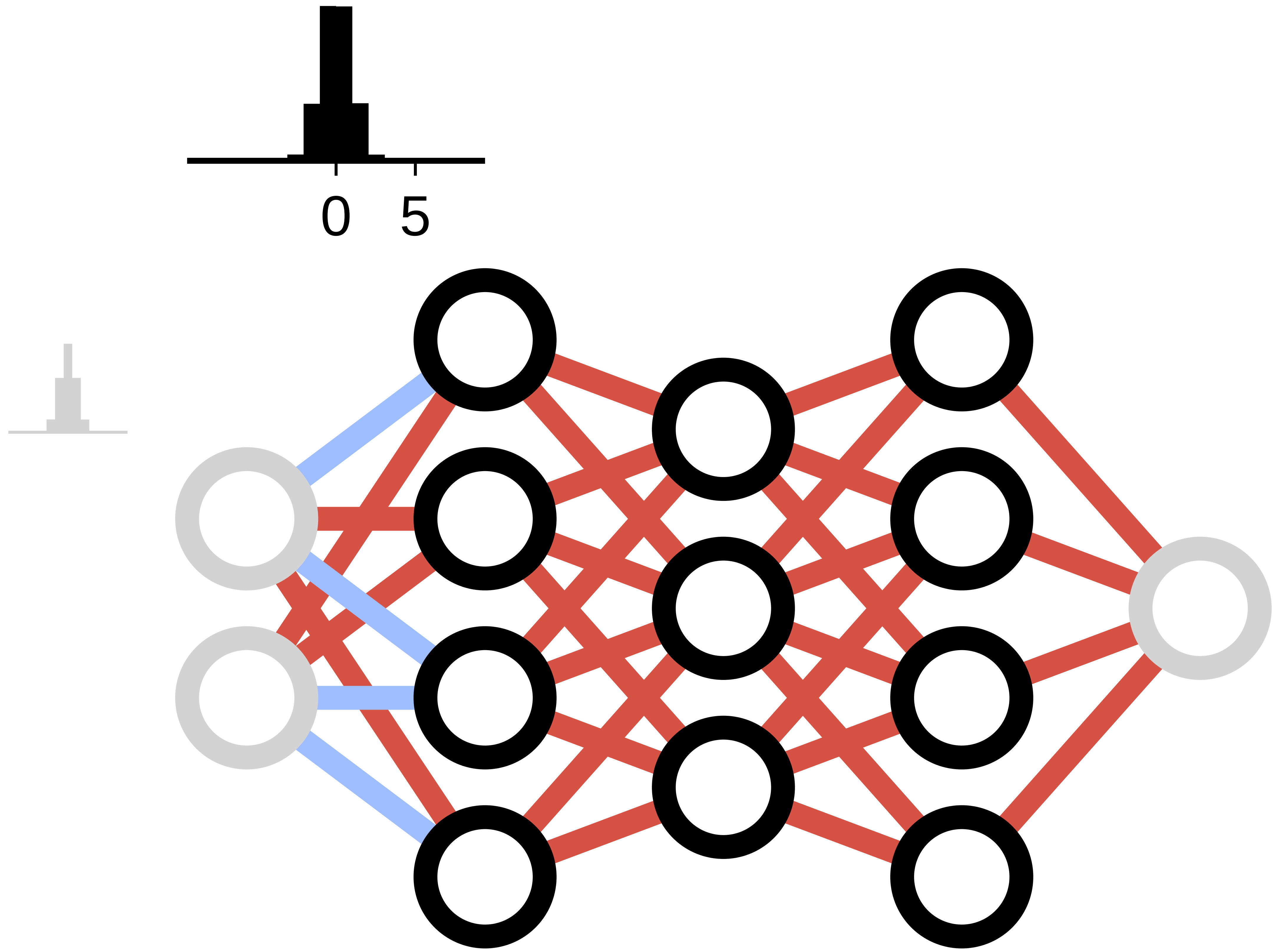

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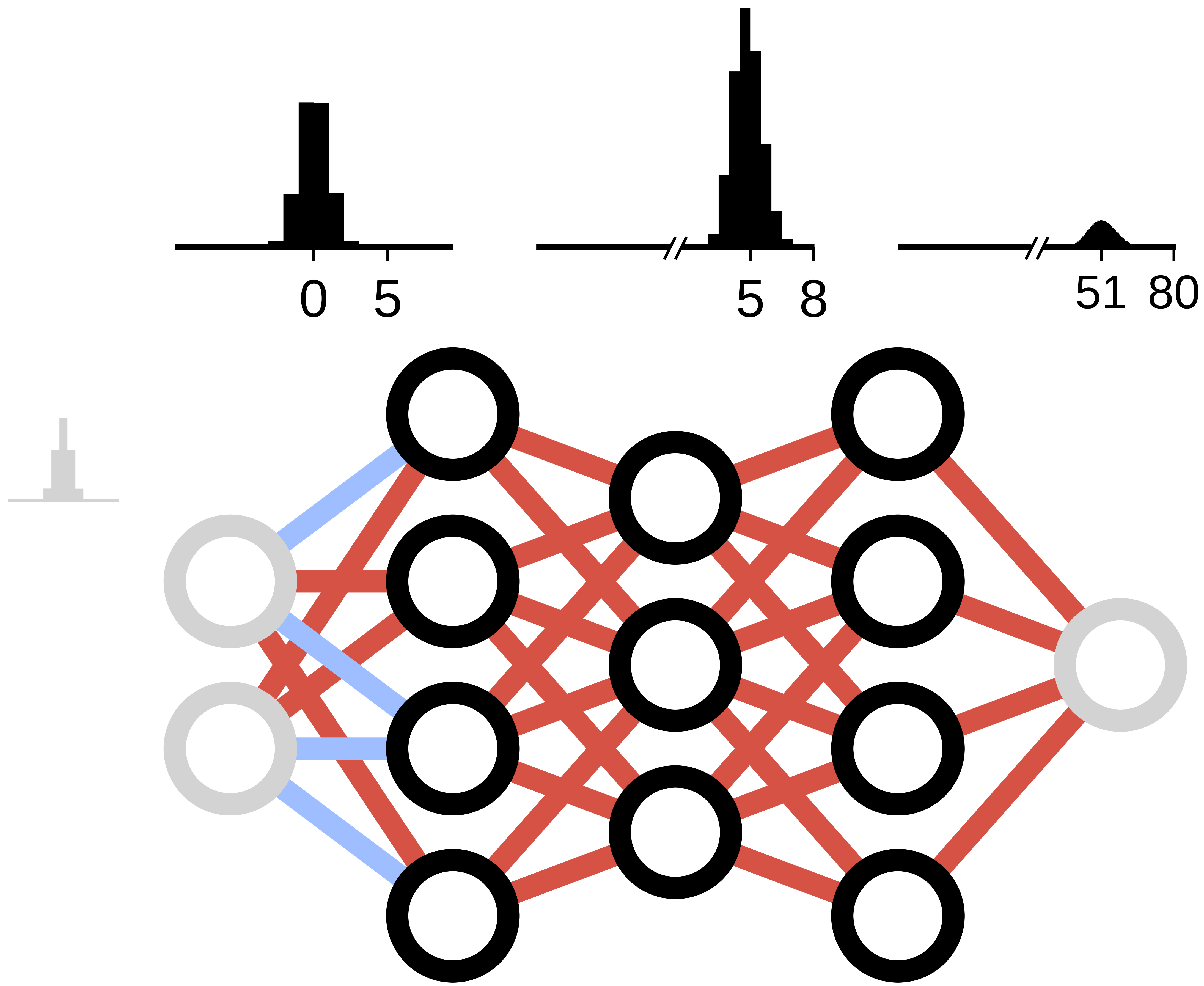
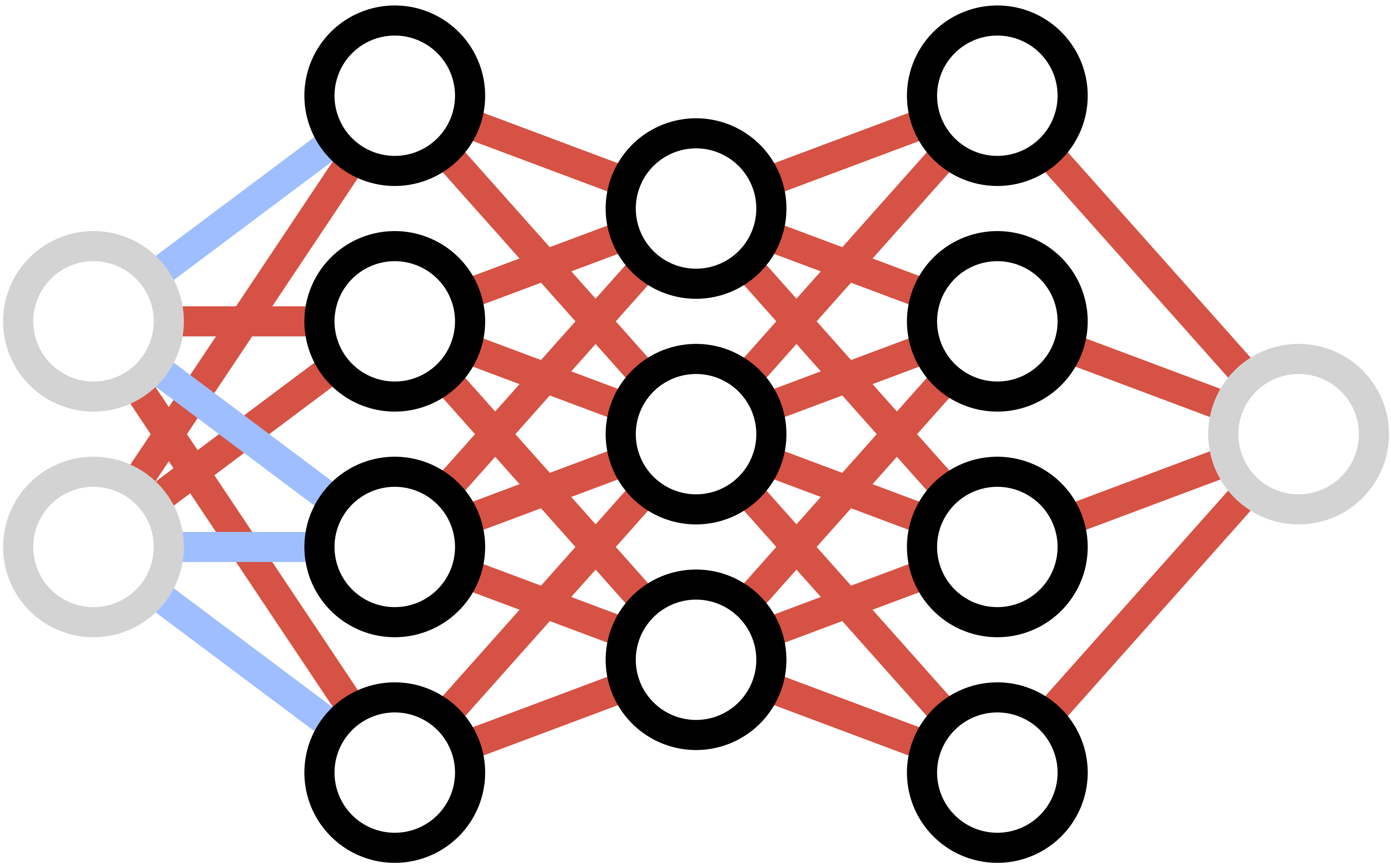


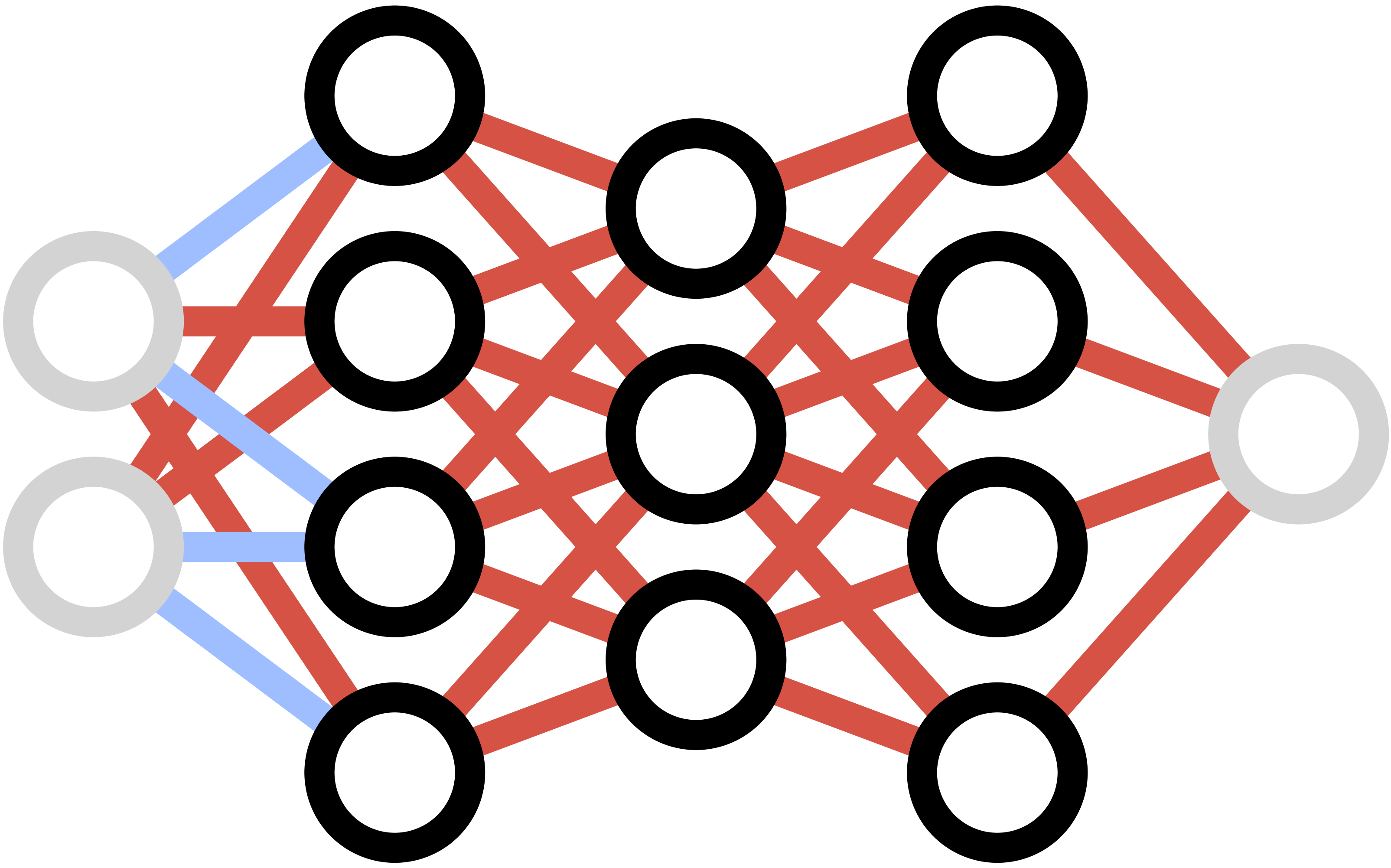
INPUT-CONVEX NETWORKS



Amos, B. et al. (2017). Input Convex Neural Networks.

INPUT-CONVEX NETWORKS

- Positive weights
- Convex non-linearities



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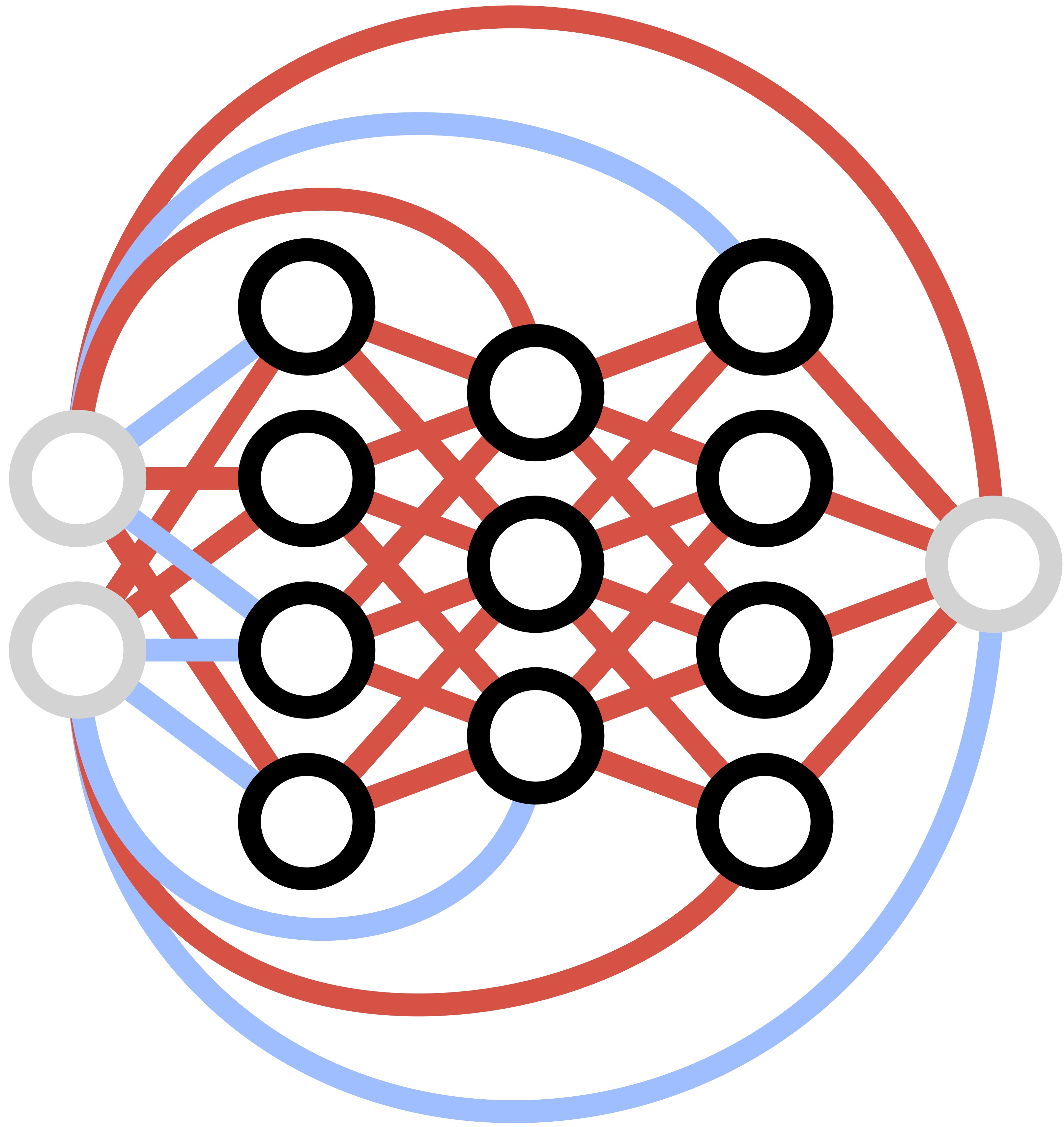
INPUT-CONVEX NETWORKS

- Positive weights
- Convex non-linearities
- Skip-connections



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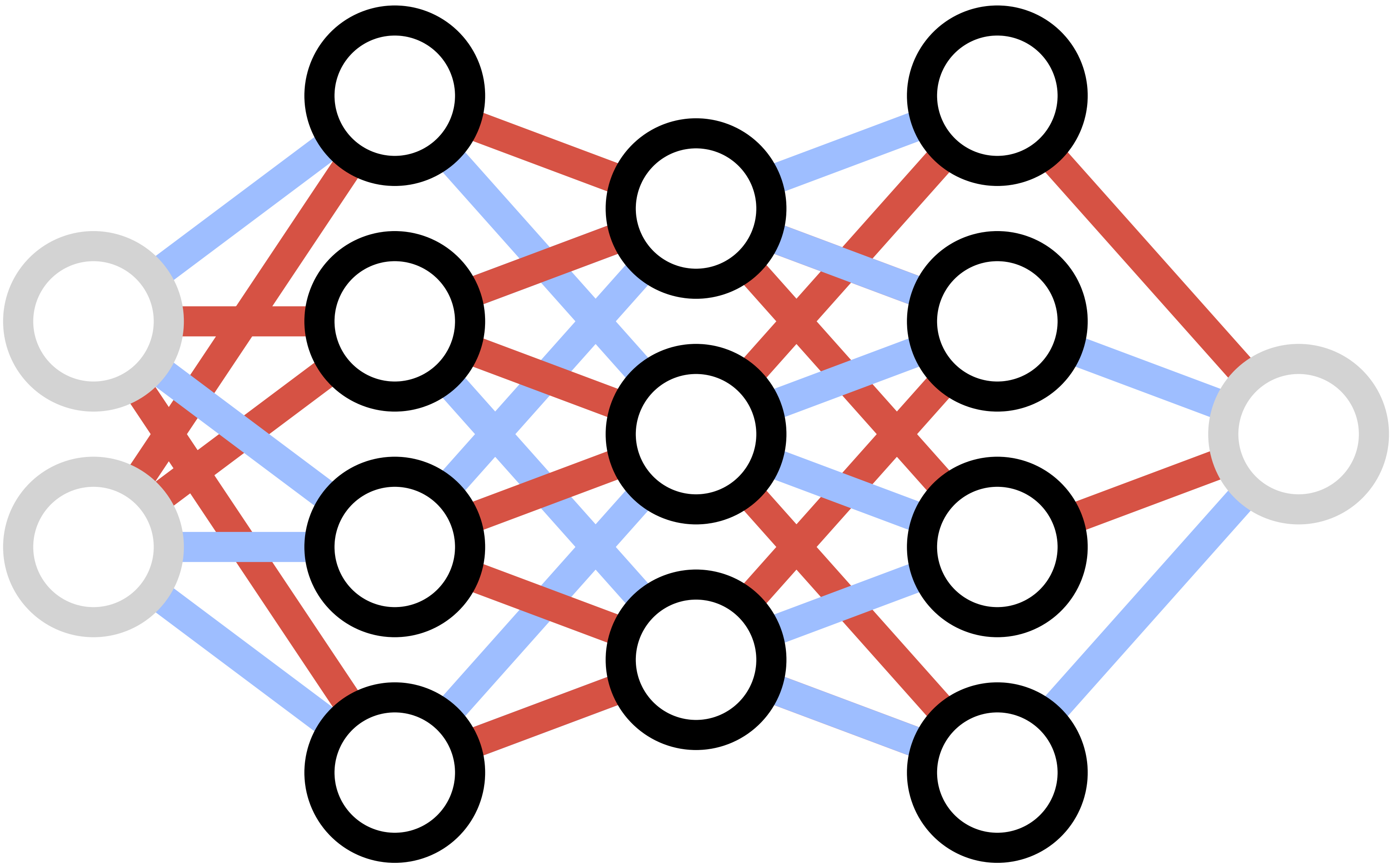
GENERALISING SIGNAL PROPAGATION
IN REGULAR NETWORKS

$$\mu_s = 0$$

$$\sigma_s^2 = {\color{purple}\sigma_w^2} \sum_k \mathbb{E}[x_k^2]$$



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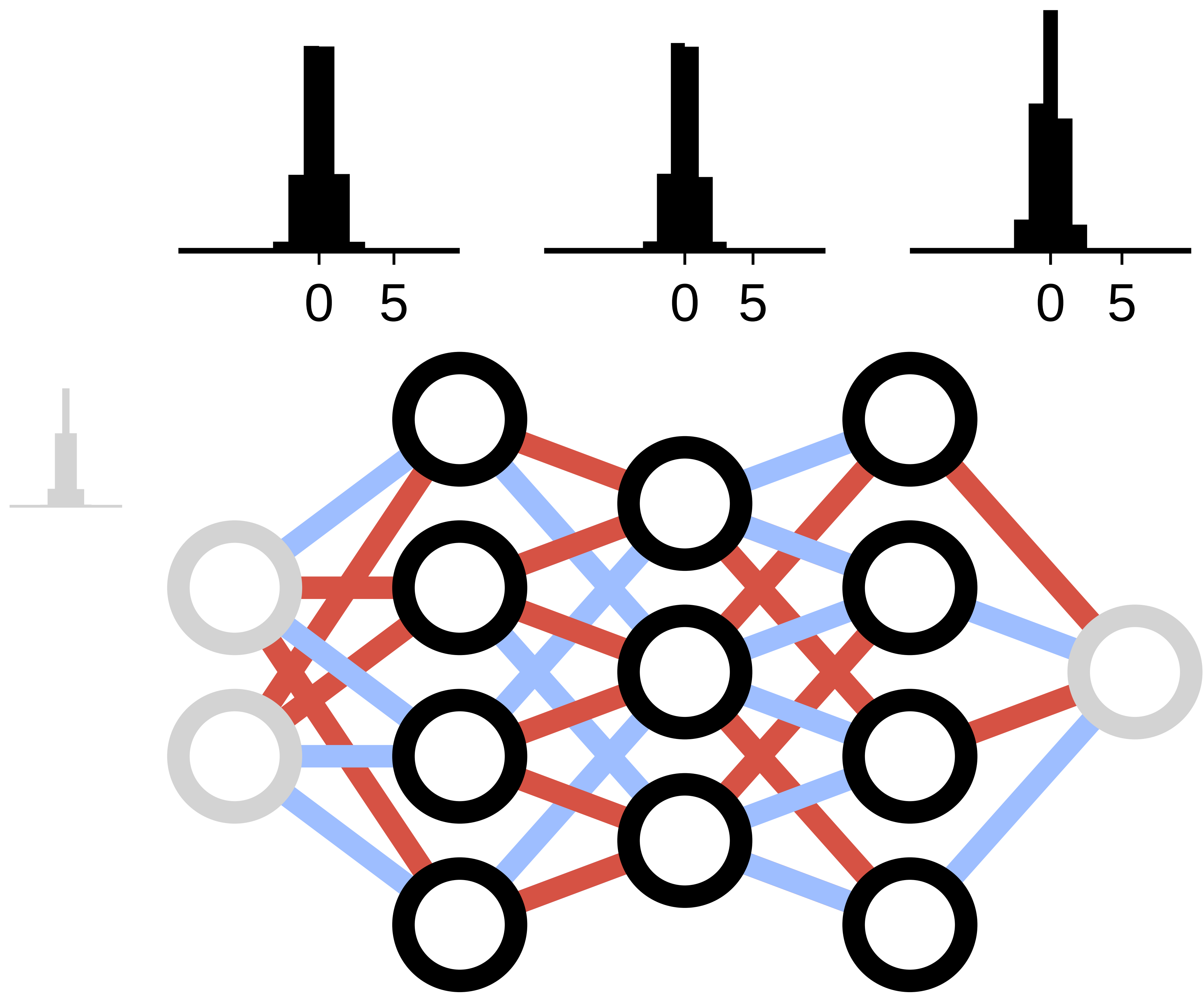

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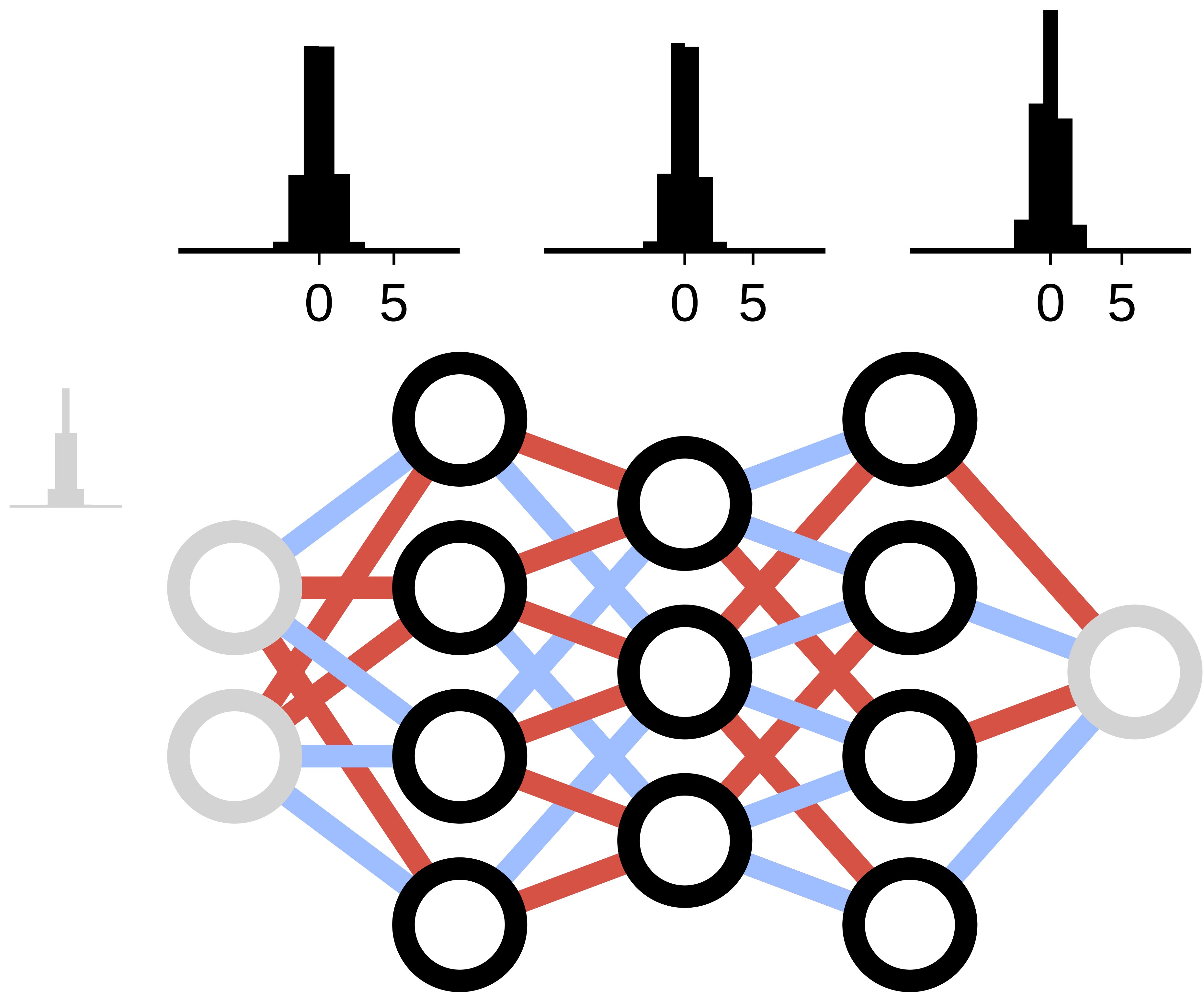

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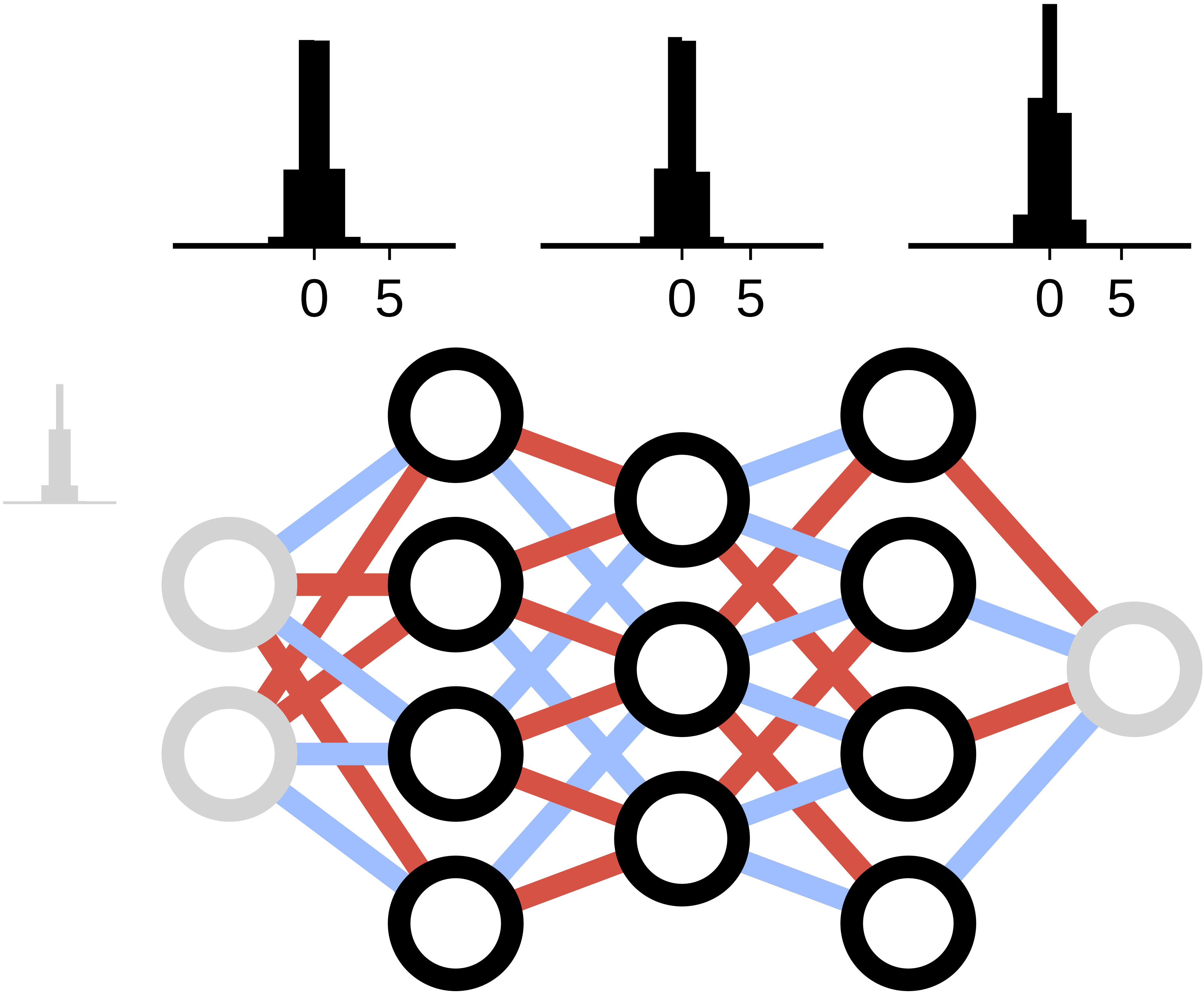

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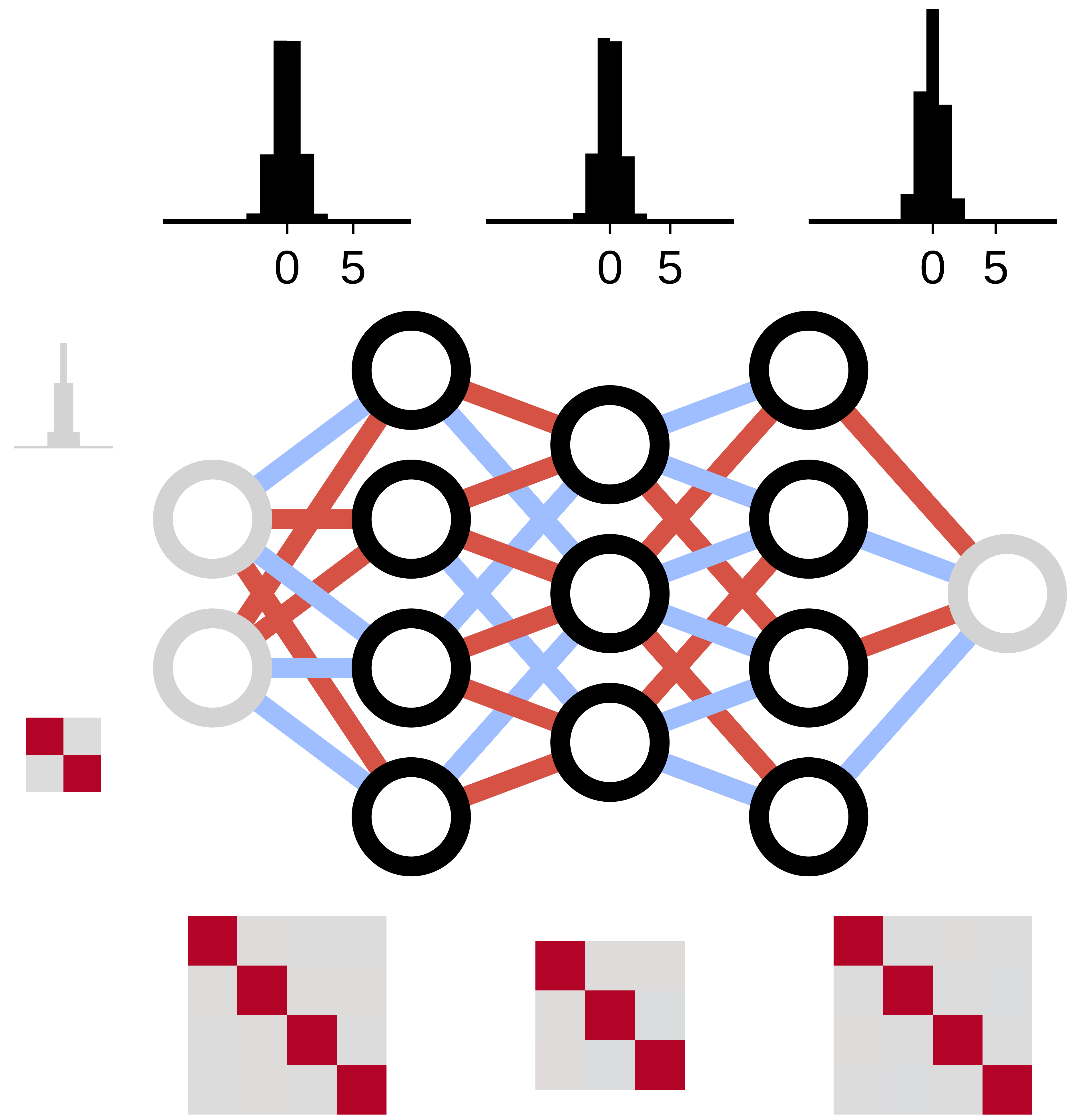

GENERALISING SIGNAL PROPAGATION IN REGULAR NETWORKS

$$\mu_s = \mu_w \sum_k \mathbb{E}[x_k] = 0$$

$$\sigma_s^2 = \sigma_w^2 \sum_k \mathbb{E}\left[x_k^2\right] + c_s$$

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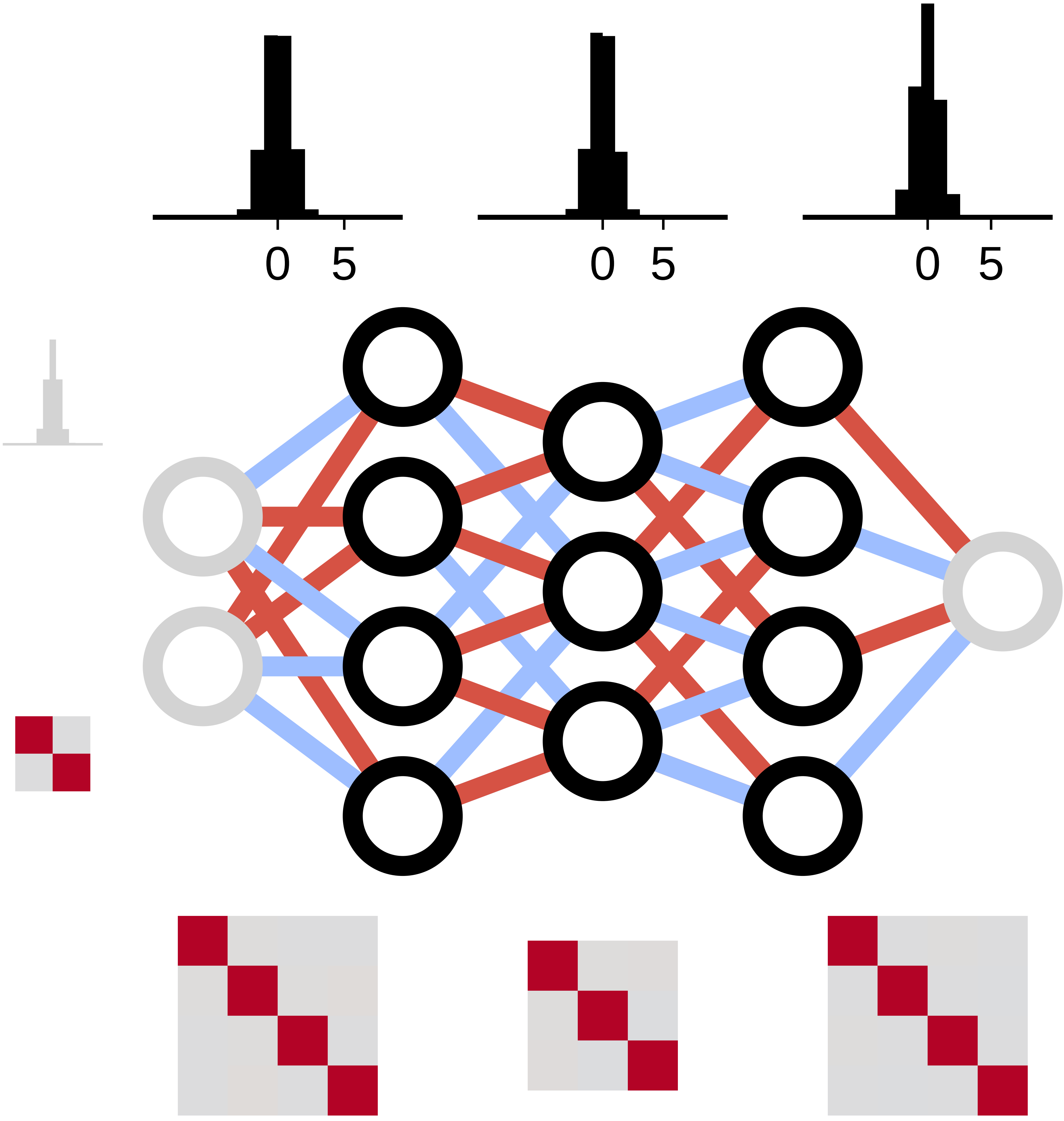

GENERALISING SIGNAL PROPAGATION
IN INPUT-CONVEX NETWORKS

$$\mu_s = \mu_w \sum_k \mathbb{E}[x_k] \neq 0$$

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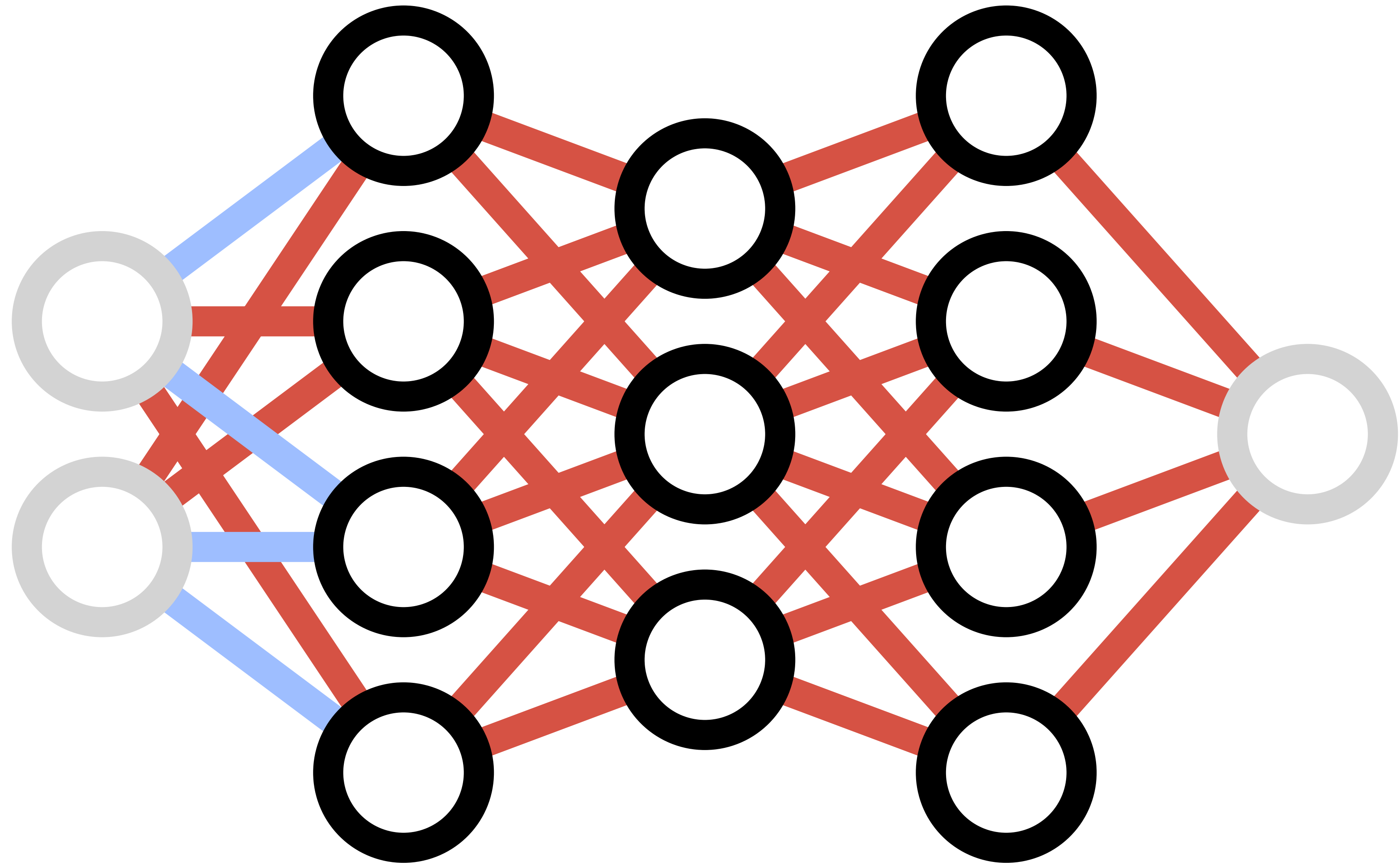

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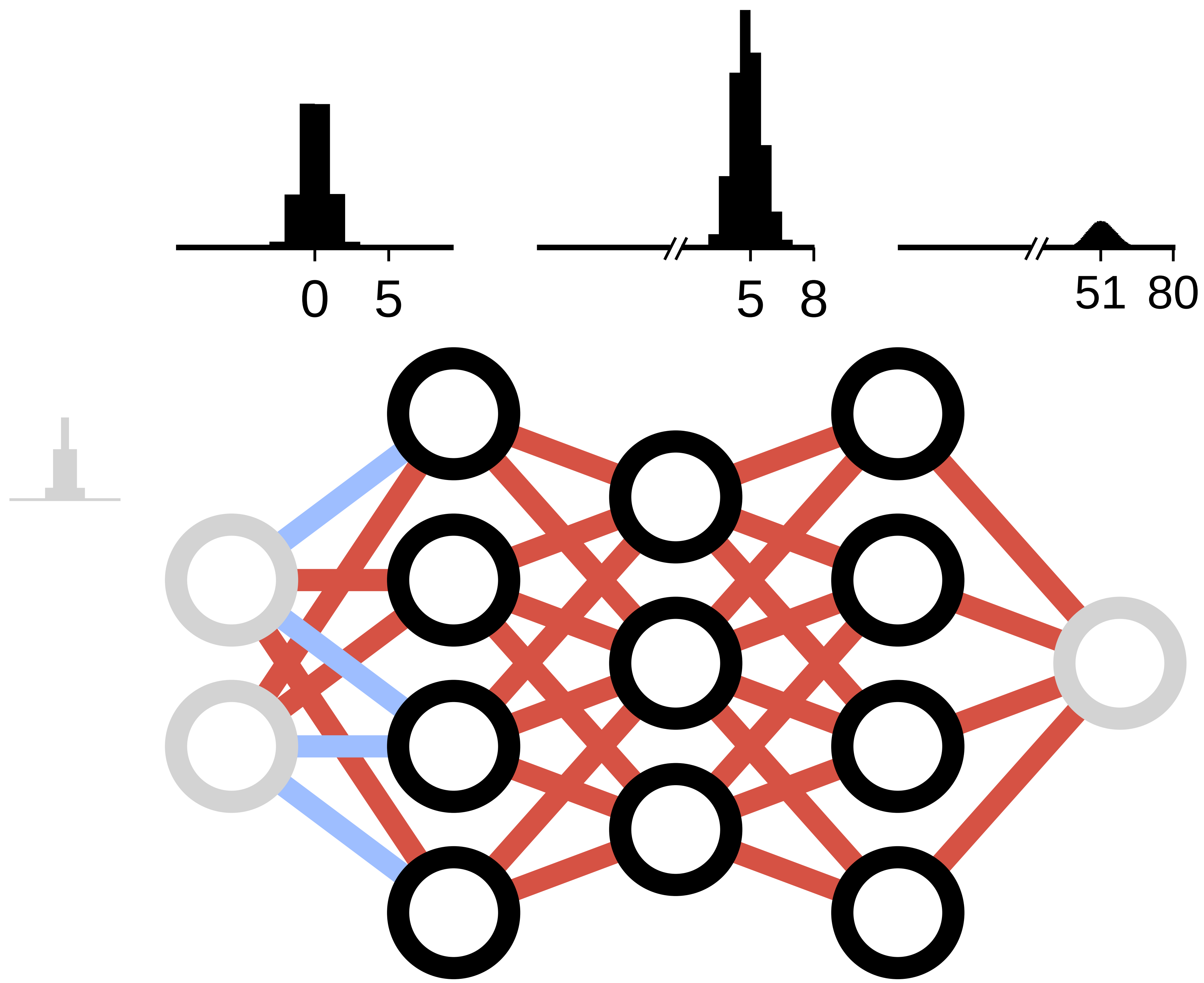

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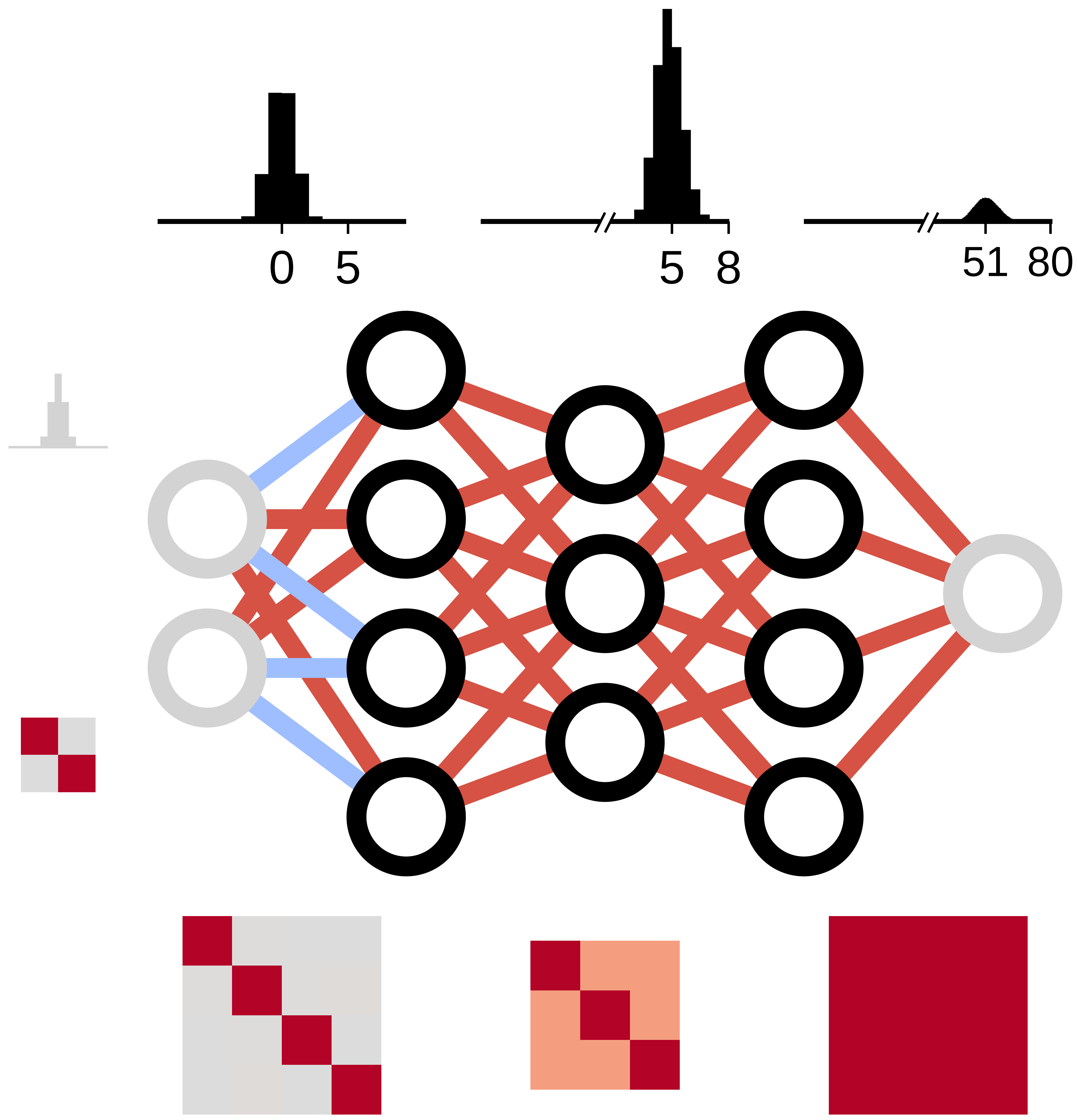

PRINCIPLED INITIALISATION FOR ICNN
FIXED POINT EQUATIONS

$$\mu_s = \mu_w \sum_k \mathbb{E}[x_k]$$

$$\sigma_s^2 = \sigma_w^2 \sum_k \mathbb{E}\left[x_k^2\right] + c_s$$

$$c_s = \mu_w^2 \sum_{k,k'} \mathrm{Cov}[x_k, x_{k'}]$$

PRINCIPLED INITIALISATION FOR ICNN
FIXED POINT EQUATIONS

$$\mu_s = \mu_w \sum_k \mathbb{E}[\phi(s_k^-)]$$
$$\sigma_s^2 = \sigma_w^2 \sum_k \mathbb{E}\left[\phi(s_k^-)^2\right] + c_s$$
$$c_s = \mu_w^2 \sum_{k,k'} \mathrm{Cov}[\phi(s_k^-), \phi(s_{k'}^-)]$$

PRINCIPLED INITIALISATION FOR ICNN
FIXED POINT EQUATIONS

$$\mu_s = \mu_w \sum_k \mathbb{E}[\phi(z_k)]$$

$$\sigma_s^2 = \sigma_w^2 \sum_k \mathbb{E}\left[\phi(z_k)^2\right] + c_s$$

$$c_s = \mu_w^2 \sum_{k,k'} \mathrm{Cov}[\phi(z_k), \phi(z_{k'})]$$

$$\begin{bmatrix} z_k \\ z_{k'} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_s \\ \mu_s \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & c_s \\ c_s & \sigma_s^2 \end{bmatrix}\right)$$

PRINCIPLED INITIALISATION FOR ICNN
FIXED POINT EQUATIONS

$$\mu_s = \mu_w \sum_k \mathbb{E}[\phi(z_k)] + \mu_b = 0$$

$$\sigma_s^2 = \sigma_w^2 \sum_k \mathbb{E}\left[\phi(z_k)^2\right] + c_s$$

$$c_s = \mu_w^2 \sum_{k,k'} \mathrm{Cov}[\phi(z_k), \phi(z_{k'})]$$

$$\begin{bmatrix} z_k \\ z_{k'} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & c_s \\ c_s & \sigma_s^2 \end{bmatrix}\right)$$

PRINCIPLED INITIALISATION FOR ICNN

FIXED POINT SOLUTIONS

$$\mu_b = -\mu_s$$

$$\sigma_w^2 = \frac{2}{N}(1 - \rho^*)$$

$$\mu_w = \sqrt{\rho^* f(\rho^*)^{-1}}$$

PRINCIPLED INITIALISATION FOR ICNN
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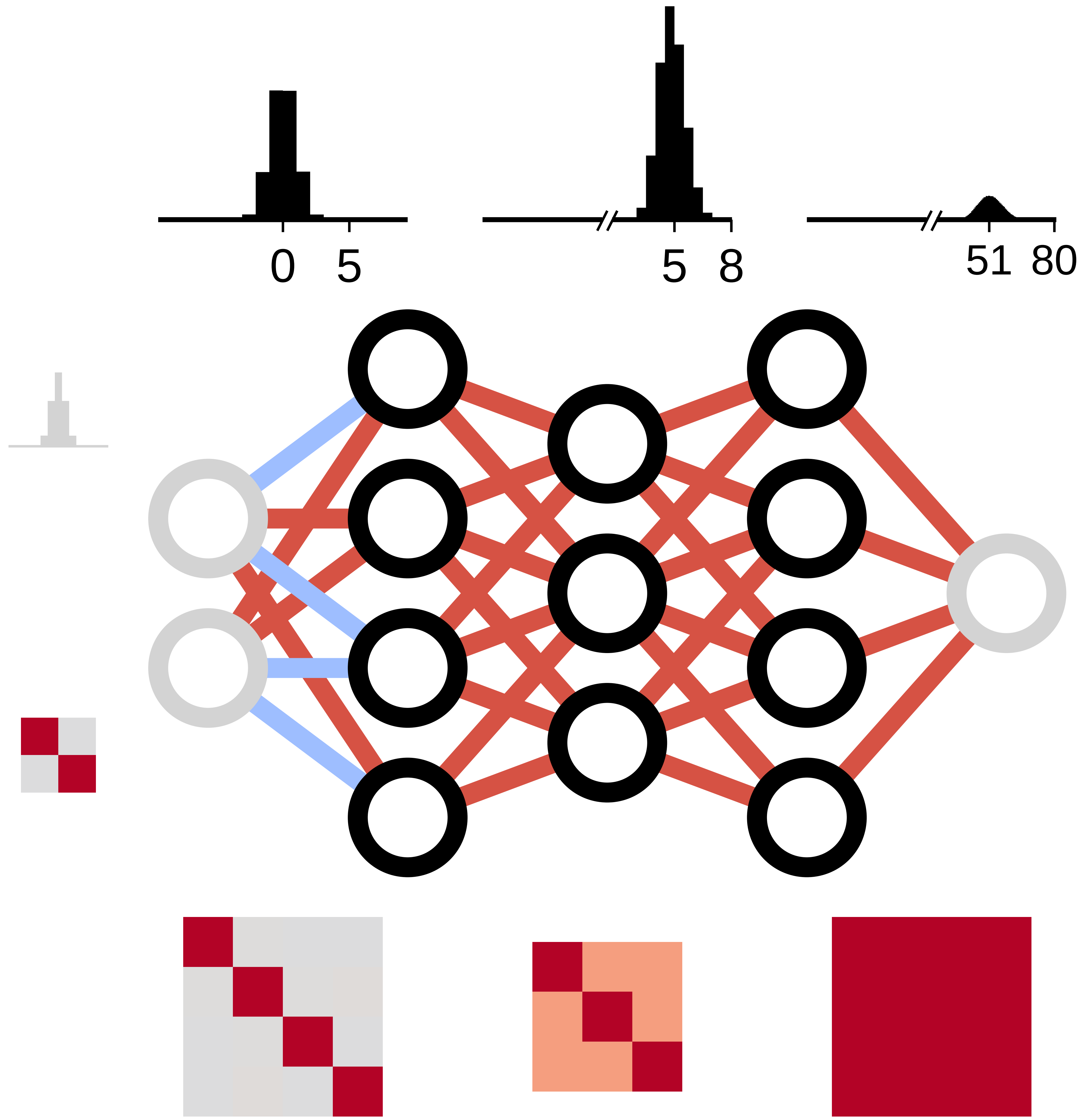

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PRINCIPLED INITIALISATION FOR ICNN
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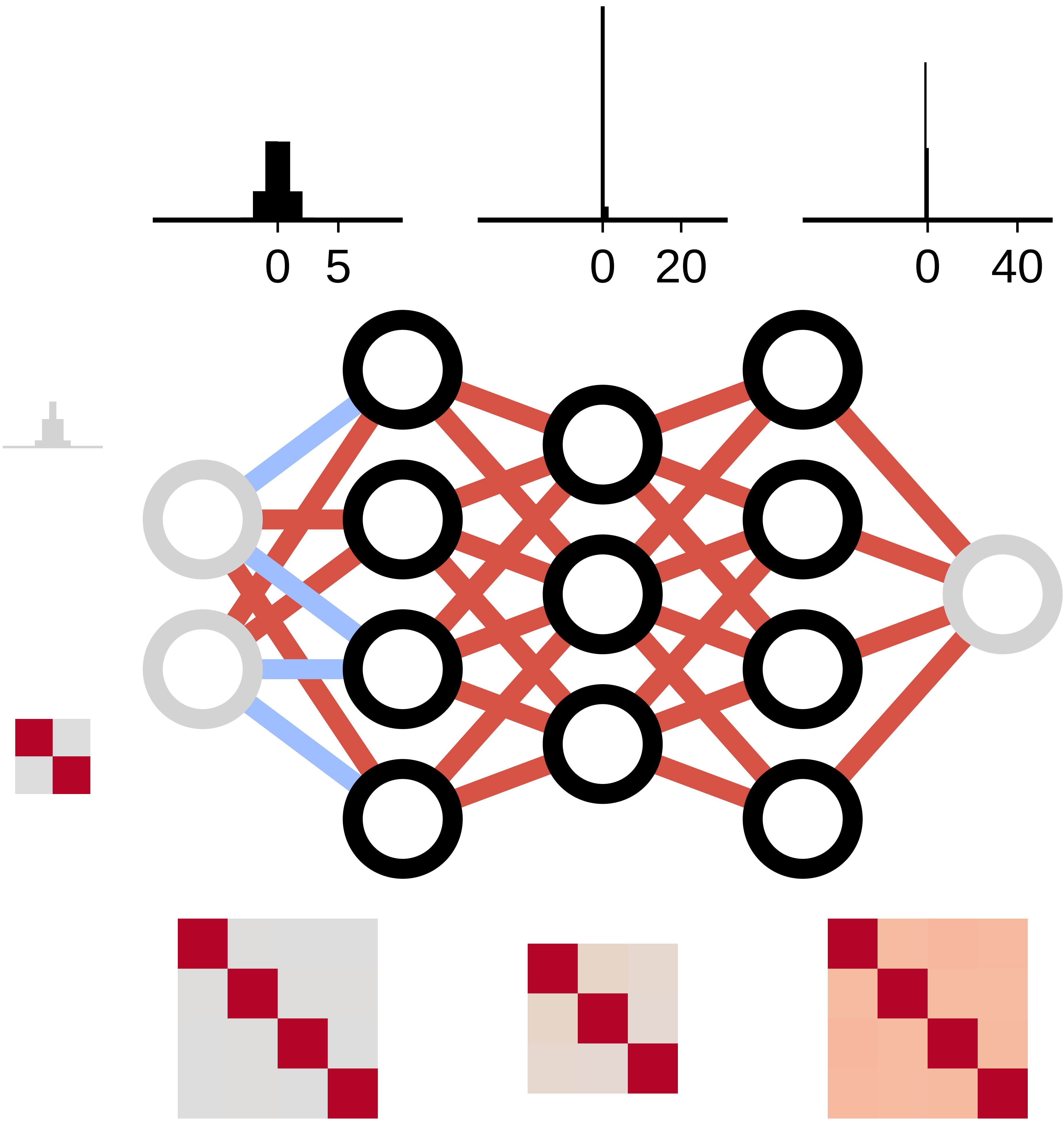
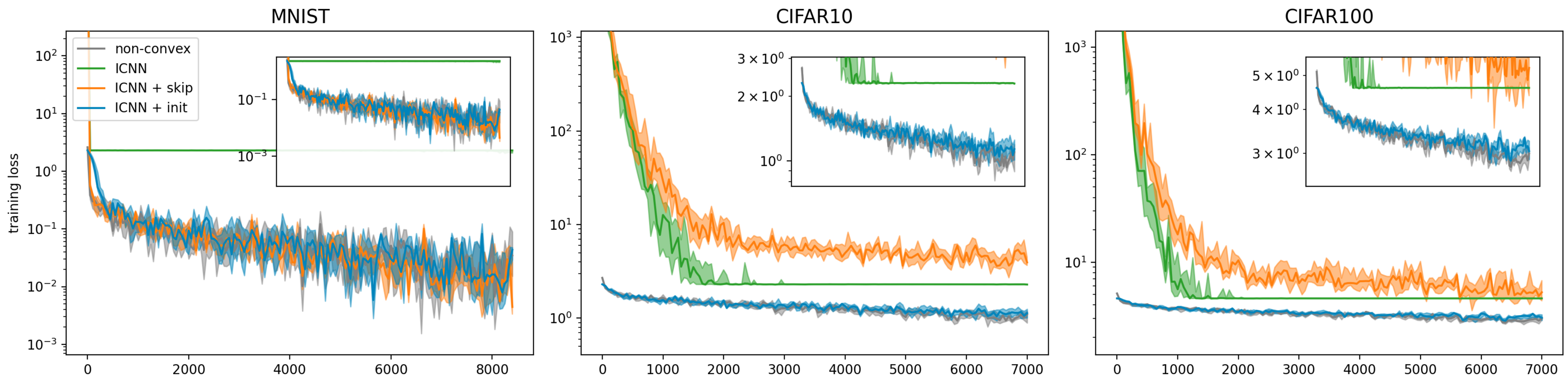


PRINCIPLED INITIALISATION FOR ICNN
LEARNING CURVES



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TL;DR: CONVEX INITIALISATION

- Generalisation of Signal Propagation
- Initialisation for ICNNs
- Faster Training of ICNNs ...
- ... without Skip-connections

code: https://github.com/ml-jku/convex-init

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