Robust Model Reasoning and Fitting via Dual Sparsity Pursuit

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Background

- Existing Geometric model fitting

  ✓ RANSAC-like

$$\max_{\theta, \mathcal{I} \subseteq \mathcal{X}} |\mathcal{I}|$$
$$\text{s.t.} \quad r(\mathbf{p}_i|\theta) \leq \epsilon, \quad \forall \mathbf{p}_i \in \mathcal{I}.$$

  Parmeters are essentially estimated with DLT solution with 8- or 4-Point Alg.

$$\min_{\theta} \|\mathbf{M}^\top \theta\|_2^2, \quad s.t. \quad \|\theta\|_2 = 1,$$

  ✓ Global optimization with robust loss

$$\min_{\theta} \|\mathbf{M}^\top \theta\|_1, \quad s.t. \quad \|\theta\|_2 = 1.$$

Background

- Geometric Constraints:-- Algebraic form $\mathbf{M}^\top \theta = \mathbf{0}$

Data Embedding: $\mathbf{m}_i = \Phi_{\mathcal{M}}(\mathbf{p}_i, \mathbf{p}_i')$

Parameter Vector: $\theta = vec(\mathcal{M}) \in \mathbb{R}^D$

Ask for predefining the model type !

$$F = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$$

$$\mathbf{p}_i'^\top \mathbf{F} \mathbf{p}_i = 0. \qquad \Phi_{\mathbf{F}}(\mathbf{p}_i, \mathbf{p}_i')^\top vec(\mathbf{F}) = 0$$

$$\Phi_{\mathbf{F}}(\mathbf{p}_i, \mathbf{p}_i')^\top = (u_i'u_i, u_i'v_i, u_i', v_i'u_i, v_i'v_i, v_i', u_i, v_i, 1)$$

$$H = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$$

$$[\mathbf{p}_i']_\times \mathbf{H} \mathbf{p}_i = \mathbf{0} \qquad \Phi_{\mathbf{H}}(\mathbf{p}_i, \mathbf{p}_i')^\top vec(\mathbf{H}) = \mathbf{0}$$

$$\Phi_{\mathbf{H}}(\mathbf{p}_i, \mathbf{p}_i')^\top = \begin{bmatrix} u_i & v_i & 1 & 0 & 0 & 0 & -u_i'u_i & -u_i'v_i & -u_i' \\ 0 & 0 & 0 & u_i & v_i & 1 & -v_i'u_i & -v_i'v_i & -v_i' \end{bmatrix}$$

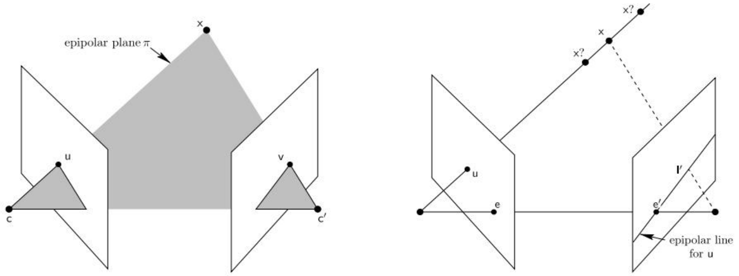
$$H_A = \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{p}_i' = \mathbf{A}\mathbf{p}_i, \qquad \Phi_{\mathbf{A}}(\mathbf{p}_i, \mathbf{p}_i')^\top vec(\mathbf{A}) = \mathbf{0}$$

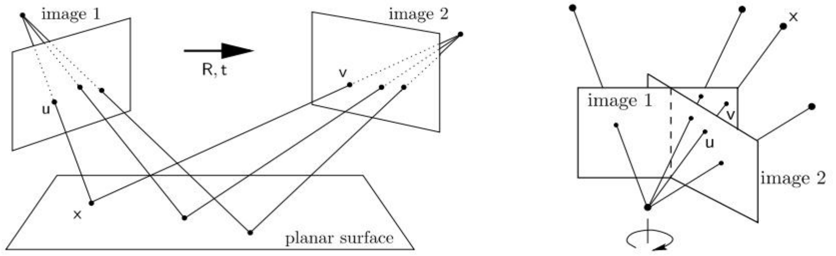
$$\Phi_{\mathbf{A}}(\mathbf{p}_i, \mathbf{p}_i')^\top = \begin{bmatrix} u_i & v_i & 1 & 0 & 0 & 0 & -u_i' \\ 0 & 0 & 0 & u_i & v_i & 1 & -v_i' \end{bmatrix}$$

Motivation & New Insight

- Model type varies a lot, and is tricky to predefine

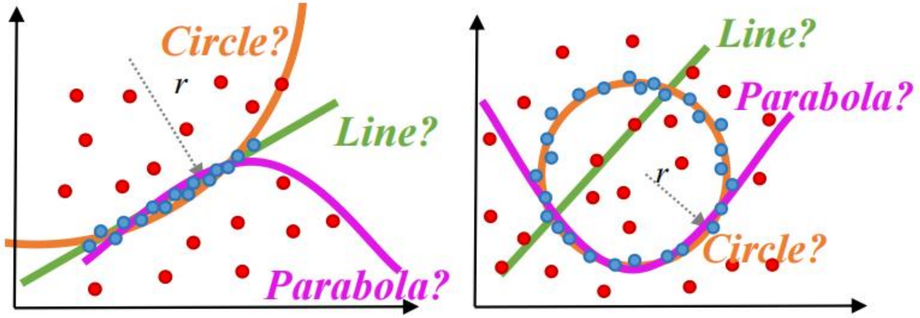


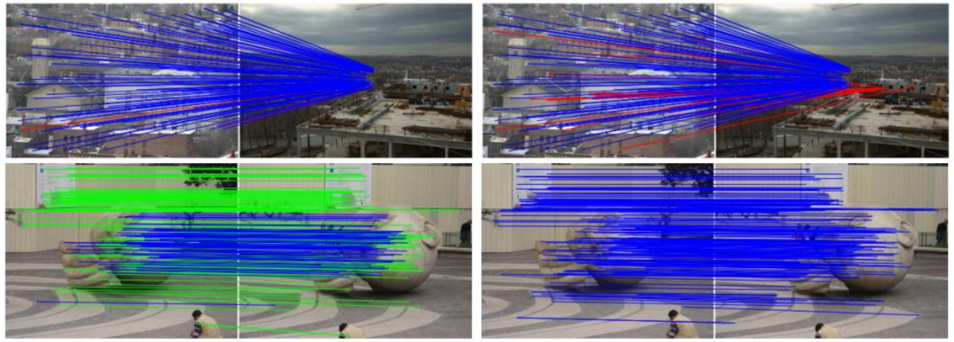
Fundamental Model: F/E



Homography Model: H

- A wrong Model would produce failed estimation





(a) Estimate *Homography* Model (b) Estimate *Fundamental* Model

Simultaneously solve:

i) Outlier Rejection, ii) Model Reasoning, iii) Parameter Estimation

Problem Formulation

- Sparse Subspace Recovery （SSR） Theory

  Recovering multiple sparse hyperplanes under an over embedded data space

$$\widetilde{\Phi}(\mathbf{p}_i, \mathbf{p}'_i)^\top \Psi(\mathcal{M}) = \mathbf{0}, \quad \mathcal{M} \in \{\mathbf{F}, \mathbf{H}, \mathbf{A}\}$$

Common Data Embed. Model Embed.

- *e.g.* Line fitting:

$$\Phi_{Line}(x_i, y_i) = [x_i, y_i, 1]$$

Under SSR :

Over Embedding $\Phi_{Ellipse}(x_i, y_i) = [x_i^2, y_i^2, x_i, y_i, 1]^\top$

Sparse Subspace $\theta = [0, 0, a, b, c]^\top$

	x_i^2	y_i^2	x_i	y_i	1
Line			✓	✓	✓
Parabola	✓		✓	✓	✓
Ellipse	✓	✓	✓	✓	✓

Problem Formulation

- SSR for Two-View Geometry

Model (req.)	DLT (data & model type:D,d,r)	DSP for known model fitting (data & model type:D,d,r)	DSP for unknown model fitting (data type:D)
Fund.	$\mathbf{m}_i^\top = (u'_i\mathbf{p}_i^\top, v'_i\mathbf{p}_i^\top, \mathbf{p}_i^\top)_{1\times 9}$ $\mathbf{x} = vec(\mathbf{F})$ $\mathbf{M}^\top\mathbf{x} = \mathbf{0}_{N\times 1}, G = (9,8,1,0)$	$\mathbf{m}_i^\top = (u'_i\mathbf{p}_i^\top, v'_i\mathbf{p}_i^\top, \mathbf{p}_i^\top)_{1\times 9}$ $\mathbf{x} = vec(\mathbf{F})$ $\mathbf{M}^\top\mathbf{x} = \mathbf{0}_{N\times 1}, G = (9,8,1,0)$	$\mathbf{m}_i^\top = (u'_i\mathbf{p}_i^\top, v'_i\mathbf{p}_i^\top, \mathbf{p}_i^\top)_{1\times 9}$ $\mathbf{x} = vec(\mathbf{F})$ $\mathbf{M}^\top\mathbf{x} = \mathbf{0}_{N\times 1}, G = (9,8,1,0)$
Homo.	$\mathbf{m}_i^\top = \begin{bmatrix} \mathbf{p}_i^\top, \mathbf{0}_{1\times 3}, -u'_i\mathbf{p}_i^\top \\ \mathbf{0}_{1\times 3}, \mathbf{p}_i^\top, -v'_i\mathbf{p}_i^\top \end{bmatrix}_{2\times 9}$ $\mathbf{x} = vec(\mathbf{H})$ $\mathbf{M}^\top\mathbf{x} = \mathbf{0}_{2N\times 1}, G = (9,8,1,0)$	$\mathbf{m}_i^\top = (u'_i\mathbf{p}_i^\top, v'_i\mathbf{p}_i^\top, \mathbf{p}_i^\top)_{1\times 9}$ $\mathbf{X} = \begin{bmatrix} \mathbf{0}_{1\times 3}, \mathbf{h}_3^\top, -\mathbf{h}_2^\top \\ -\mathbf{h}_3^\top, \mathbf{0}_{1\times 3}, \mathbf{h}_1^\top \\ \mathbf{h}_2^\top, -\mathbf{h}_1^\top, \mathbf{0}_{1\times 3} \end{bmatrix}^\top$ $\mathbf{M}^\top\mathbf{X} = \mathbf{0}_{N\times 3}, G = (9,6,3,3)$	$\mathbf{m}_i^\top = (u'_i\mathbf{p}_i^\top, v'_i\mathbf{p}_i^\top, \mathbf{p}_i^\top)_{1\times 9}$ $\mathbf{X} = \begin{bmatrix} \mathbf{0}_{1\times 3}, \mathbf{h}_3^\top, -\mathbf{h}_2^\top \\ -\mathbf{h}_3^\top, \mathbf{0}_{1\times 3}, \mathbf{h}_1^\top \\ \mathbf{h}_2^\top, -\mathbf{h}_1^\top, \mathbf{0}_{1\times 3} \end{bmatrix}^\top$ $\mathbf{M}^\top\mathbf{X} = \mathbf{0}_{N\times 3}, G = (9,6,3,3)$
Affine	$\mathbf{m}_i^\top = \begin{bmatrix} \mathbf{p}_i^\top, \mathbf{0}_{1\times 3}, -u'_i \\ \mathbf{0}_{1\times 3}, \mathbf{p}_i^\top, -v'_i \end{bmatrix}_{2\times 7}$ $\mathbf{x} = vec(\mathbf{A})$ $\mathbf{M}^\top\mathbf{x} = \mathbf{0}_{2N\times 1}, G = (7,6,1,0)$	$\mathbf{m}_i^\top = (u_i, v_i, u'_i, v'_i, 1)_{1\times 5}$ $\mathbf{X} = \begin{bmatrix} a_{11}, a_{12}, -1, 0, a_{13} \\ a_{21}, a_{22}, 0, -1, a_{23} \end{bmatrix}^\top$ $\mathbf{M}^\top\mathbf{X} = \mathbf{0}_{N\times 2}, G = (5,3,2,1)$	$\mathbf{m}_i^\top = (u'_i\mathbf{p}_i^\top, v'_i\mathbf{p}_i^\top, \mathbf{p}_i^\top)_{1\times 9}$ $\mathbf{X} = \begin{bmatrix} 0, 0, a_{11}, 0, 0, a_{12}, -1, 0, a_{13} \\ 0, 0, a_{21}, 0, 0, a_{22}, 0, -1, a_{23} \end{bmatrix}^\top$ $\mathbf{M}^\top\mathbf{X} = \mathbf{0}_{N\times 2}, G = (9,7,2,5)$

Common Data Embed.

Model Embed.

Pros: Avoid exact data embed., directly obtain model type from solution X

Problem Formulation

- Known Model Fitting for Clean Data

$$\min_{\mathbf{X}\in\mathbb{R}^{D\times r}} \quad \|\mathbf{X}\|_0, \qquad s.t. \quad \mathbf{M}^\top\mathbf{X}=\mathbf{0}, \quad \mathrm{rank}(\mathbf{X})=r,$$

- Unknown Model Fitting with *Noises* ***G*** & *Outliers* ***E***

$$\min_{\mathbf{X},\mathbf{E},r} \quad \overbrace{\frac{1}{2}\|\mathbf{G}\|_F^2}^{\text{Gaussian Noise}\downarrow} + \lambda\overbrace{\|\mathbf{X}\|_0}^{\text{Model Sparsity}\downarrow} + \gamma\overbrace{\|\mathbf{E}\|_{2,0}}^{\text{Outlier Entry}\downarrow} - \tau\overbrace{\mathrm{rank}(\mathbf{X})}^{\text{Basis Number}\uparrow},$$

$$s.t. \quad \mathbf{M}^\top\mathbf{X}-\mathbf{G}-\mathbf{E}=\mathbf{0}, \quad \underbrace{\boxed{\mathbf{X}^\top\mathbf{X}=\mathbf{I}_{r\times r},}}_{\text{Orthogonal Constraint}}$$

Solution

- Convex Approximation : L_0 --> L_1

$$\min_{\mathbf{X},\mathbf{E},r} \frac{1}{2}\|\mathbf{M}^\top\mathbf{X}-\mathbf{E}\|_F^2+\lambda\|\mathbf{X}\|_1+\gamma\|\mathbf{E}\|_{2,1}-\boxed{\tau\mathrm{rank}(\mathbf{X})},$$
$$s.t.\ \|\mathbf{x}_i\|_2=1,\quad \mathbf{x}_i^\top\mathbf{x}_j=0,\ \forall\, i,j=1,2,...,r,\ \ i\neq j.$$

- Problem Conversion

Progressively estimate a **new sparse basis** orthomotric to all given bases **B** up to $\mathcal{L}(\mathbf{M},\hat{\mathbf{x}}_i,\hat{\mathbf{e}}_i)<\tau$ not holds

$$\min_{\mathbf{x},\mathbf{e}}\mathcal{L}(\mathbf{M},\mathbf{x},\mathbf{e})=\frac{1}{2}\|\mathbf{M}^\top\mathbf{x}-\mathbf{e}\|_2^2+\lambda\|\mathbf{x}\|_1+\gamma\|\mathbf{e}\|_1,$$
$$s.t.\ \|\mathbf{x}\|_2=1,\ \mathbf{x}^\top\mathbf{y}=0,\ \forall\mathbf{y}\in\mathbf{B}.$$

Solution: *Alternative Optimization*

- Given $\boldsymbol{x}^{k-1}$ Update $\boldsymbol{e}^{k}$: standard Threshold Shrinkage Operation

$$\mathbf{e}^k = \mathbf{M}^\top \mathbf{x}^{k-1} - \gamma \mathrm{sgn}(\mathbf{e}^k) = \mathcal{T}_\gamma(\mathbf{M}^\top \mathbf{x}^{k-1})$$

$$\mathcal{T}_\lambda(q) = \begin{cases} q-\lambda, & q > \lambda, \\ q+\lambda, & q < -\lambda, \\ 0, & \text{else.} \end{cases}$$

- Given $\boldsymbol{e}^{k}$ Update $\boldsymbol{x}^{k}$:

$$\mathbf{x}^k = \arg\min_{\mathbf{x}} \left\{ \frac{L}{2} \|\mathbf{x} - (\mathbf{x}^{k-1} - \frac{1}{L}\nabla f(\mathbf{x}^{k-1}))\|_2^2 + \lambda \|\mathbf{x}\|_1 \right\},$$

$$\mathbf{x}^k = \mathcal{T}_{\frac{\lambda}{L}}(\mathbf{q}_L(\mathbf{x}^{k-1})), \;\; \mathbf{q}_L(\mathbf{x}^{k-1}) = \mathbf{x}^{k-1} - \frac{1}{L}\nabla f(\mathbf{x}^{k-1}).$$

- Constraint Projection:

$$\mathbf{x} \leftarrow \frac{\mathbf{x}}{\|\mathbf{x}\|_2} \quad + \quad \mathbf{x} \leftarrow (\mathbf{I} - \mathbf{B}\mathbf{B}^\top)\mathbf{x}.$$

Sphere Proj. Orthogonal Proj.

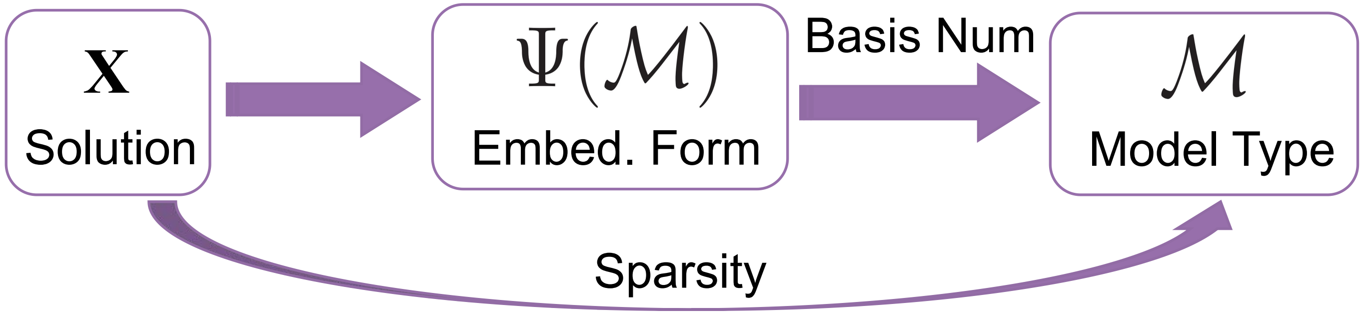
Implementation

- How to reason out the model type?

Back to our formulation:

$$\underbrace{\widetilde{\Phi}(\mathbf{p}_i, \mathbf{p}'_i)}_{\text{Common Data Embed.}}{}^\top \underbrace{\Psi(\mathcal{M})}_{\text{Model Embed.}} = \mathbf{0}, \quad \mathcal{M} \in \{\mathbf{F}, \mathbf{H}, \mathbf{A}\}$$

Given the solved $\mathbf{X}$, reason out the model type--based on the *Sparsity* and *Basis Number*



Implementation

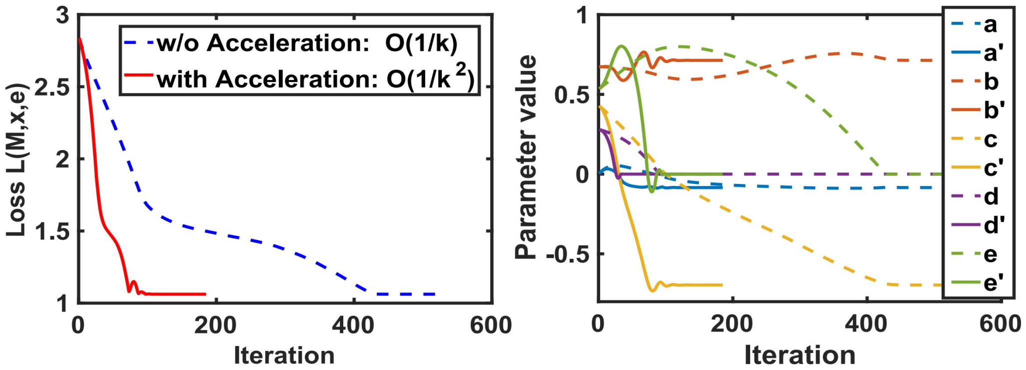
- How to reason out the model type?

2D points

	x_i^2	y_i^2	x_i	y_i	1
Line			✓	✓	✓
Parabola	✓		✓	✓	✓
Ellipse	✓	✓	✓	✓	✓

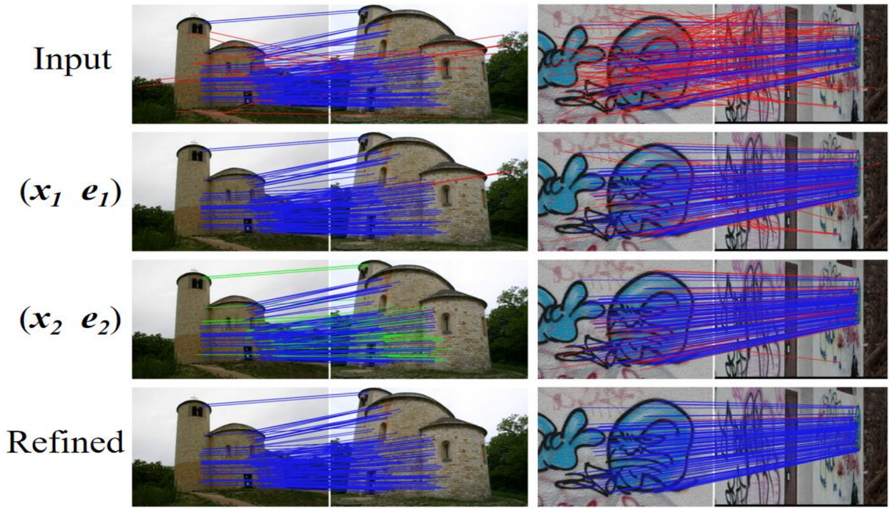
Common Embed. $\mathbf{m}_i^\top = [x_i^2, y_i^2, x_i, y_i, 1]$

Line Fitting $\theta = [0, 0, a, b, c]^\top$

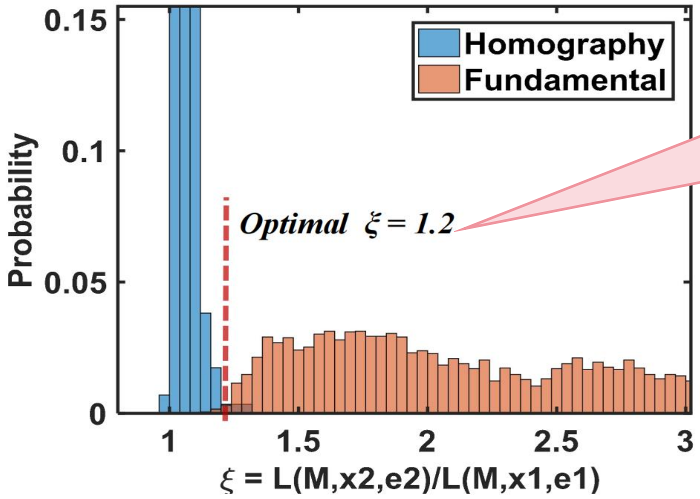


DSP for *Line* fitting from noisy 2D points

Two-view geometry



Fund. Homo.



Best Choice to distinguish H and F model

Statistic Results on All Real Data

Experiments

- Model Reasoning on Synthesized Datasets

EAS (TPAMI'22) + Selection criteria: AIC BIC GRIC

OR	Data / Model	AIC	BIC	GRIC	DSP (ours)
20%	F,100	100	62	100	100
	H,100	100	98	100	100
	A,100	100	96	100	100
50%	F,100	100	0	100	100
	H,100	100	95	100	100
	A,100	99	97	98	100
80%	F,100	85	0	93	100
	H,100	95	96	98	99
	A,100	96	92	95	98
	all	97.2	70.7	98.4	99.7

Select GRIC for Further Comparison

Experiments

- Unknown Model Fitting on Real Image Datasets

  * indicates using GRIC to select the best model

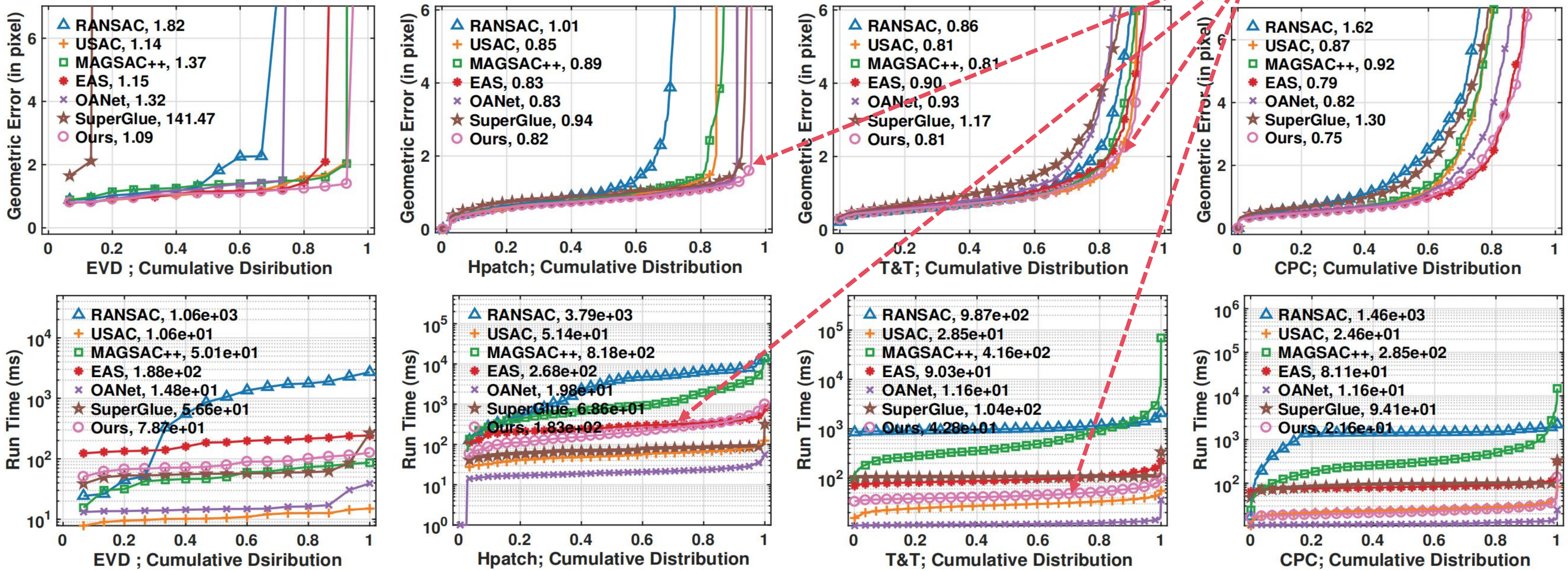
Data\Methods		RANSAC*	USAC*	MAGSAC++*	EAS*	OANet*	SuperGlue*	DSP (ours)
Fund.	*E_med*	0.8478	0.6545	<u>0.6260</u>	0.6900	0.7579	0.8926	**0.6017**
	Time(s)	0.5666	<u>0.0239</u>	0.3918	0.2037	**0.0152**	0.0721	0.0551
	FR	0.2925	<u>0.1992</u>	0.2077	0.2142	0.2632	0.2709	**0.1136**
Homo.	*E_med*	1.0428	0.8744	0.8978	<u>0.8472</u>	0.8480	0.9392	**0.8227**
	Time(s)	2.010	<u>0.0550</u>	1.3419	0.4463	**0.0342**	0.0716	0.2794
	FR	0.3021	0.2604	0.1424	0.0972	<u>0.0903</u>	0.1181	**0.066**
All	*E_med*	0.8794	0.6711	<u>0.6383</u>	0.7091	0.7670	0.9026	**0.6249**
	Time(s)	0.6638	<u>0.0260</u>	0.4558	0.22	**0.0130**	0.0721	0.0703
	FR	0.2932	0.2043	<u>0.2033</u>	0.2064	0.2515	0.2606	**0.1123**

Best Accuracy, Lowest Failure Rate, Competitive Running Time

Experiments

- Known Model Fitting (Predefine the Model Type)

Our DSP



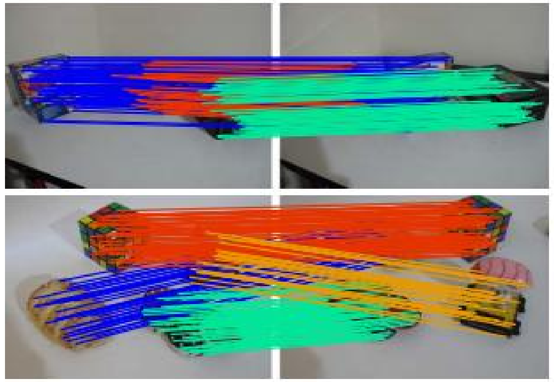
Homography: EVD，Hpatch

Fundamental: T&T，CPC

Lowest Geometric Error & Competitive Running Time

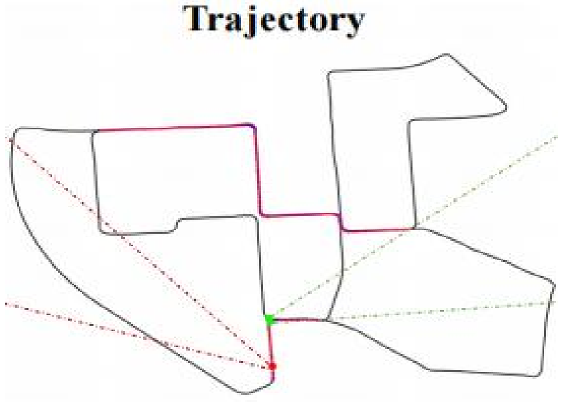
Applications

- Multi-model fitting:



	Tlink	RCMSA	RPA	MCT	MLink	RFM-SCAN	Ours
ME_ave	27.65	10.05	5.28	11.36	6.04	<u>2.63</u>	**1.64**
ME_med	27.88	6.087	4.35	1.21	4.22	<u>1.20</u>	**0**
RT(s)	1.95	2.16	10.84	6.44	9.72	**0.01**	<u>0.18</u>

- Loop Closure Detection:



	RSAC	USAC	MSAC++	EAS	OANet	Ours
K00	0.9112	0.9118	0.9105	0.9086	0.7910	**0.9162**
K02	0.7632	0.7796	0.7757	0.7664	0.6441	**0.7882**

Thanks!

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Project & Code & Demo

https://github.com/StaRainJ/DSP