

An Optimization-based Approach To Node Role Discovery in Networks

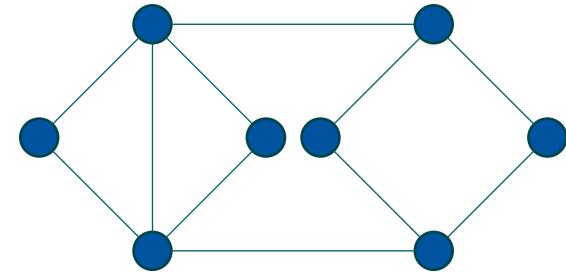
Approximating Equitable Partitions

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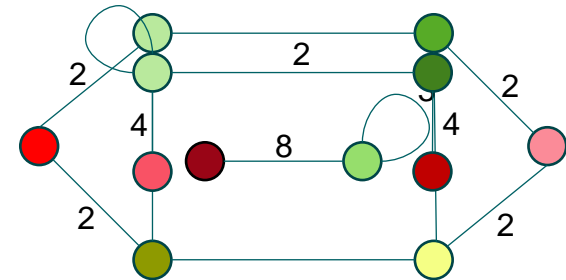
NeurIPS'23

What are roles?

- Low rank descriptors of network structure
 - Complementary to Community Detection
- Large number of different definitions:
 - Structural Equivalence
 - Regular Equivalence
 - Automorphic Equivalence
- Strongest Computationally feasible:
 - Equitable Partition



G



G^π

Equitable Partitions

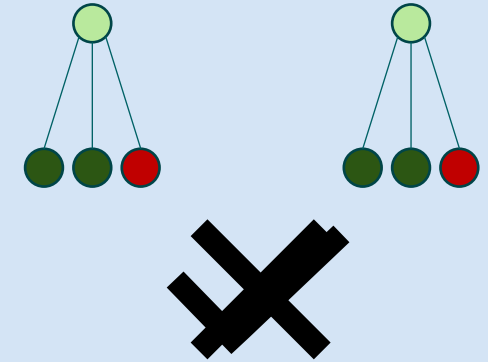
Definition: Equitable partition (EP).

A partition $\mathcal{C} = \{C_1, \dots, C_k\}$ is *equitable* if for all nodes $v, u \in C_i$:

$$|N(v) \cap C_j| = |N(u) \cap C_j|, \quad j \in \{1, \dots, k\}$$

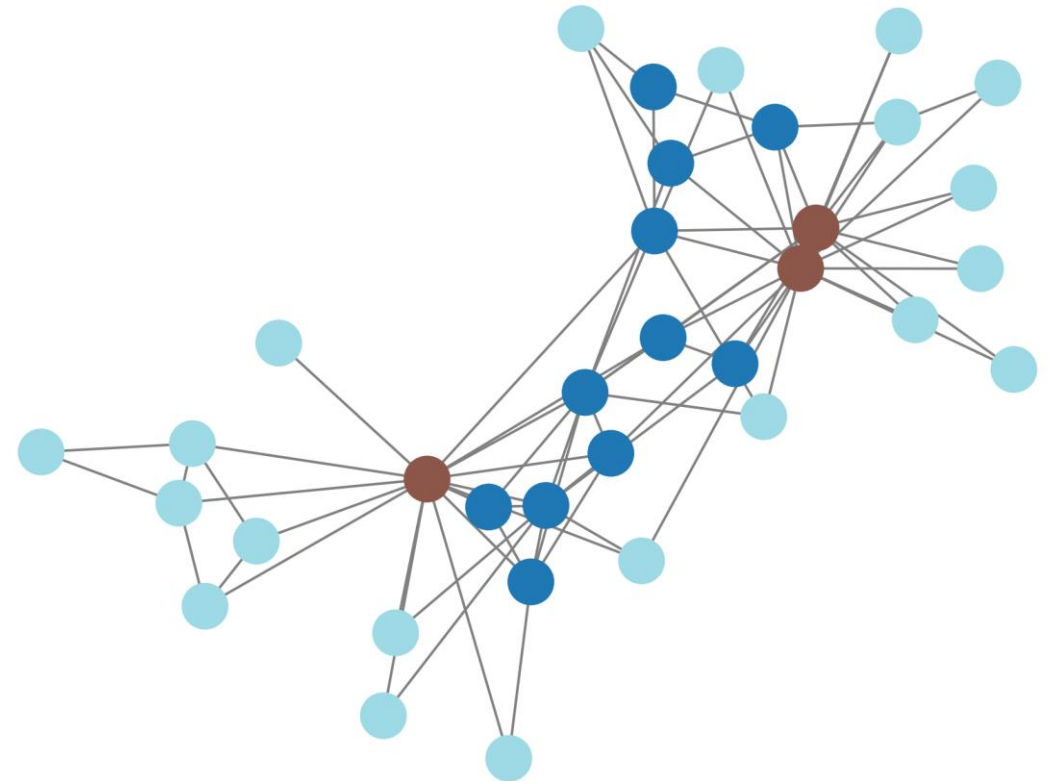
Or equivalently: Let $H \in \{0,1\}^{n \times k}$ with $H\mathbf{1} = \mathbf{1}$, s.t: $H_{v,j} = 1 \Leftrightarrow v \in C_j$, then

$$AH = HA^\pi$$



- Equitable partitions:
 - Describe the the global structure of a graph
 - Determine the behaviour of many (non-)linear dynamical systems
 - Centralities like PageRank, Diffusion-type dynamical systems, etc.
 - “Upper-bound” the expressive power of GNNs
 - Encode descriptive information about the surroundings of a node

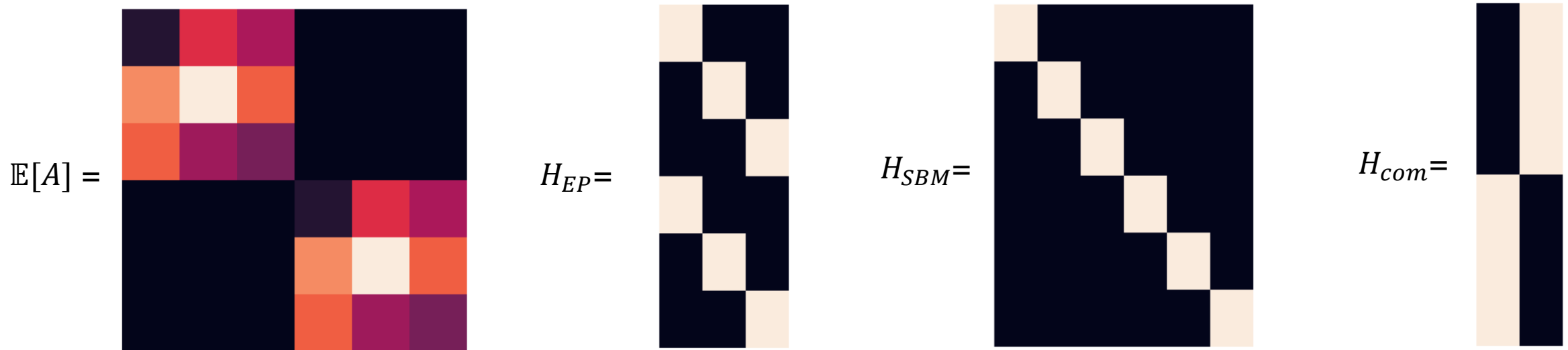
The problem with Equitable Partitions



The Fix

Definition: Stochastic Role.

Two nodes $u, v \in V$ have the same *stochastic role* if they are assigned to the same class in the EP of $\mathbb{E}[A]$.



Defining the Quality of a partition

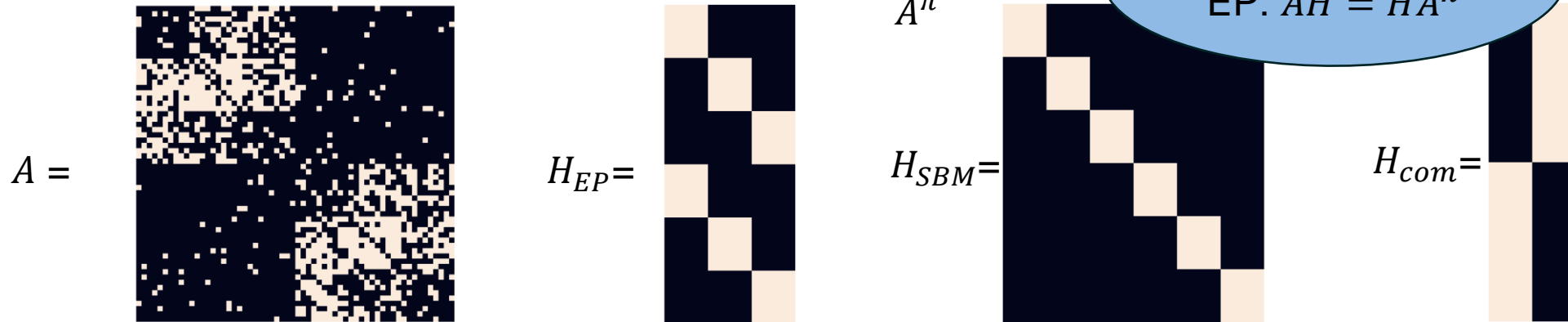
Definition: Short term cost:

For an adjacency matrix $A \in \mathbb{R}^{n \times n}$, a partition indicated by $H \in \{0,1\}^{n \times k}$, and a normalization $D = \text{diag}(\mathbb{1}H)$, the *short-term cost* is defined as:

$$\|AH - HD^{-1}H^T AH\|$$

A^π

Remember:
EP: $AH = HA^\pi$



Modularity	-0.08	0.05	<u>0.3</u>
Neg. Log-likelihood	1902	<u>1384</u>	1558
Short-term cost*k	<u>60.3</u>	115.8	78.6

Thanks for your attention

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