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A Curriculum-Training-Based Strategy for Distributing Collocation Points during Physics-Informed Neural Network Training

Setup

Reconstruct plasma environment around spacecraft trajectory

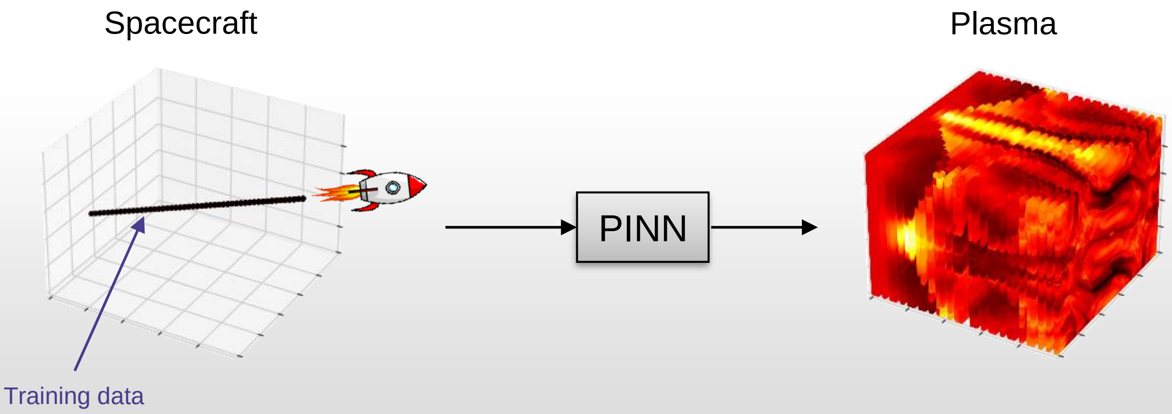
- PINN predicts 2D MHD solutions U_{net} given partial linear samples of the original data
- Approximated solution follows both a physical constraint (1) and a boundary data constraint (2)

(1) $$\frac{\partial \mathbf{U}_{net}}{\partial t} + F(\mathbf{U}_{net}, \frac{\partial \mathbf{U}_{net}}{\partial x}) + G(\mathbf{U}_{net}, \frac{\partial \mathbf{U}_{net}}{\partial y}) = 0$$

(2) $$\mathbf{U}_{net}(x_i, y_i, t_i) = \mathbf{U}_i^{st},\ i = 1 \dots N_d$$

for N_d coordinates of (x, y, t) in space-time

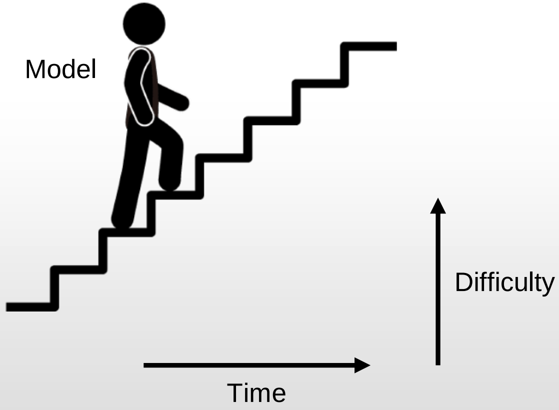
- F and G are the 2D MHD fluxes



Curriculum Strategies

Curriculum Learning

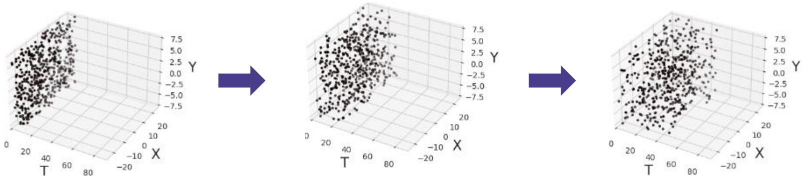
- Human-like learning
- Start easy
- Stepwise increase difficulty



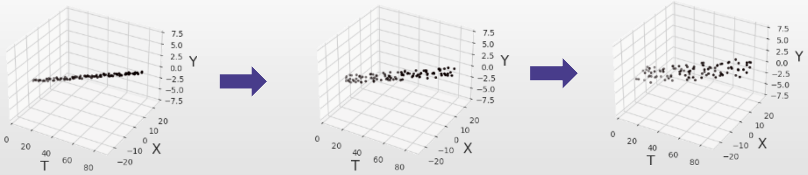
Approaches

Schedule collocation point distributions:

⟶ **Cuboid**: learn evolution over time by expanding a cuboid that covers the whole spatial domain over the time axis



⟶ **Cylinder**: easier predictions close to spacecraft trajectory; expand collocation point distribution in concentric bubbles around the trajectory data

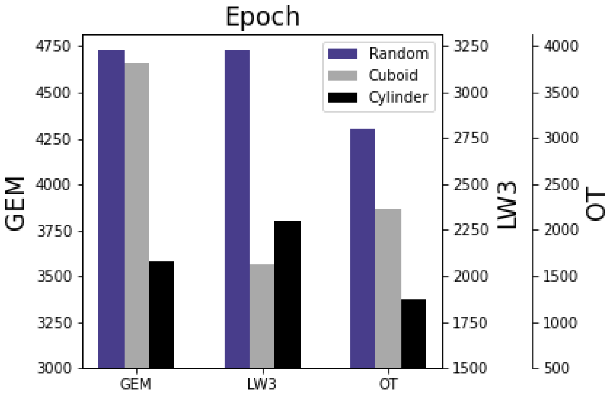


Results

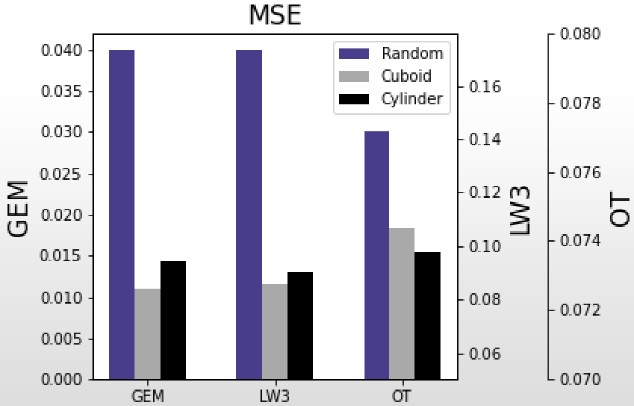
Compare to a randomly sampling baseline on three datasets:

1. An MHD reconnection benchmark (GEM)
2. A 2D Riemann Problem (LW3)
3. An MHD vortex designed to study turbulence (OT)

Convergence



Accuracy



Conclusion:

Scheduling the collocation point distributions significantly enhances PINN MHD reconstruction, simultaneously boosting accuracy and reducing convergence speed. However, we note that results depend on the scenario and the models‘ initializations.