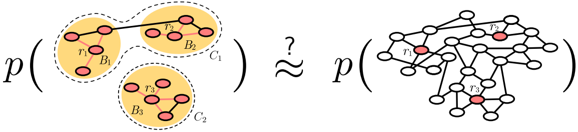
Learning on Random Balls is Sufficient for Estimating (Some) Graph Parameters



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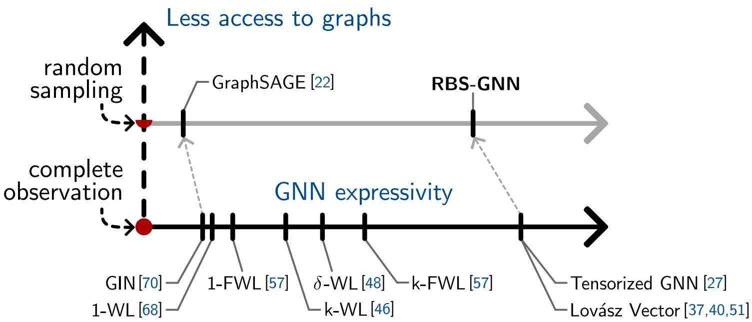
GNN Universality

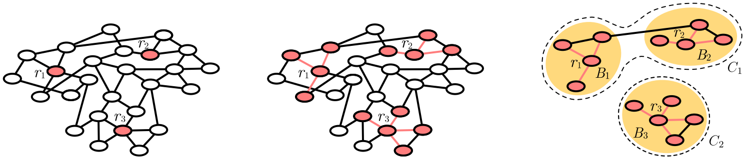


❗ Additional dimensionality to the discussion: What if we do not have complete observations over the graphs?

Citation numbers are consistent with our manuscript.

Computational Model

We study the following computational model, which can be found in large-scale systems (or methods like GraphSAGE).



Step 1: Sample random roots ▸ Step 2: Sample random neighbors ▸ Step 3: Get induced subgraphs and connected components

- **RBS-GNN** is defined to be the class of universal GNNs which takes Step 3 as input.
- Computational model defines a notion of *continuity* in the space of all graphs.

Main Result

Theorem 1 (RBS-GNN Universality)

If a graph parameter $p : \mathcal{G} \to \mathbb{R}$ is estimable (in the random neighborhood computational model), then it is estimable by a RBS-GNN.

This theorem connects GNN literature and graph proptery testing literature. Furthermore, we define the following distance:

$$d(G, H) = \sum_{r=1}^{\infty} 2^{-r} d_{TV}(z_r(G), z_r(H)). \quad (1)$$

Theorem 2 (Estimable iff Continuous)

Graph parameter p is estimable iff continuous w.r.t. the distance defined by Equation 1.

Example 3 (Estimable Parameters)

Edge density, triangle density, and the local clustering coefficient are estimable.

Example 4 (Non-estimable Parameters)

Connectivity, number of connected components, and min/max-degrees are non-estimable.

! Theorem 2 can be used to prove a parameter is estimable or not using continuity.

Theorem 5
Estimable functions are size-generalizable in the approximation-theoretic sense.

❗ Theorem 2 can also prove the main theorem of GraphSAGE:

Lemma 6
The mini-batch version of the GraphSAGE can estimate the local clustering coefficient.

❗ See our manuscript for other results on size-generalization by domain adaptation and non-universal RBS-GNN.

Thank you for listening!