

# Friendship paradox

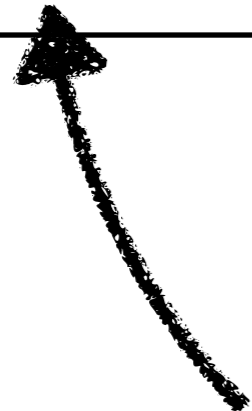
You have more friends than I do  
(at least on average)



A



B

A rectangular box representing a friend list entry. It contains a gray circular placeholder for a profile picture on the left and three horizontal lines for text on the right.A rectangular box representing a friend list entry. It contains a gray circular placeholder for a profile picture on the left and three horizontal lines for text on the right.A rectangular box representing a friend list entry. It contains a gray circular placeholder for a profile picture on the left and three horizontal lines for text on the right.A rectangular box representing a friend list entry. It contains a gray circular placeholder for a profile picture on the left and three horizontal lines for text on the right.

In which friend list am I likely to appear?

A



A rectangular box containing a gray circle on the left and three horizontal lines on the right, representing a friend list entry.

B



A rectangular box containing a gray circle on the left and three horizontal lines on the right, representing a friend list entry.

A rectangular box containing a gray circle on the left and three horizontal lines on the right, representing a friend list entry.

A rectangular box containing the 'kissing face with heart' emoji on the left and three horizontal lines on the right, representing a friend list entry.

**3 times more likely  
to appear in the  
B's friend list**

# Friendship paradox

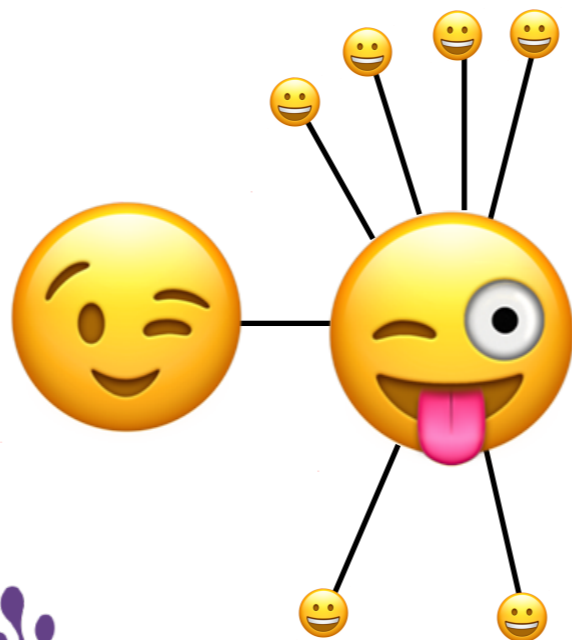
Your friends have more friends than you do  
(on average)

...because more friends someone has,  
more likely the someone appears in your friend list.



Can introduce *biases* in graph embeddings





# Residual2Vec: Debiasing graph embedding using random graphs



arXiv.org

<https://arxiv.org/abs/2110.07654>



 **Sadamori Kojaku**



**Jisung Yoon**

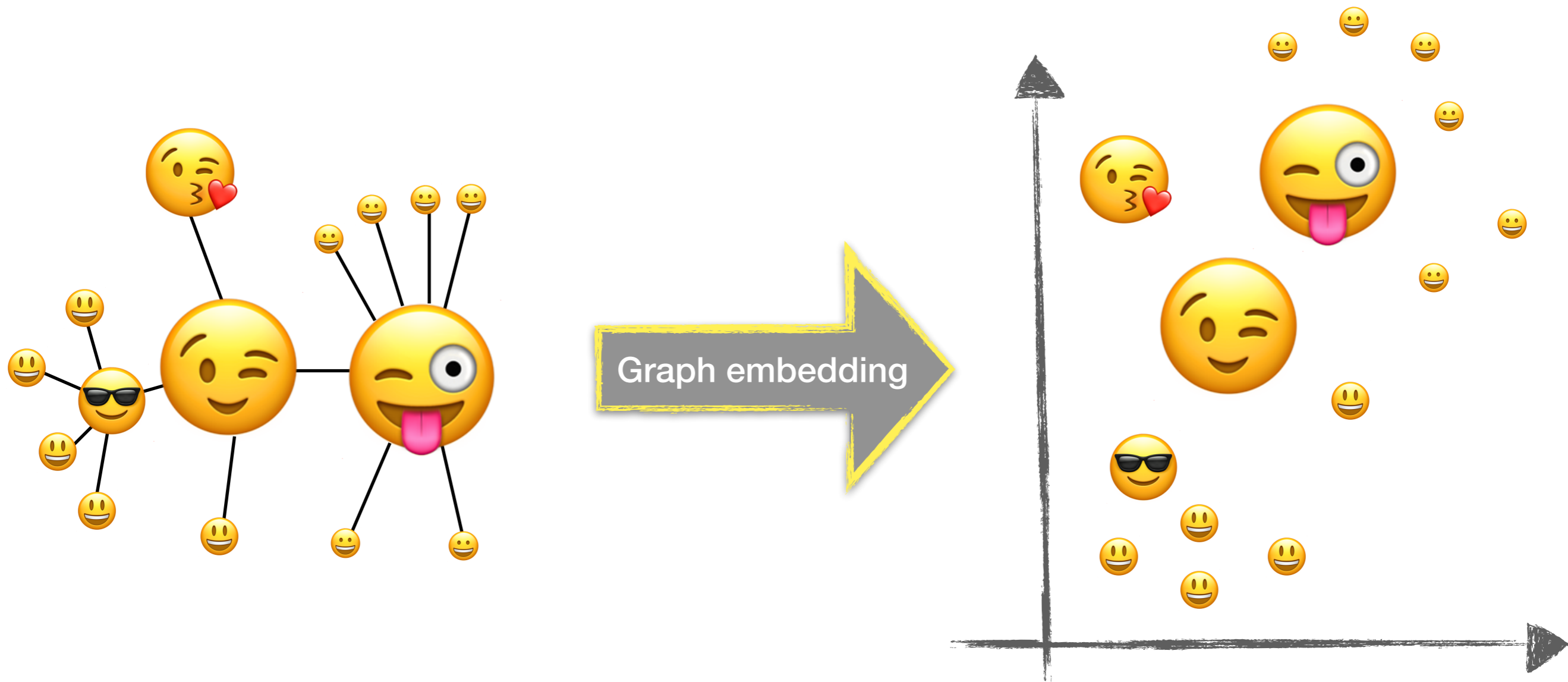


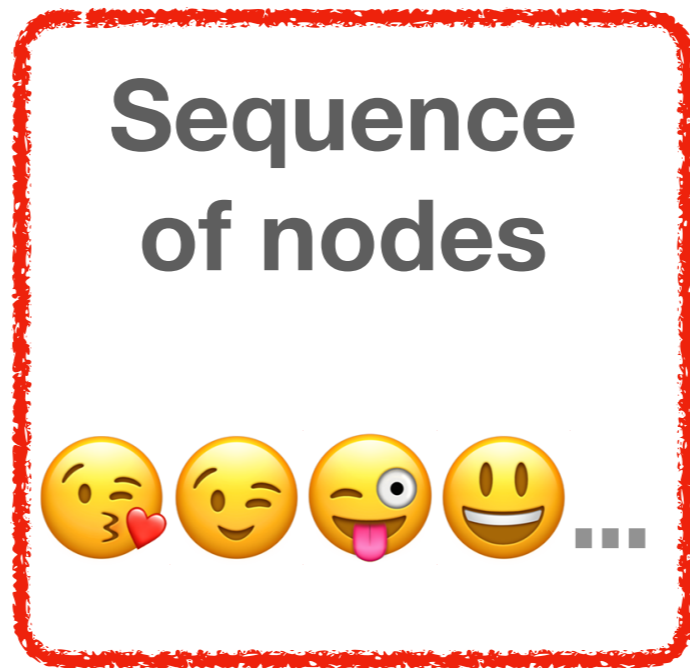
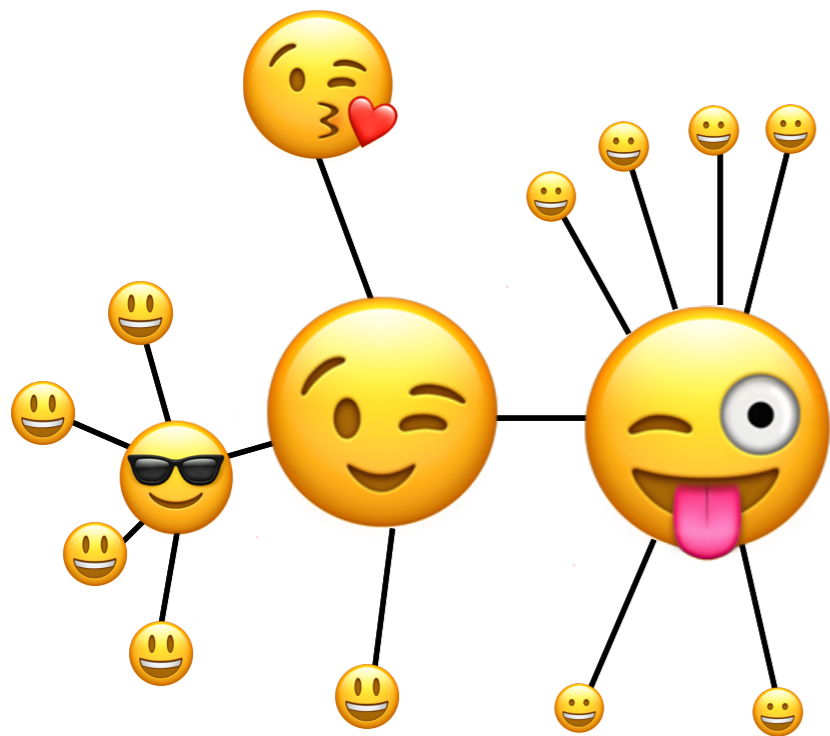
**Isabel Constantino**



**Yong-Yeol Ahn**

# Graph embedding





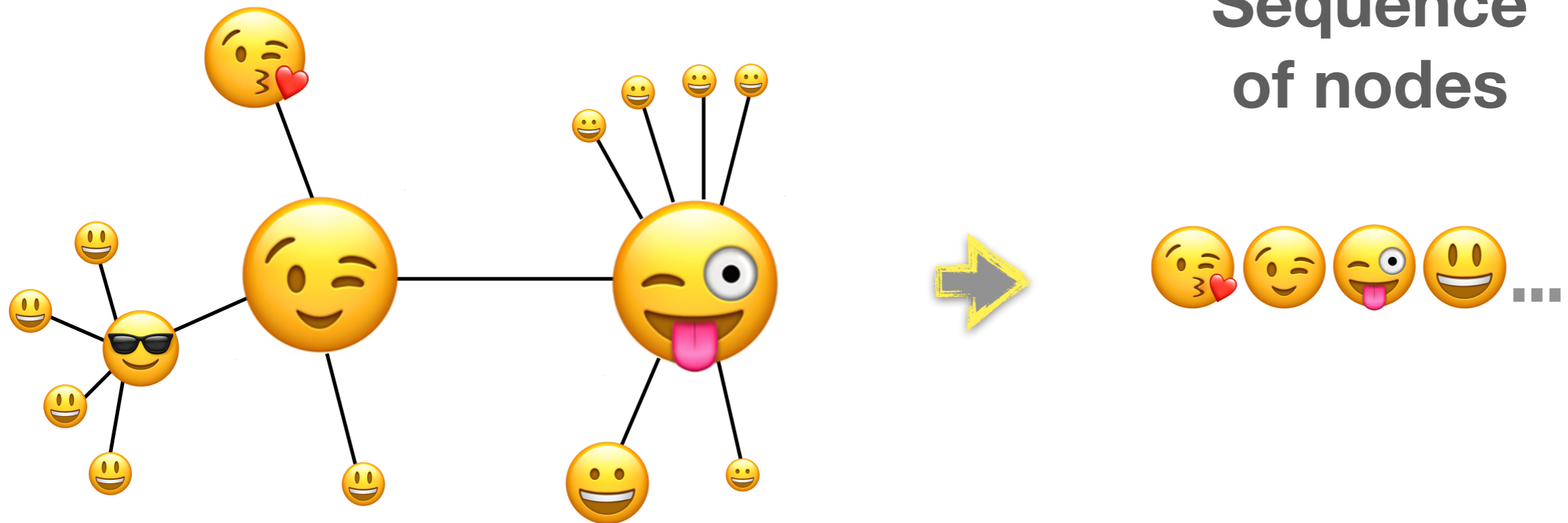
Embed by word2vec



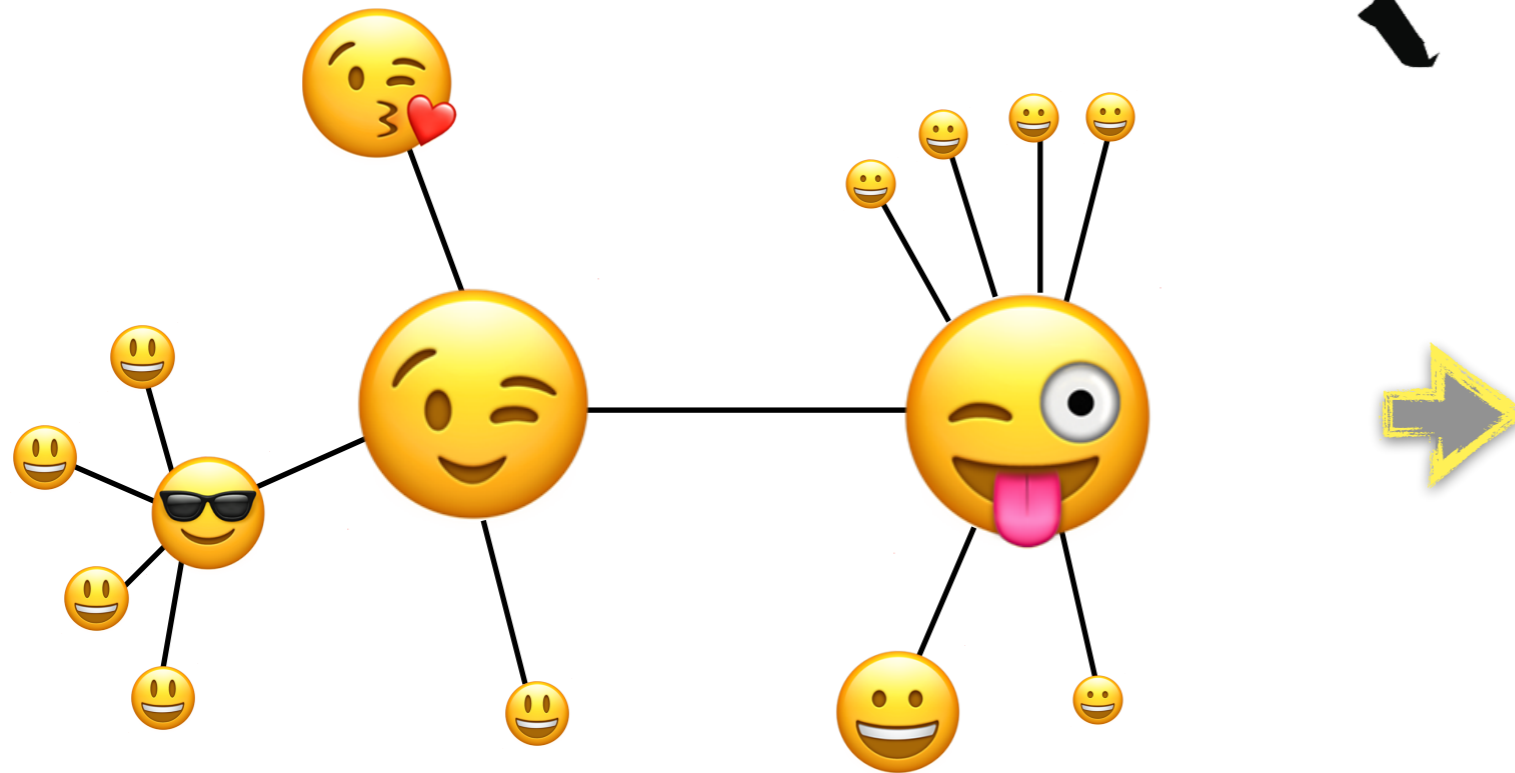
Word embedding

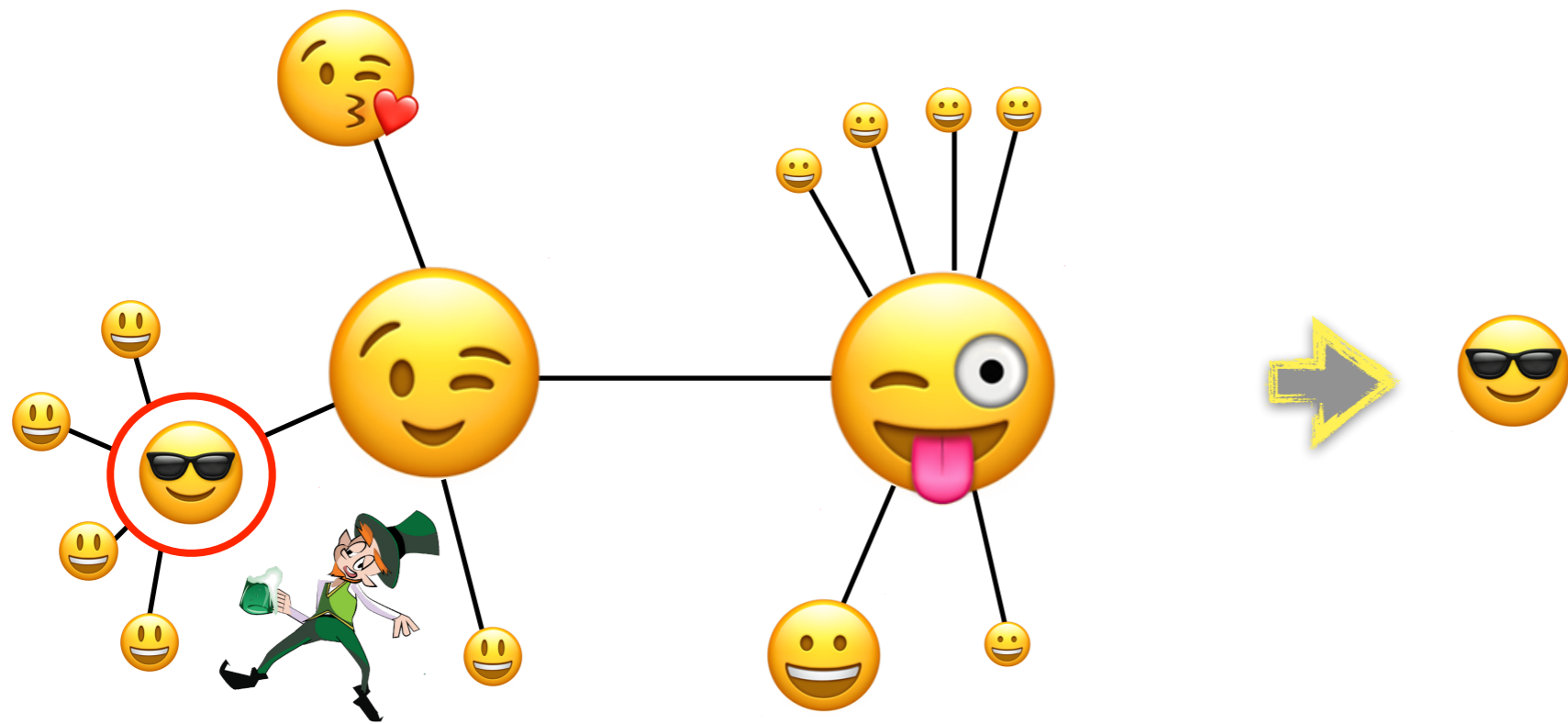
signal.  
binary code with which the present  
ls may take various forms, all of  
e property that the symbol (or  
'representing each number (or signi  
differs from the ones representi  
er and the next higher number (o  
litude) in only one digit (or puls  
Because this code in its primar  
built up from the conventional  
a sort of reflection process and l  
rms may in turn be built up fro  
form in similar fashion, the c  
, which has as yet no recognized  
ated in this specification and  
s the "reflected binary code."  
a receiver station, reflected bina

# How to generate the sequence from a graph?

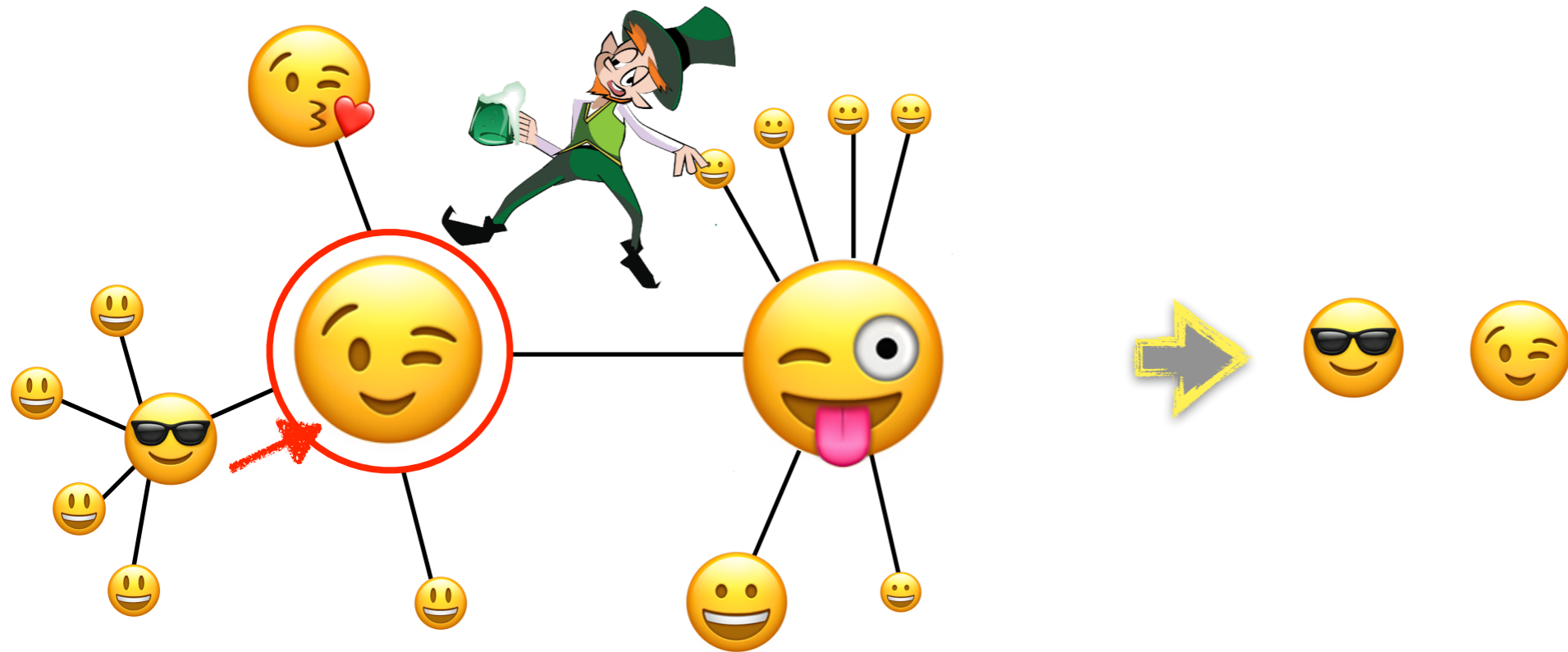


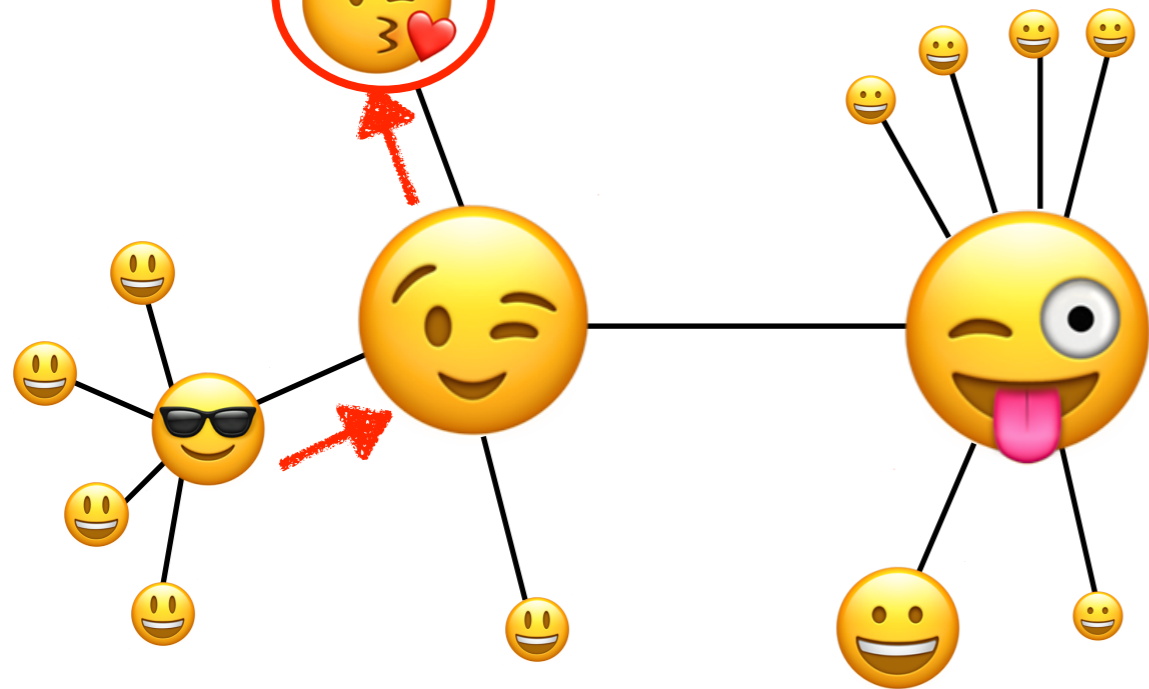
# Random walker



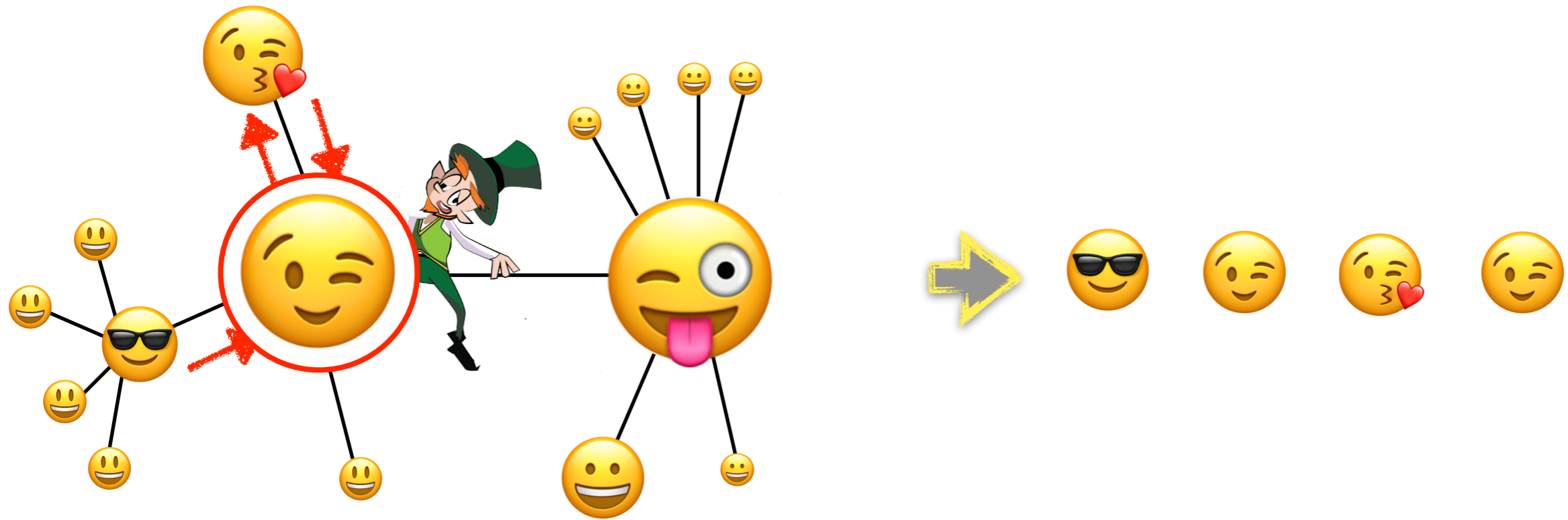


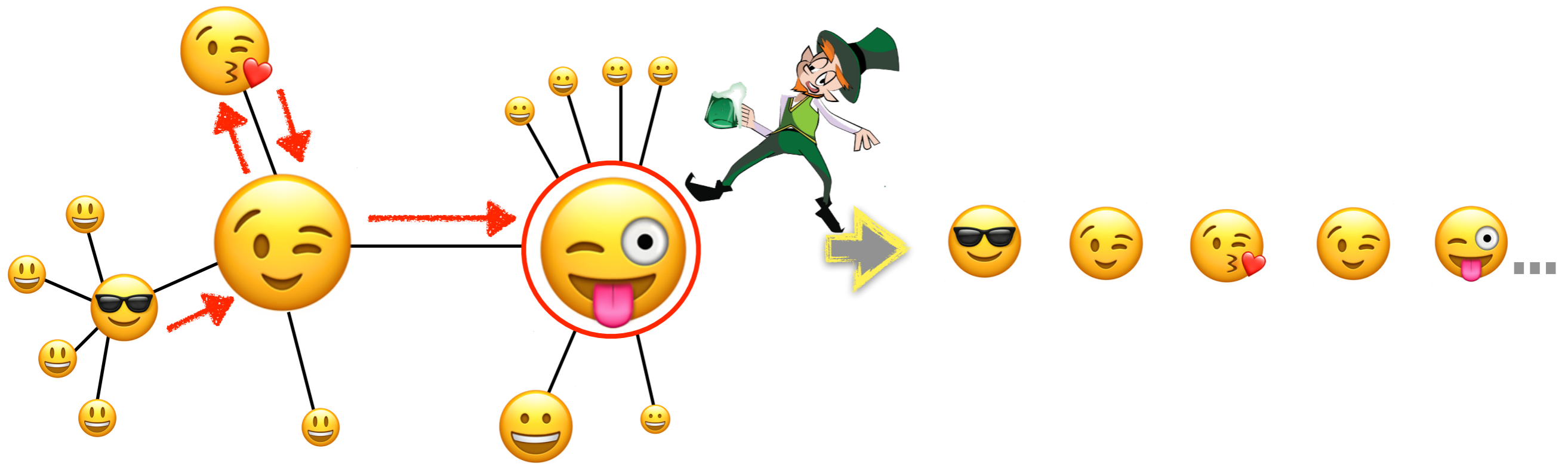




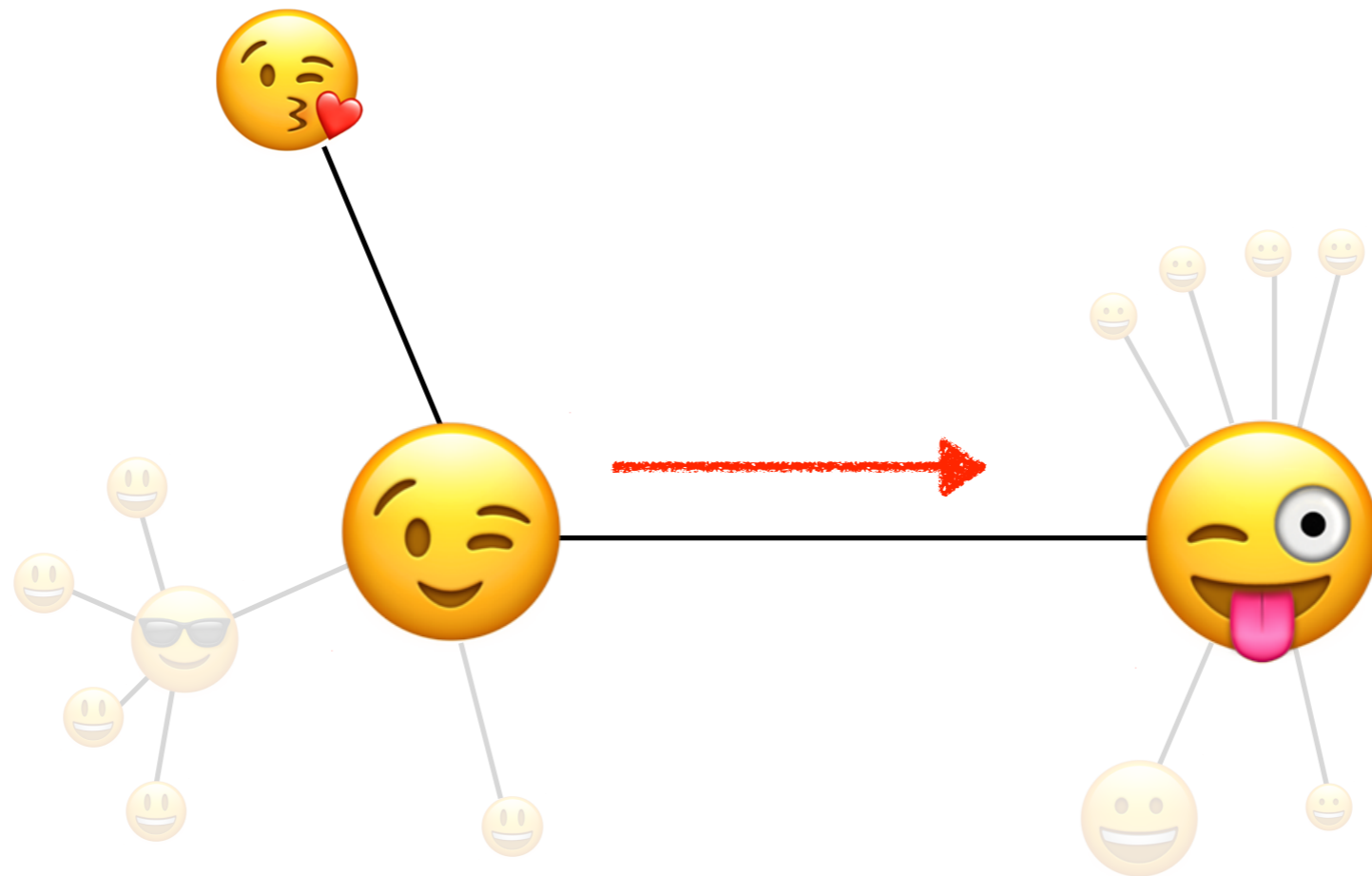








# Friendship paradox

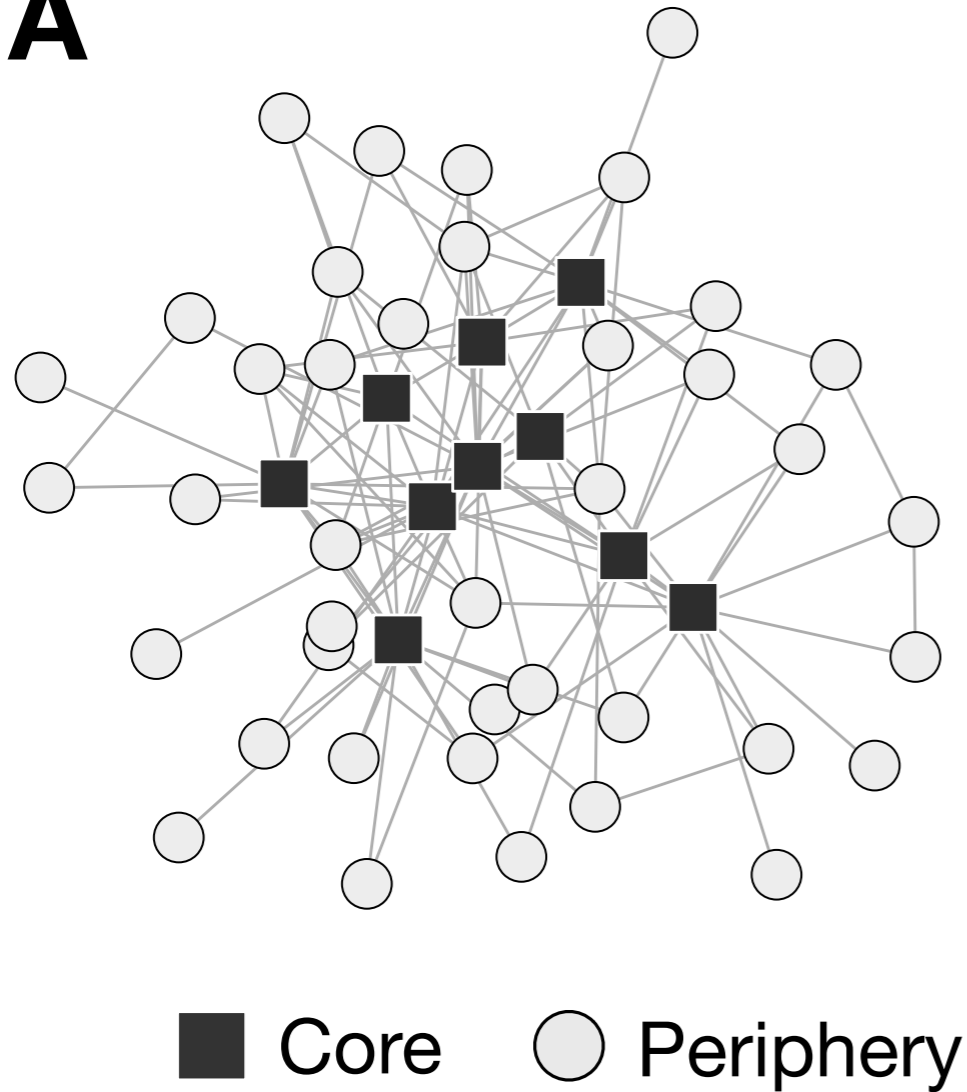


The walker is likely to visit *a node with many friends*.

Following edges is a **biased** sampling that preferentially leads the walker to nodes with many neighbors.

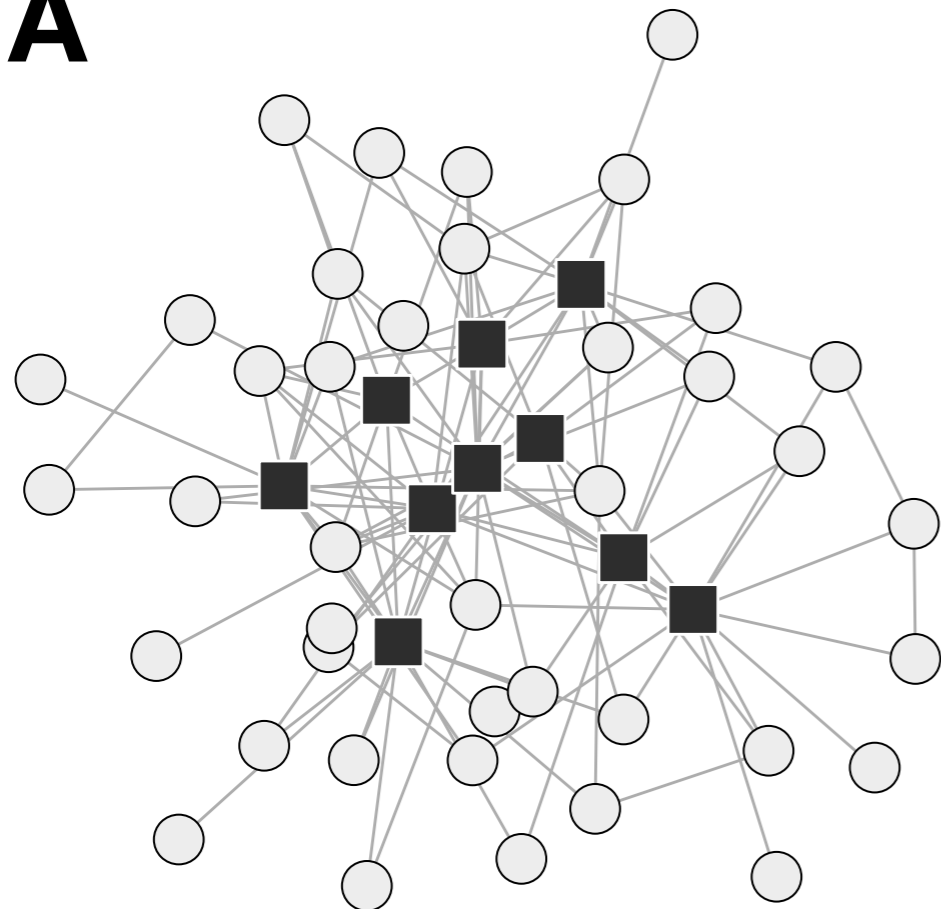
# Toy example

**A**



# Toy example

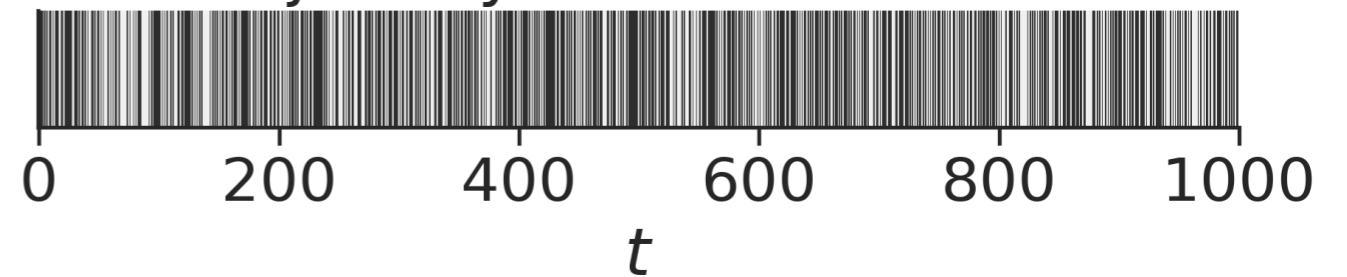
**A**



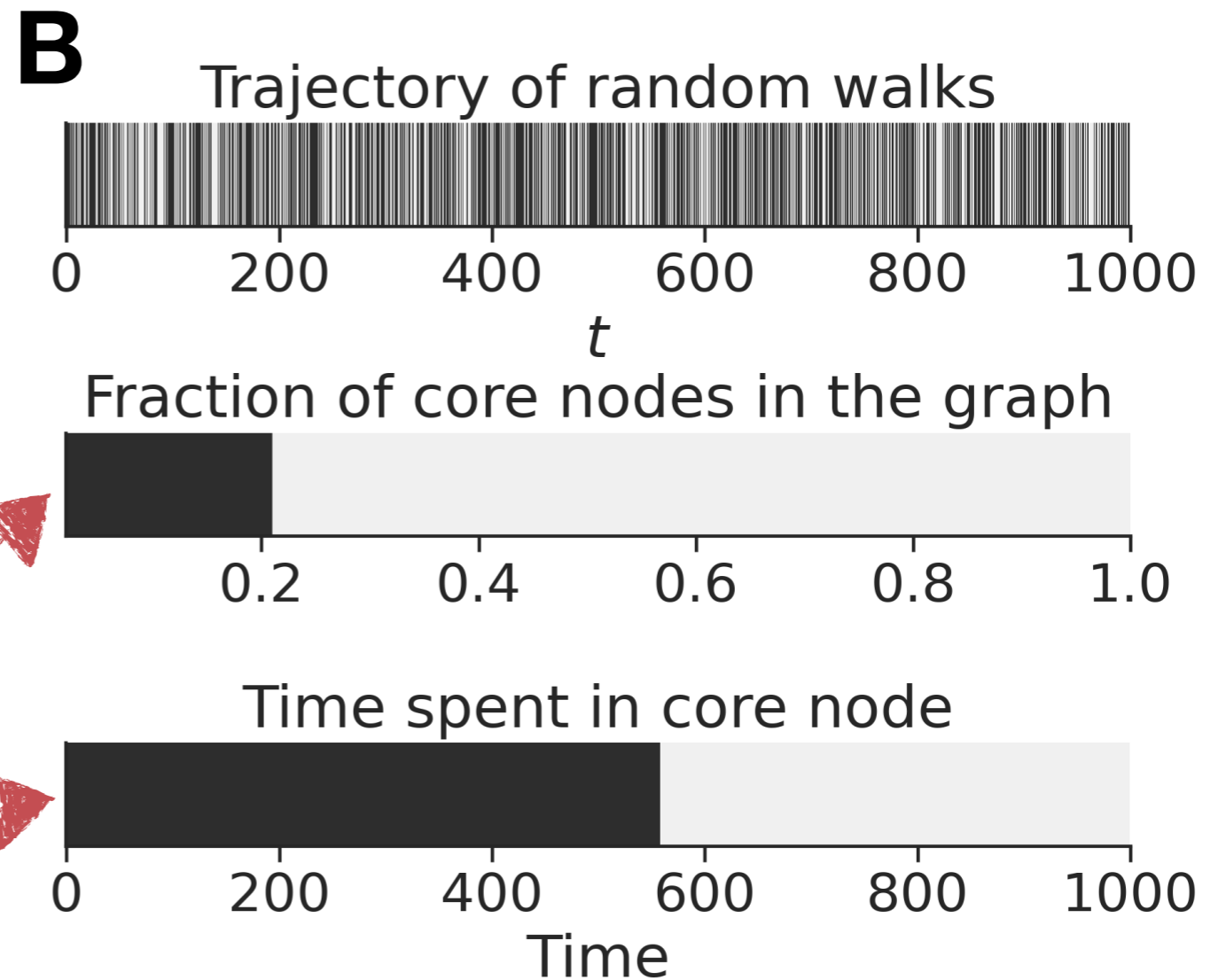
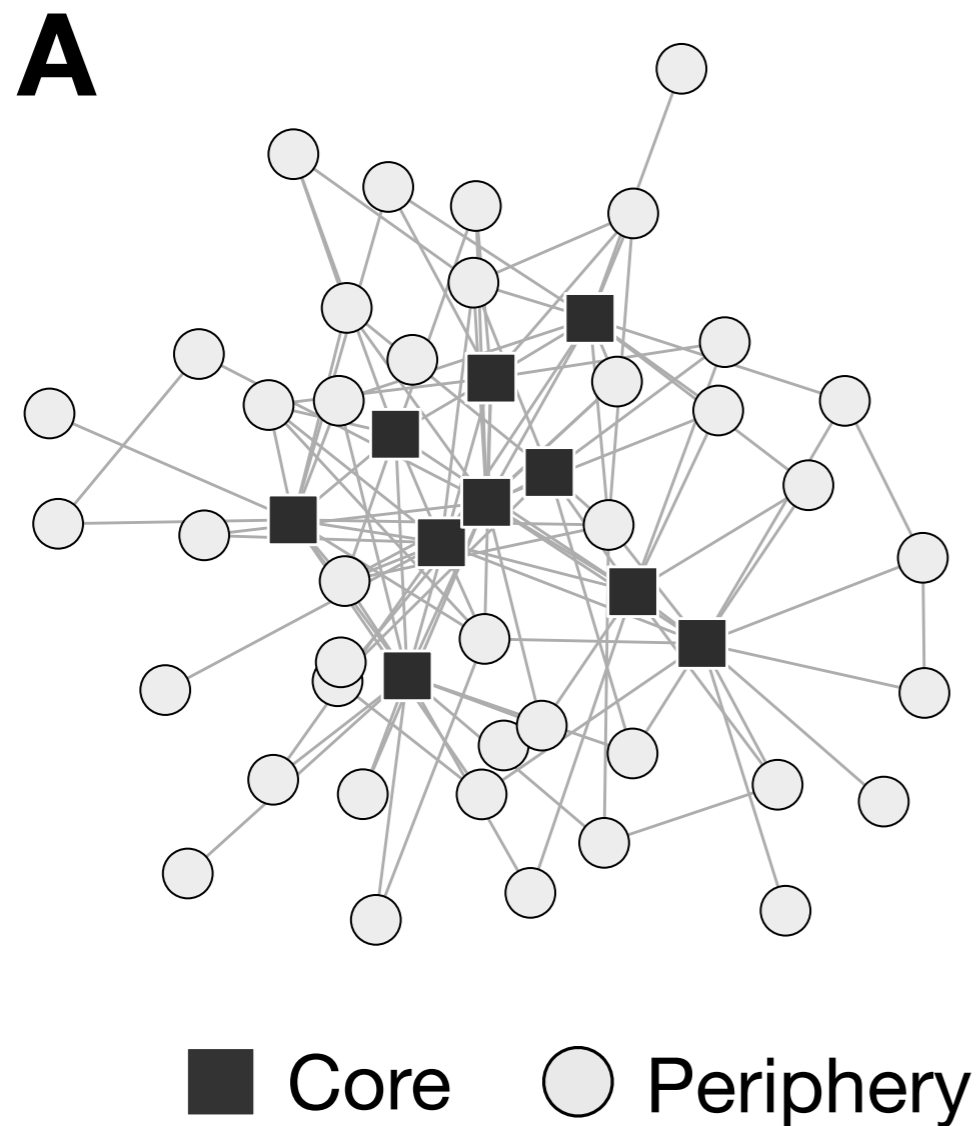
■ Core    ○ Periphery

**B**

Trajectory of random walks



# Toy example

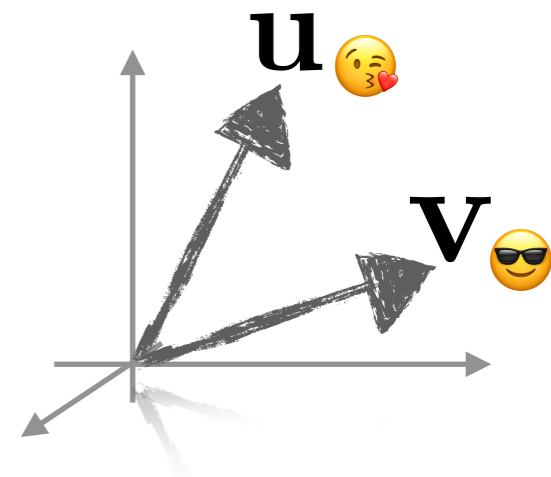
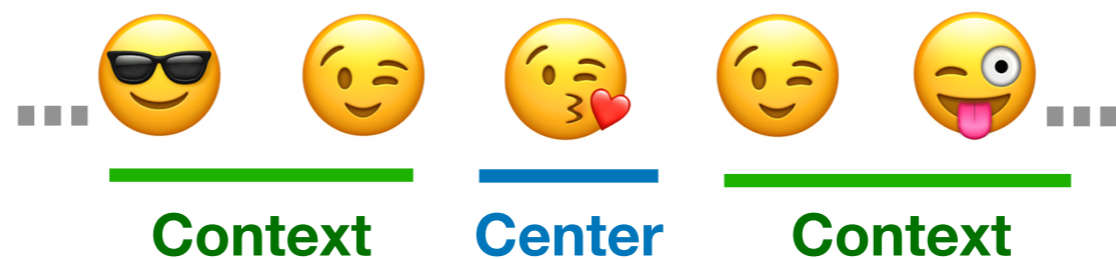


**Core nodes are overrepresented  
due to *the friendship paradox!***

**Does the sampling bias have  
negative impact?**

**word2vec trained with negative  
sampling has **an overlooked built-  
in debiasing feature!****

**We demonstrate **how to leverage  
this feature to debias other biases!****



$$P_{w2v} \left( \text{Context} \mid \text{Center} \right) = \frac{1}{Z} \exp \left( \mathbf{u}^T \mathbf{v} \right)$$

$$Z = \sum_j \exp \left( \mathbf{u}^T \mathbf{v}_j \right)$$

**Negative Sampling** is used for training word2vec\*

↑ Simplified

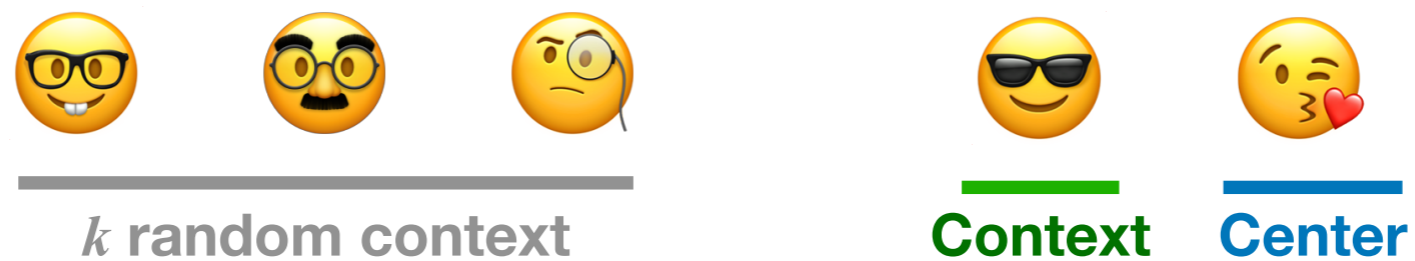
# 💪 Noise Contrastive Estimation (NCE) 💪

\*Mikolov, Tomas, Ilya Sutskever, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013. "InDistributed Representations of Words and Phrases and Their Compositionality." In *Advances in Neural Information Processing Systems*, 1389–99.

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)



# 💪 Noise Contrastive Estimation (NCE) 💪



Noise distribution

$$\sim P_0(\text{👓}) \propto \{\text{Frequency of 👓}\}^\gamma$$

\*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

# 🦾 Noise Contrastive Estimation (NCE) 🦾



$$Y_{\text{glasses}} = Y_{\text{mustache}} = Y_{\text{thinking}} = 0 \quad Y_{\text{sunglasses}} = 1$$

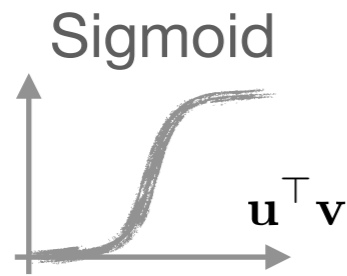
$$P(Y_{\text{sunglasses}} = 1 | \text{🕶️}) \propto \frac{P(\text{🕶️} | Y_{\text{sunglasses}} = 1)}{P(\text{🕶️})} P(Y_{\text{sunglasses}} = 1)$$

*Posterior*
*Likelihood*
*Prior*

$$P(\text{🕶️} | Y = 1) = \begin{cases} P_{w2v}(\text{🕶️} | \text{👄}) & (Y = 1) \\ P_0(\text{🕶️}) & (Y = 0) \end{cases} \quad P(Y = 1) = \frac{1}{k + 1}$$

\*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

# 🦾 Noise Contrastive Estimation (NCE) 🦾



$$P(Y_{\text{😎}} = 1 | \text{😎}) = \frac{1}{1 + \exp(-\mathbf{u}_{\text{😎}}^{\top} \mathbf{v}_{\text{😎}} + \ln P_0(\text{😎}) + c)}$$

**NCE is asymptotically unbiased for**

$$P_{w2v}(\underbrace{\text{😎}}_{\text{Context}} | \underbrace{\text{😘}}_{\text{Center}}) = \frac{1}{Z} \exp(\mathbf{u}_{\text{😘}}^{\top} \mathbf{v}_{\text{😎}})$$

\*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

# Noise Contrastive Estimation (NCE)

$$P(Y_{\text{smiley}} = 1 | \text{smiley}) = \frac{1}{1 + \exp(-\mathbf{u}^{\top} \mathbf{v} + \ln P_0(\text{smiley}) + c)}$$

asymptotically unbiased for

$$P_{w2v}(\text{smiley} | \text{smiley}) = \frac{1}{Z} \exp(\mathbf{u}^{\top} \mathbf{v})$$

$$P(Y = 1 | x) = \frac{1}{1 + \exp(-f(x) + \ln P_0(x) + c)}$$

asymptotically unbiased for

$$P(x) = \frac{1}{Z} \exp(f(x))$$

Also unbiased for a general model

\*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

## Noise Contrastive Estimation (NCE)

$$P(Y = 1|x) = \frac{1}{1 + \exp(-f(x) + \ln P_0(x) + c)}$$

asymptotically unbiased for

$$P(x) = \frac{1}{Z} \exp(f(x))$$

## Negative sampling\*

$$P(Y = 1|x)$$

$$= \frac{1}{1 + \exp(-f(x) + \ln P_0(x) + c)}$$

$$= \frac{1}{1 + \exp[-(f(x) + \ln P_0(x)) + \ln P_0(x) + c]}$$

asymptotically unbiased for

$$P(x) = \frac{1}{Z'} \exp(f'(x))$$

$$P(x) = \frac{1}{Z'} P_0(x) \exp(f(x))$$

word2vec trained with Skip-gram **Negative sampling**  
 (SGNS) is asymptotically unbiased for

$$P_{w2v}(w|c) \neq \frac{1}{Z} \frac{1}{Z'} P_0(w) \exp(f(\mathbf{u}_w^\top \mathbf{v}_c))$$

*Baseline probability*

*Residual from the baseline*

$\sim P_0(\text{😎}) \propto \{\text{Frequency of } \text{😎}\}^\gamma$

Information other than frequency

**Built-in debiasing feature for frequency bias!**



$$P_{w2v}(\text{😎} | \text{😘}) = \frac{1}{Z} P_0(\text{😎}) \exp(\mathbf{u}_{\text{😎}}^\top \mathbf{v}_{\text{😘}})$$

$$\sim P_0(\text{😎}) \propto \{\text{Frequency of 😎}\}^\gamma$$

Sampling bias due to *the friendship paradox*\*

$$\left\{ \text{Frequency of 😎 in the sentence} \right\} \propto \left\{ \# \text{ neighbors of 😎 in the graph} \right\}$$

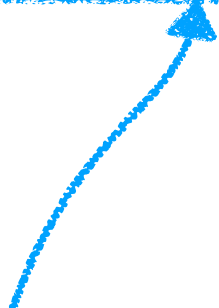
The friendship paradox has *no effect* thanks to *the built-in debiasing feature* of SGNS word2vec if  $\gamma = 1$ .



\*Masuda, Naoki, Mason A. Porter, and Renaud Lambiotte. "Random walks and diffusion on networks." *Physics reports* 716 (2017): 1-58.

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)

# Residual2Vec

$$P_{r2v}(\text{😎} | \text{😘}) = \frac{1}{Z} \underbrace{P_0(\text{😎} | \text{😘})}_{\text{baseline}} \exp(\mathbf{u}^{\text{😘}} \mathbf{v}^{\text{😎}})$$


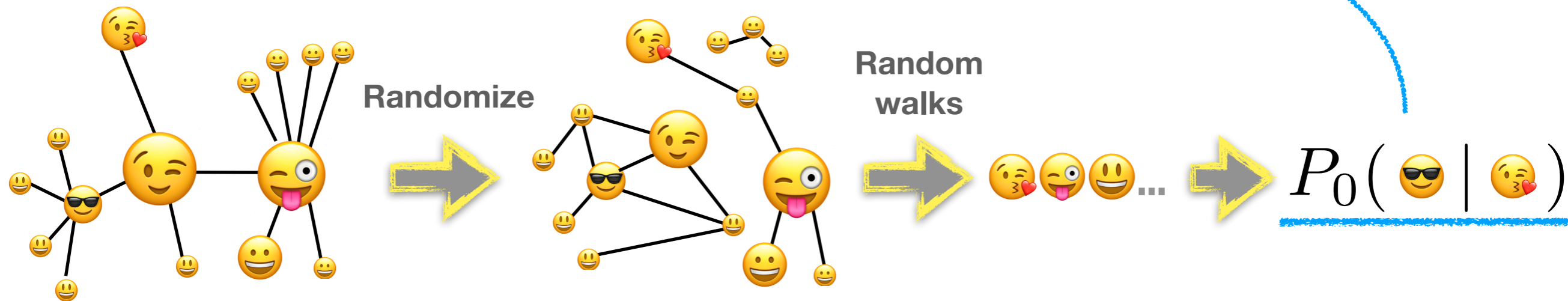
**Model the baseline  
explicitly to control bias**



# Residual2Vec

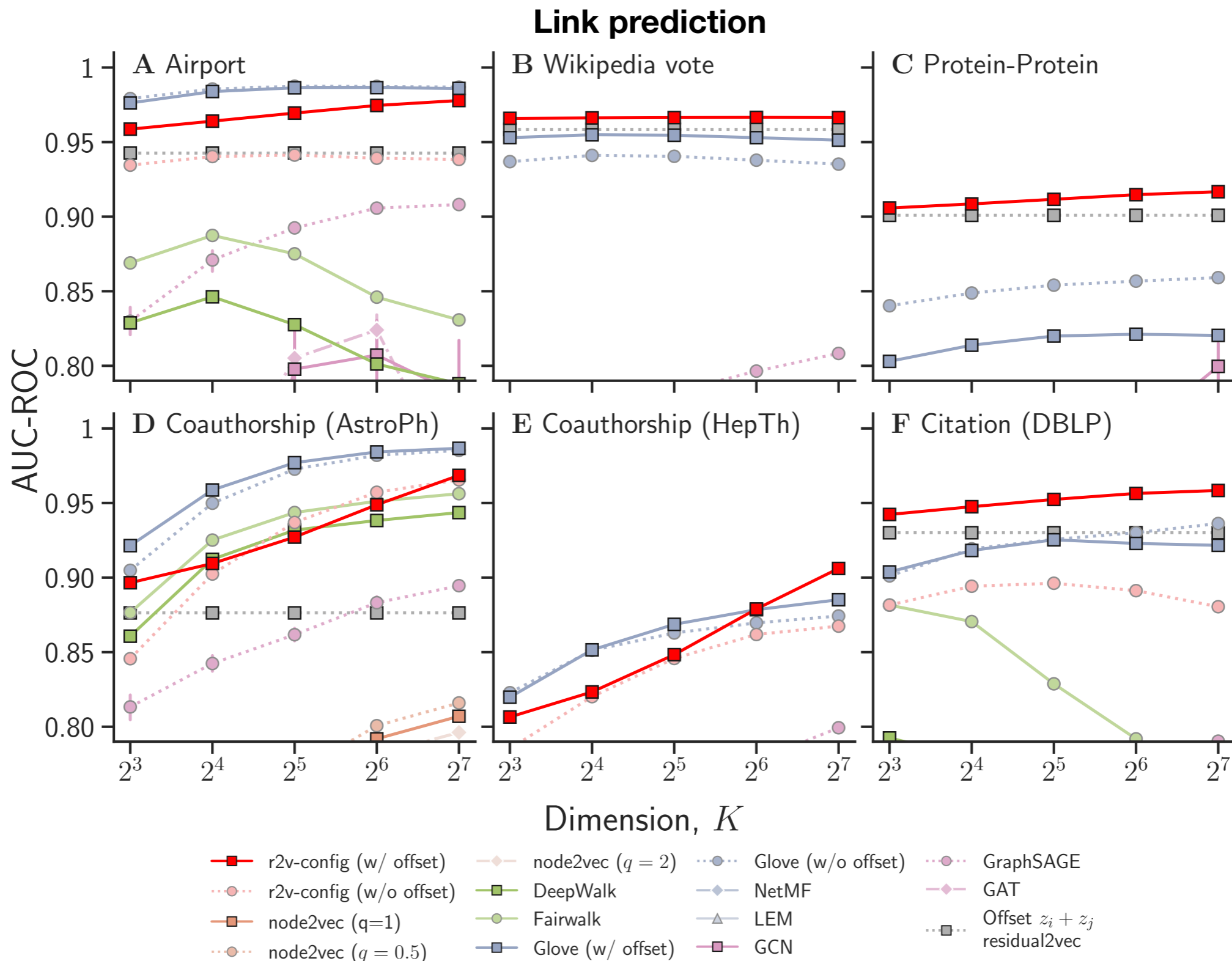
How can we model the baseline?

$$P_{r2v}(\text{😎} | \text{😘}) = \frac{1}{Z} P_0(\text{😎} | \text{😘}) \exp(\mathbf{u}_{\text{😘}}^\top \mathbf{v}_{\text{😎}})$$

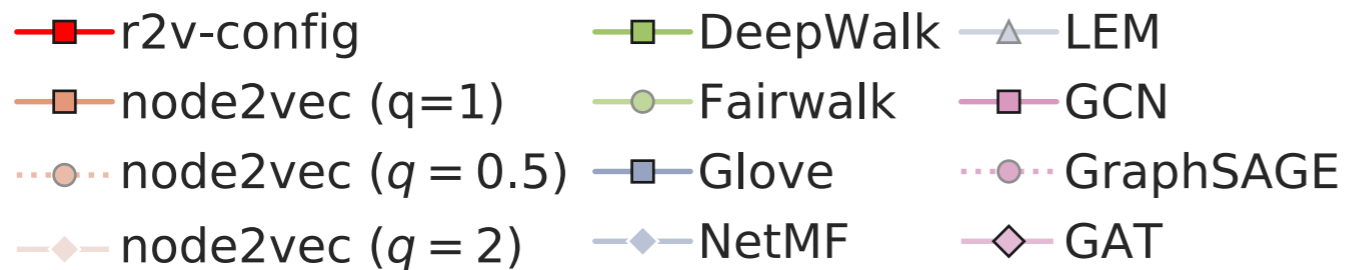


**Can be done analytically without simulations!**

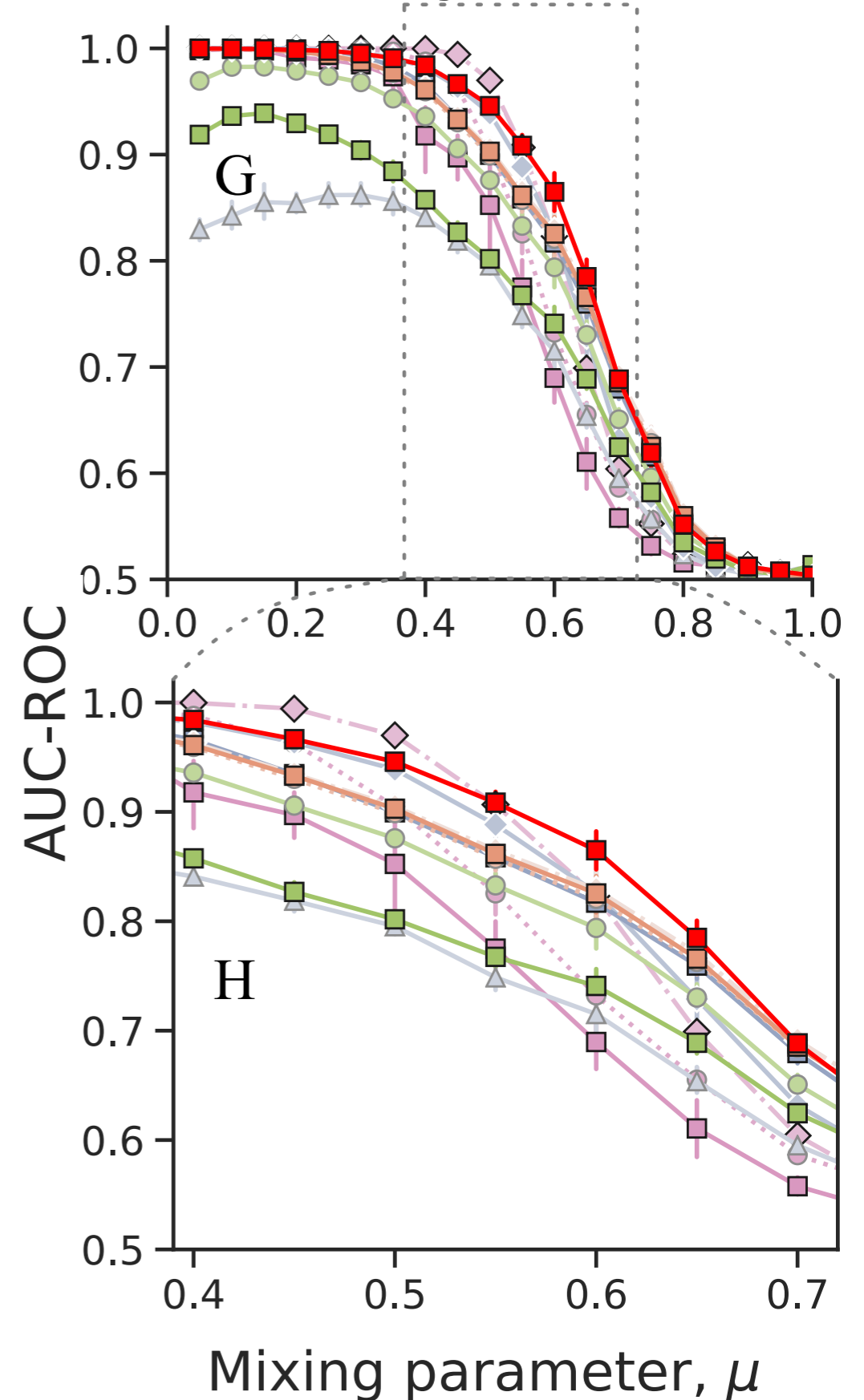
# Performed the best or nearly the best for all the six graphs of different domains



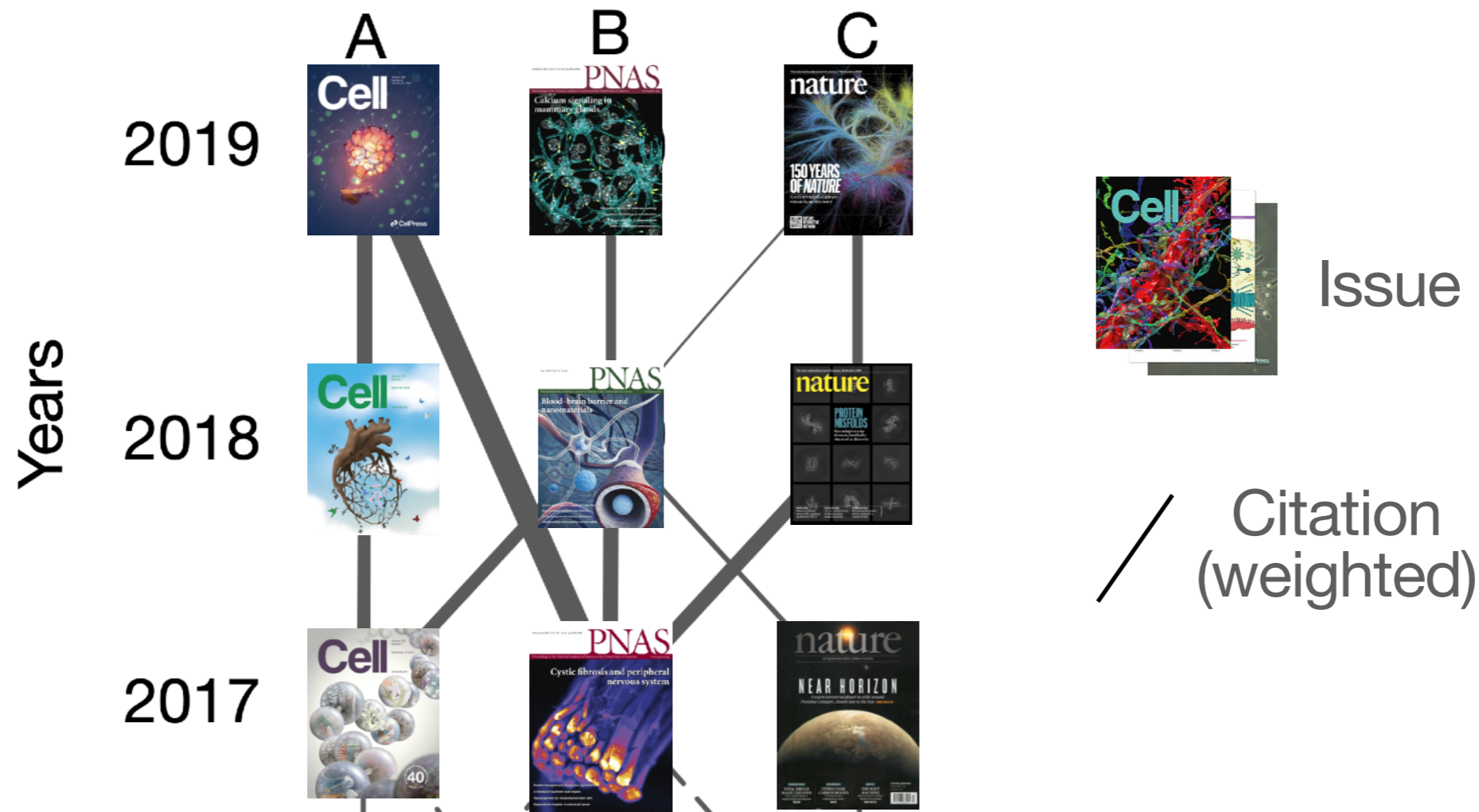
The best or the second best performer for a community detection benchmark



## Community detection



# Case studies



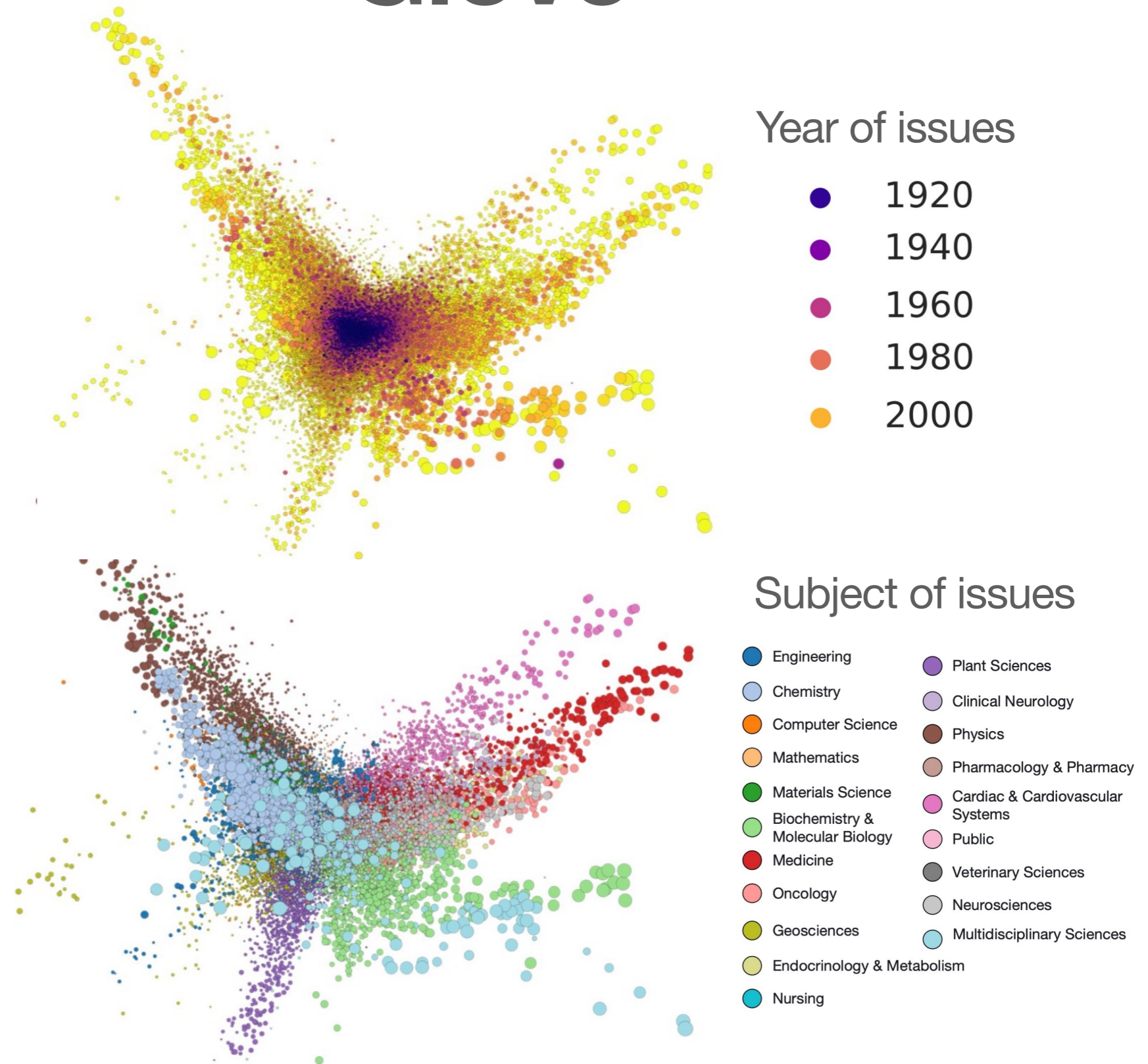
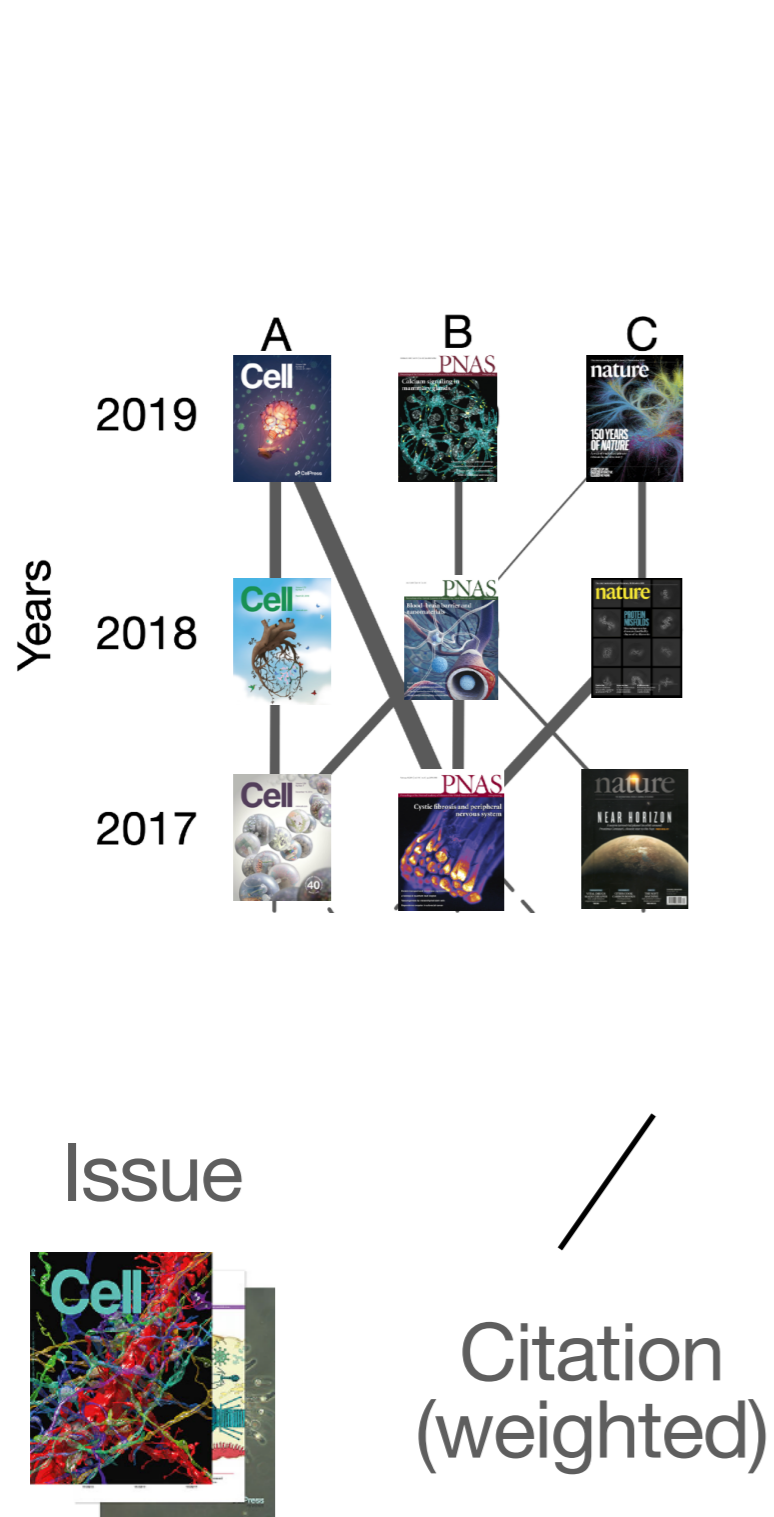
Citation graph of academic issues

Constructed using the Web of Science

240K issues and 250M citations



# Glove\*



\*Jeffrey Pennington, Richard Socher, and Christopher D. Manning. "Glove: Global vectors for word representation." *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*. 2014.

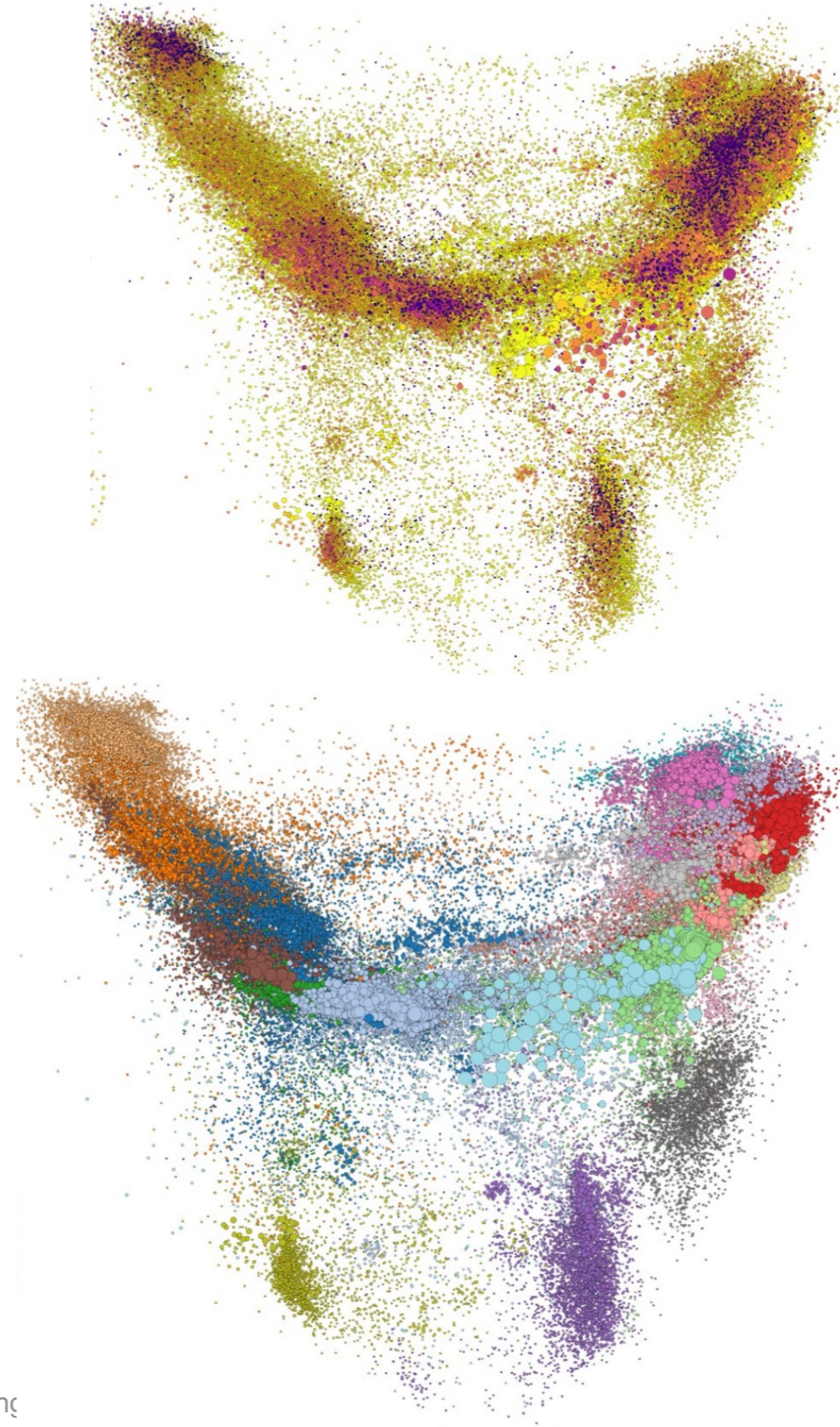
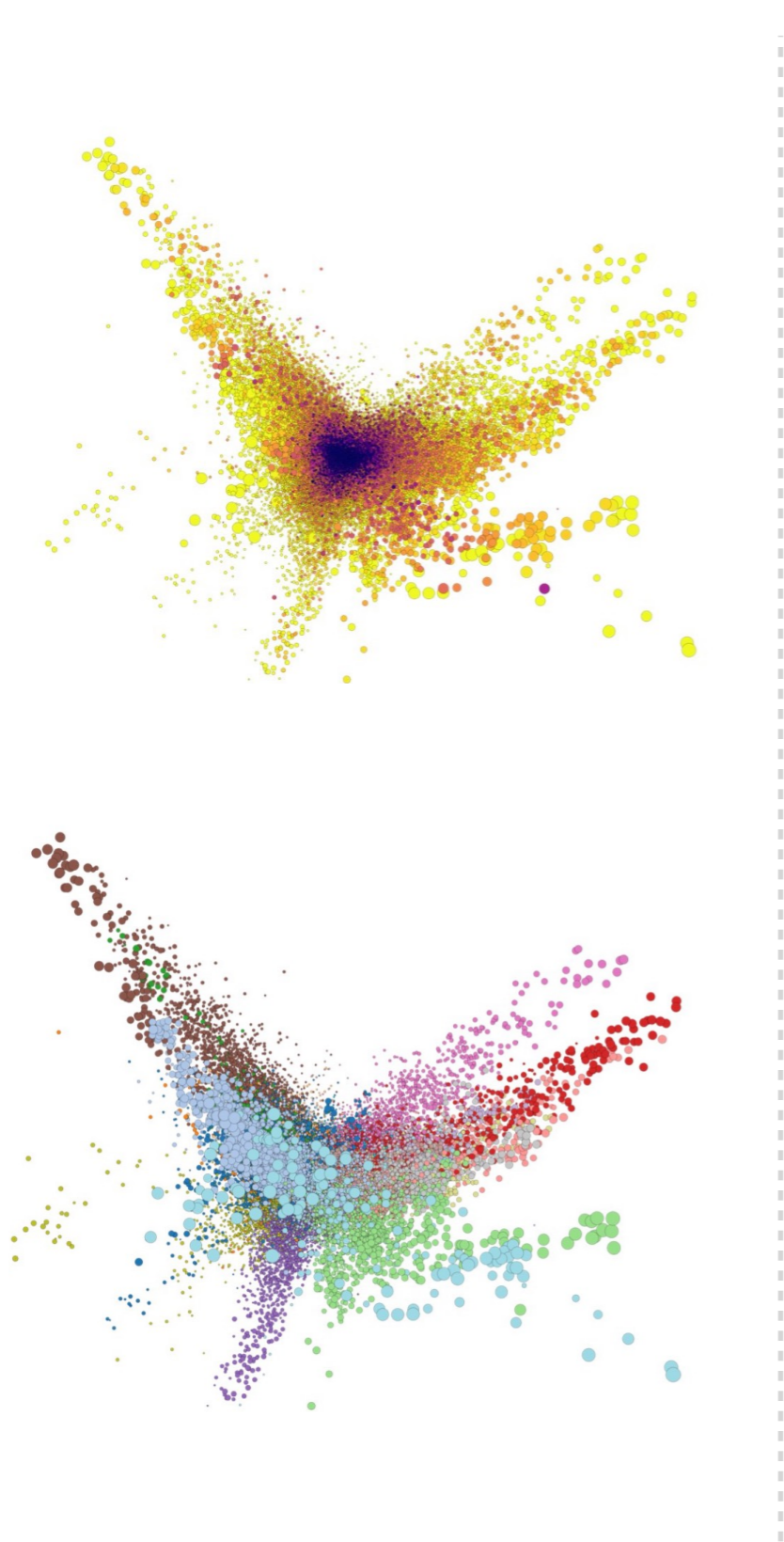
Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. *NeurIPS* (2021)



# Residual2Vec

(temporal biases are debiased)

## Glove



Year of issues

- 1920
- 1940
- 1960
- 1980
- 2000

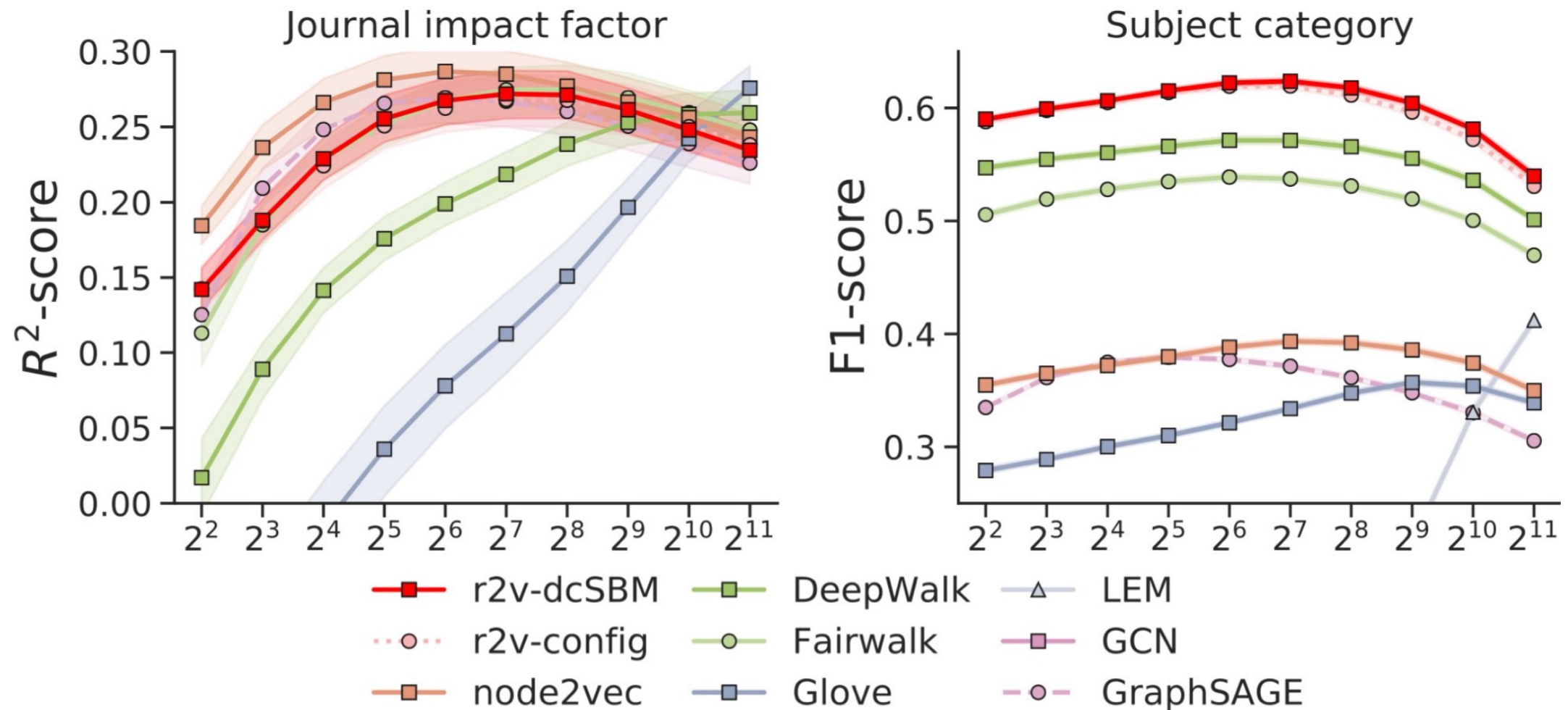
Subject of issues

- Engineering
- Chemistry
- Computer Science
- Mathematics
- Materials Science
- Biochemistry & Molecular Biology
- Medicine
- Oncology
- Geosciences
- Endocrinology & Metabolism
- Nursing
- Plant Sciences
- Clinical Neurology
- Physics
- Pharmacology & Pharmacy
- Cardiac & Cardiovascular Systems
- Public
- Veterinary Sciences
- Neurosciences
- Multidisciplinary Sciences



# Predicted the impact factor and the subject of journals well

## Prediction based on embedding

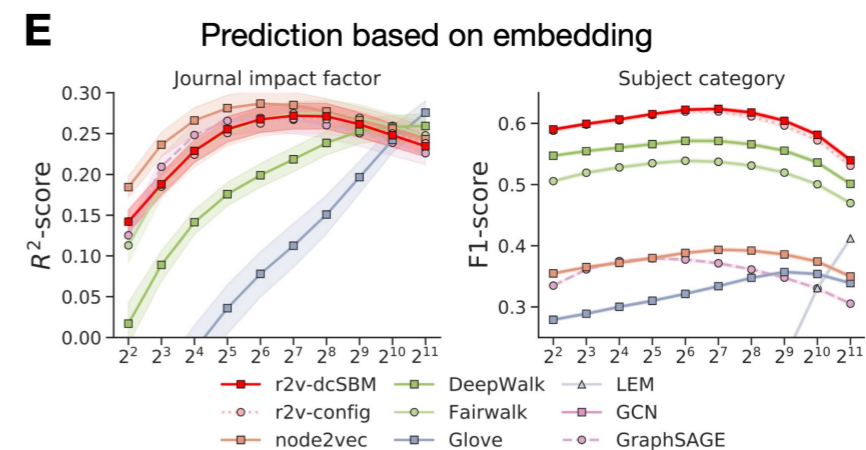
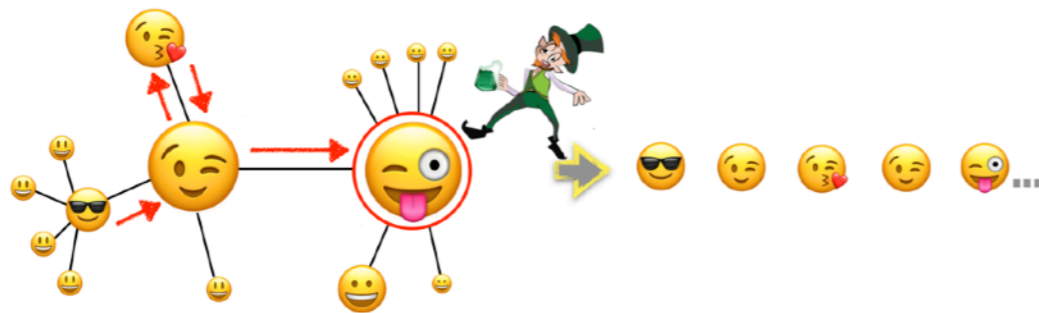


# Summary

word2vec has a built-in debiasing feature attributed to negative sampling

Inspired by this finding, propose *residual2vec* that can negate other types of structural biases

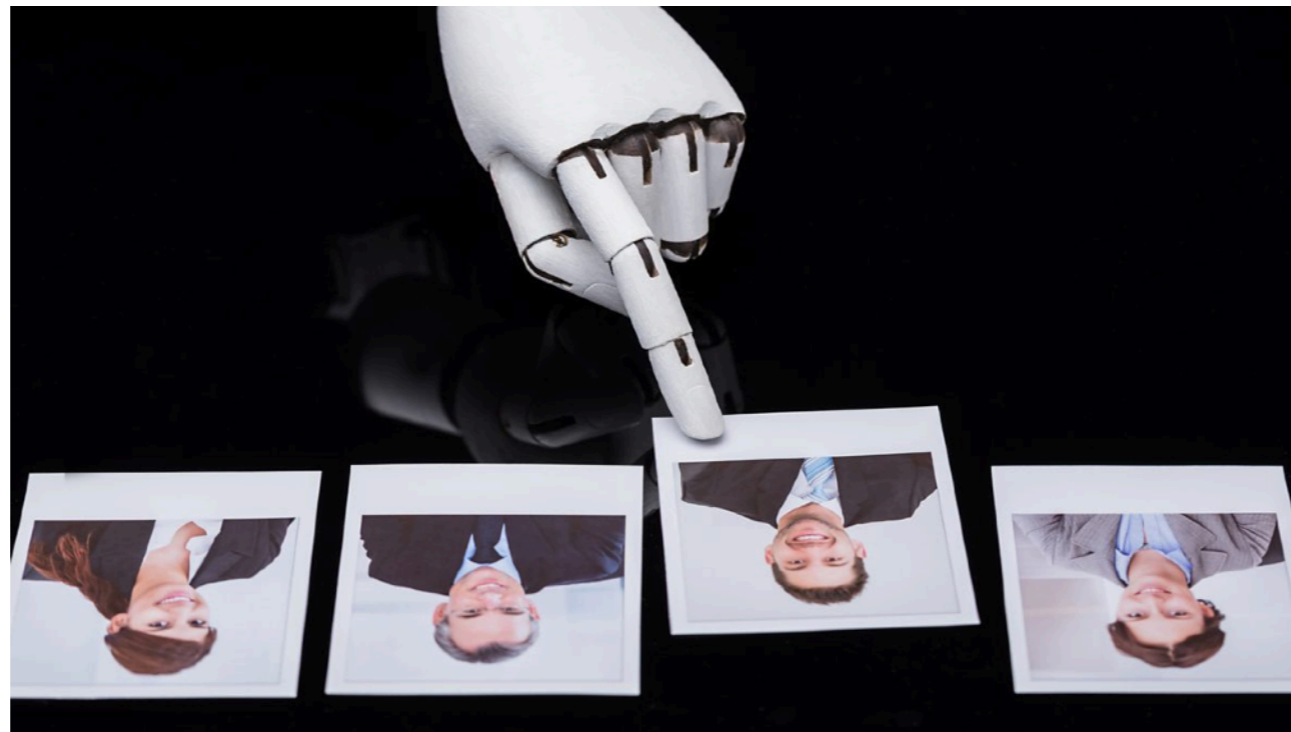
Good performance and enable more control on the biases in embedding.





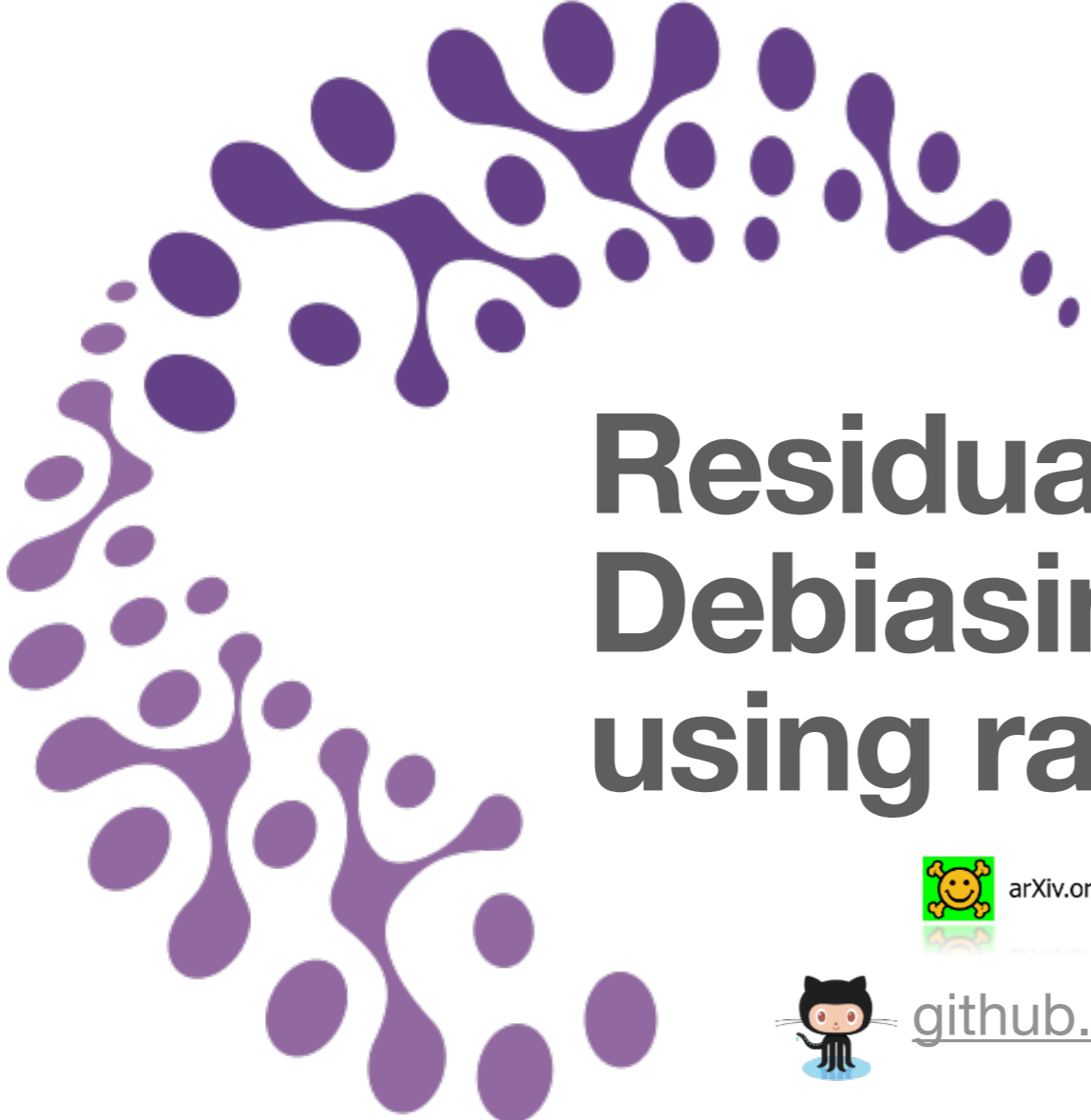
# Discussion

We demonstrated a new potential of **negative sampling** as a way to mitigate **bias in representations**



This figure is taken from <https://www.bbc.co.uk/programmes/w3ct1ls5>

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)



# Residual2Vec: Debiasing graph embedding using random graphs

 arXiv.org <https://arxiv.org/abs/2110.07654>

 [github.com/skojaku/residual2vec](https://github.com/skojaku/residual2vec)

 [skojaku@iu.edu](mailto:skojaku@iu.edu)



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**Isabel Constantino**



**Yong-Yeol Ahn**