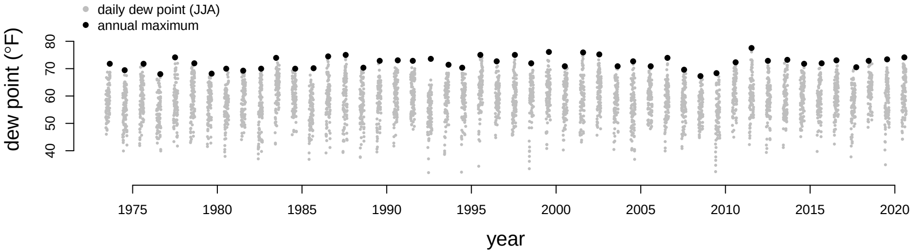
Methods for Extremes

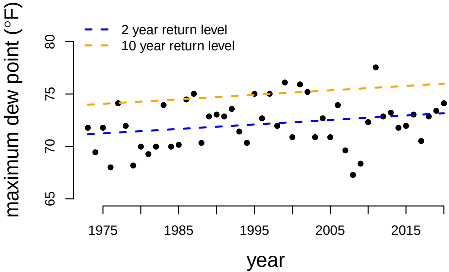
Motivation

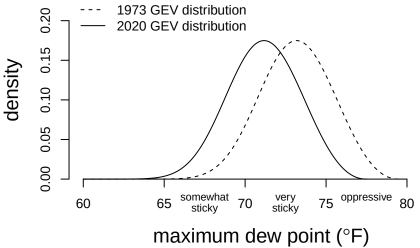
Climate change impacts depend disproportionately on extreme events (e.g. heat waves, extreme precipitation)

- Typical questions:
  - How has the distribution of extreme events changed (or how is it projected to change)?
  - What is the "fraction of attributable risk" or "risk ratio" for an event that has occurred?
- Extreme events are by definition rare, require principled methods for their characterization

Example: Block Maximum Approach (humidity in Minneapolis)







Note: this is a very naive analysis

Classical Extreme Value Theory (Fisher-Tippet-Gnedenko Theorem)

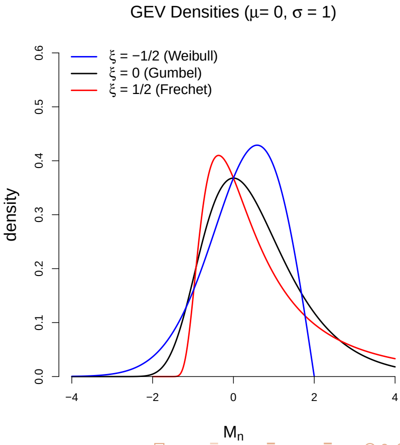
Let $X_1, ..., X_n$ be i.i.d. and $M_n = \max\{X_1, ..., X_n\}$. If there exist sequences $\{a_n\}$ and $\{b_n\}$ such that $(M_n - b_n)/a_n$ converges to a nondegenerate distribution $F(z)$, then $F(z)$ is a member of the *generalized extreme value* (GEV) distribution:

$$F(z) = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}$$

for $\mu, \xi \in (-\infty, \infty)$ and $\sigma > 0$

- i.e., maxima approximately follow a GEV distribution if n is big

See Coles (2001) and Cooley et al. (2019) for reviews



Block Maximum Approach

Typical approach:

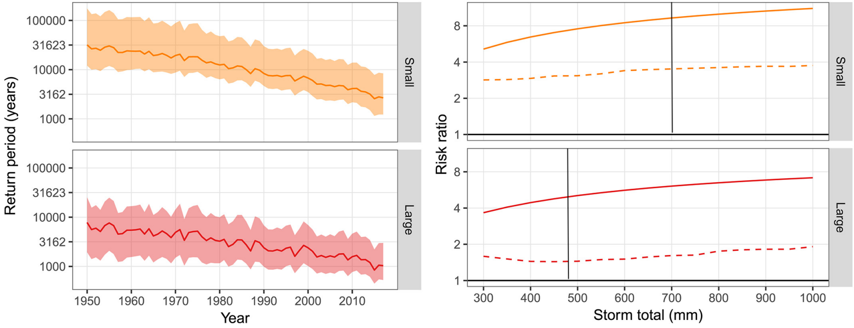
- Take maximum over a block of time (e.g., yearly maximum) and model with a GEV(μ_t, σ_t, ξ_t)
- Some or all parameters may change over time and with covariates
  - may also be spatial processes

Terminology:

- z_p is the *return level* associated with the *return period* $1/p$ if $Pr(M_{n,t} > z_p) = 1 - p$
  - i.e., the 10-year return level is the 90th percentile of the yearly maximum distribution
- The *risk ratio* $RR(z) = Pr(M_{n,t_1} > z)/Pr(M_{n,t_0} > z)$ is the ratio of exceedance probabilities for a fixed event magnitude at two time points

Example: Attribution for Hurricane Harvey Extreme Precipitation

Risser and Wehner (2017) calculate increase in risk of Hurricane Harvey precipitation accumulations due to anthropogenic climate change



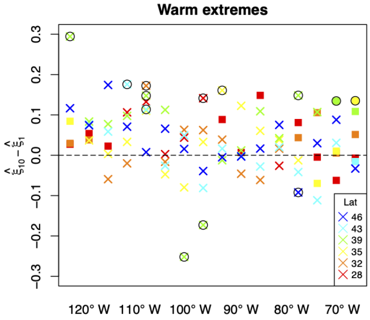
(Figure adapted from Risser and Wehner (2017))

Block Maximum Tradeoffs

The block maximum approach involves a type of *bias* - *variance* tradeoff:

- Large block size needed for asymptotics to be appropriate
- Many blocks needed for efficient statistical estimation

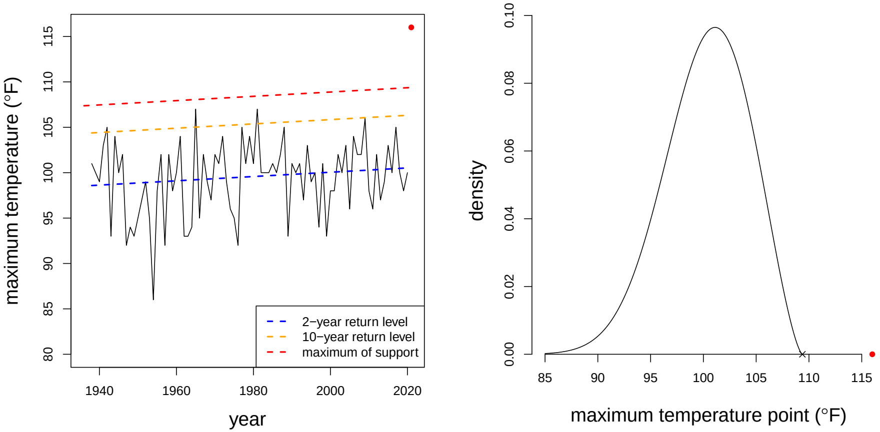
Comparing estimates of ξ using 1- vs 10-year blocks:



(image adapted from Huang et al. (2016))

Caution

A naive analysis of annual maxima temperatures at Portland International Airport, 1936-2020



The Threshold Exceedance Approach (Pickands–Balkema–De Haan theorem)

A complementary approach is to model *threshold exceedances*:

Let $X_1, ..., X_n$ be i.i.d. and suppose the maximum has a nondegenerate limiting distribution. Then the limiting distribution of $(X_i - u)|X_i > u$ is a *generalized Pareto* distribution (GPD):

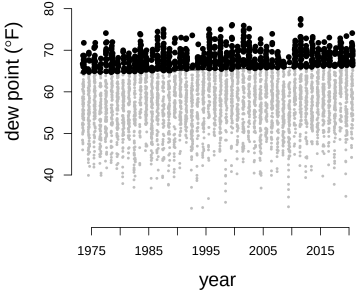
$$Pr(X_i - u \leq y | X_i > u) \approx 1 - \left(1 + \frac{\xi y}{\tilde{\sigma}}\right)^{-1/\xi}, \ y > 0$$

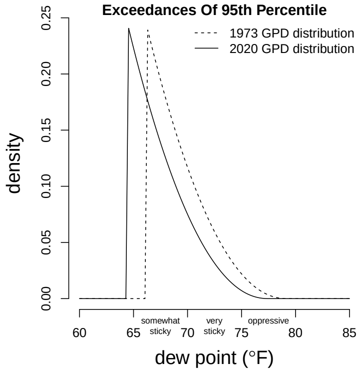
for large u, where ξ is the same parameter from the GEV distribution and $\tilde{\sigma} = \sigma + \xi(u - \mu)$.

Example: Exceedances of 95th percentile (humidity in Minneapolis)

Issues to watch out for:

- GPD approach allows you to make use of more data, but watch out for temporal dependence (cluster identification)
- Threshold choice involves same tradeoffs as block size choice





(again, a very naive analysis)

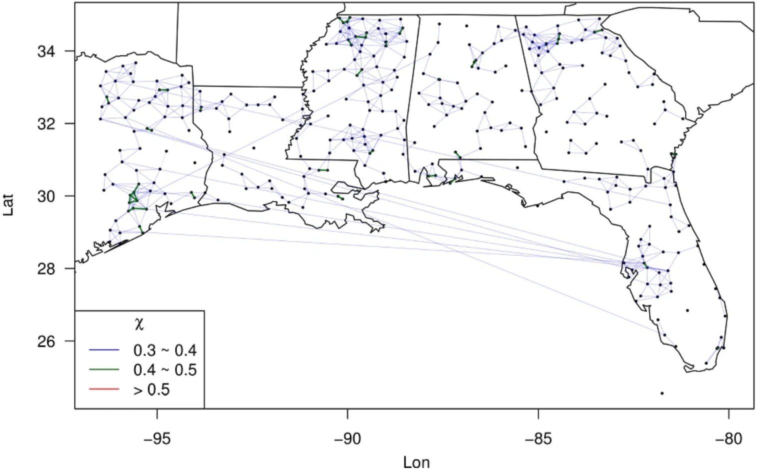
Multivariate and Spatial Extremes

Impacts may depend on extremes over a spatial domain and/or of multiple variables

- Multivariate extension to *componentwise maxima* and *exceedances*, but class of limiting distributions is wide
  - Typically choose parametric subfamily accommodating different levels of *asymptotic dependence* ($\lim_{z \to \infty} Pr(Y > z | Z > z)$)
  - May or may not address desired meaning of multivariate extreme event
- Spatial extension of GEV methodology is to *max-stable processes* (Davison et al., 2019)
  - Computational challenges in evaluating full likelihood
  - Theory does not address temporally coherent extremes (componentwise extremes)

Example: Exploring Extremal Dependence Networks

Extremal dependence for hurriance-season maximum rainfall



(Figure adapted from Huang et al. (2019))

Summary

Analyzing extreme events requires care

- Involves extrapolating into the tail of a distribution
- Classical methods rely on asymptotics for maxima or threshold exceedances, but there are subtleties in their application
- Characterizing multivariate or spatial extremes is much more challenging (conceptually, mathematically, computationally, etc.)
  - Existing theory may not always correspond to climate community's conception of extremes
- Lots of work needed in developing and applying new methodologies

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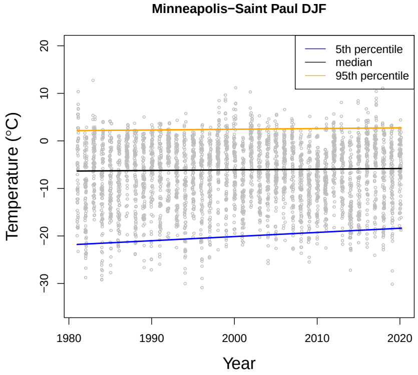
Quantile Regression for Climate Science

Motivation

Climate extremes are important, but a climate extreme isn't necessarily an extreme value analysis extreme

- **Extreme value analysis methods**: parametric (asymptotic justification), principled methods for extrapolating into tail of distribution
  - What is the e.g. "100 year event"? How has that changed?
- **Quantile regression methods**: nonparametric (more empirical), useful for less extreme but still atypical events
  - What is the e.g. 95th percentile? The 5th percentile? Have they changed in different ways?

Example: Winter Temperature Trends



Northern US and Canada winters have shown decreased temperature variability due to faster warming at the *low quantiles*

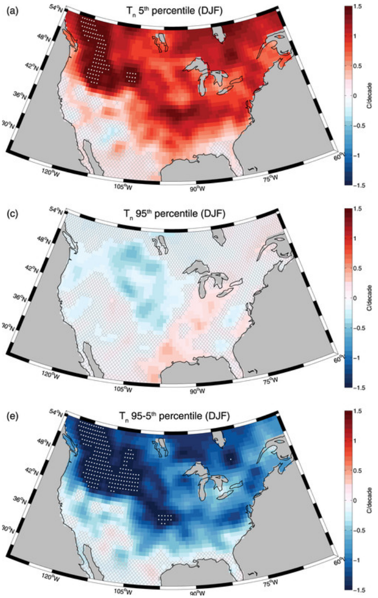
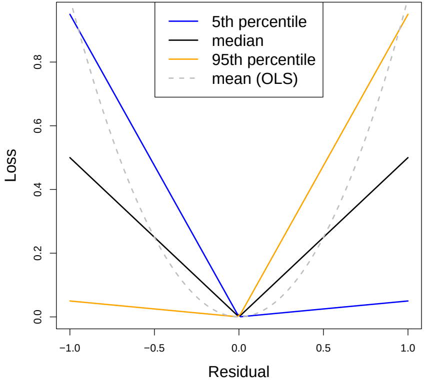


Figure adapted from Rhines et al. (2017)

Quantile Regression

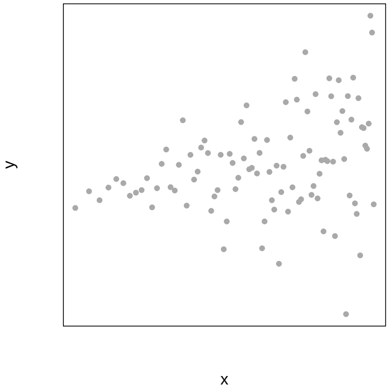
Quantile regression (Koenker and Bassett Jr, 1978) involves estimating the τth *conditional quantile function*, $q_\tau(x_i)$, of response y_i given predictors $\mathbf{x}_i$ via the minimization problem

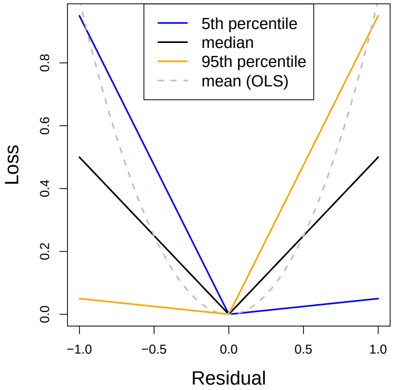
$$\arg\min \rho_\tau \left(y_i - q_\tau(x_i)\right), \text{ where } \rho_t(u) = u \times (\tau - I(u < 0))$$



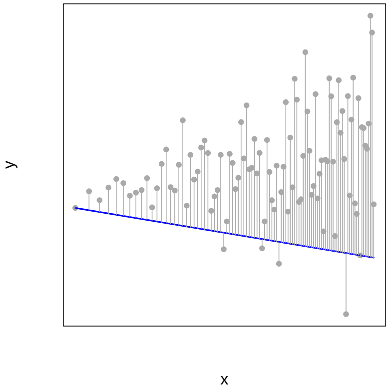
Problem is a linear program and can be solved using standard methods

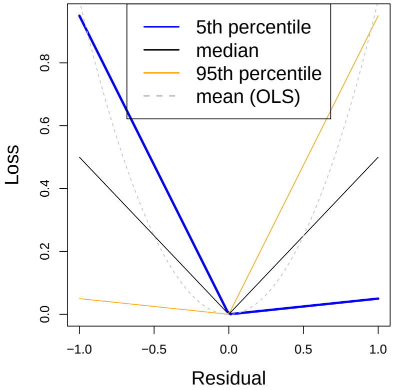
Quantile Regression Illustration



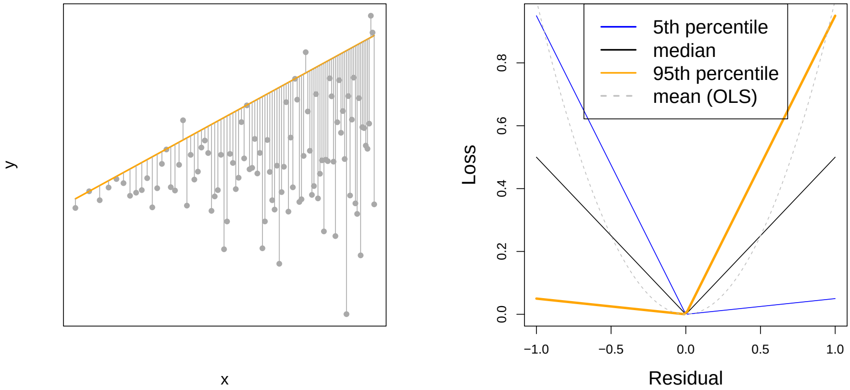


Quantile Regression Illustration





Quantile Regression Illustration



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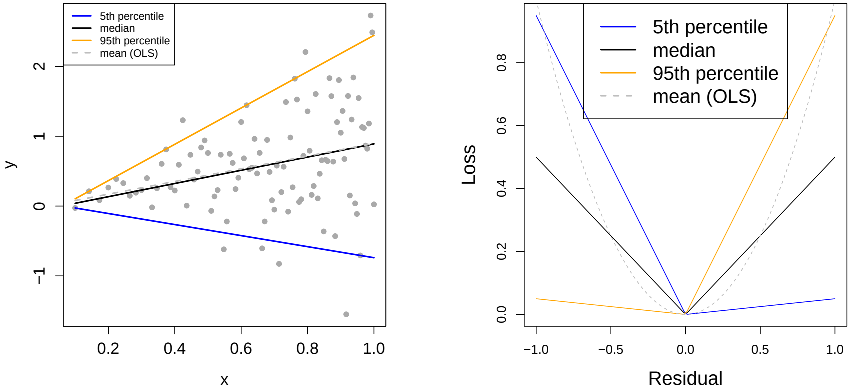
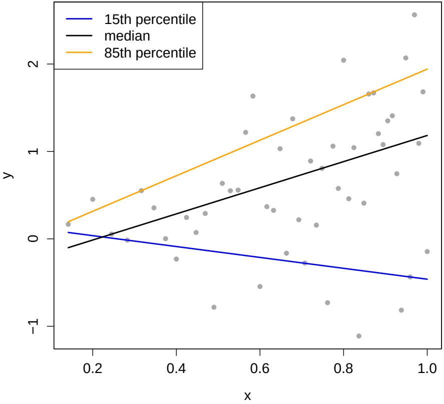


Quantile Crossing

Quantile curves estimated separately can cross. Proposed solutions include

- Reorder the estimates (Chernozhukov et al., 2010)
- Stepwise estimation, adding inequality constraints (e.g., Liu and Wu (2009))
- Simultaneous estimation, adding noncrossing constraints (e.g., Bondell et al. (2010))



Nonparametric Quantile Regression

Koenker et al. (1994) considers “quantile smoothing splines” solving

$$\min \rho_\tau \left(y_i - q_\tau(x_i)\right) + \lambda V(q'_\tau(x_i))),$$

where $V(q'_\tau(x_i))) = \sum_{i=1}^{n-1} |q'_\tau(x_{i+1}) - q'_\tau(x_i)|$ is the *total variation penalty* on the derivative of $q_\tau(x_i)$ (and λ a tuning parameter).

- solution is a linear spline with knots at each observation x_i
- convenient because problem is still a linear program

There are many alternatives, including those making use of neural networks (see e.g. Cannon (2018) for an example analyzing precipitation extremes)

Example: Hot and dry events in the Southwest

McKinnon et al. (2021) considers the model

$$\underbrace{q_\tau(t)}_{\text{specific humidity quantile}} = \beta_{0,\tau} + \underbrace{s_{0,\tau}\left(T'(t)\right)}_{\text{local temperature relationship}} + \underbrace{\beta_{1,\tau} G'(t)}_{\text{change with global temperature}} + \underbrace{G'(t) s_{1,\tau}\left(T'(t)\right)}_{\text{change in local temperature relationship}}$$

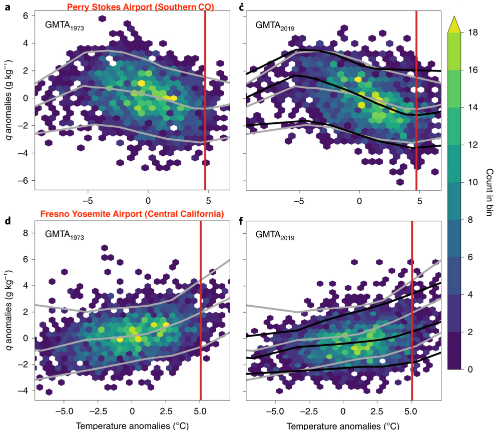


Figure adapted from McKinnon et al. (2021)

Focus is on the lower quantiles on historically hot days

- See a *decrease* in the 5th percentile of specific humidity on historically hot days

Example: Hot and dry events in the Southwest

$$\underbrace{q_\tau(t)}_{\text{specific humidity quantile}} = \beta_{0,\tau} + \underbrace{s_{0,\tau}\left(T'(t)\right)}_{\text{local temperature relationship}} + \underbrace{\beta_{1,\tau} G'(t)}_{\text{change with global temperature}} + \underbrace{G'(t) s_{1,\tau}\left(T'(t)\right)}_{\text{change in local temperature relationship}}$$

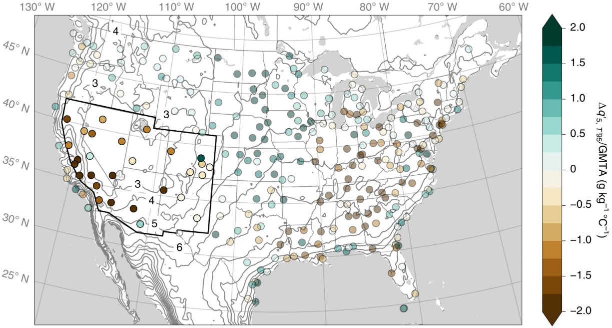


Figure adapted from McKinnon et al. (2021)

Dry days that are hot have gotten *even dryer* in the Southwest

- explained by decreases in June soil moisture

Summary

Quantile regression provides an additional framework for studying changes in more extreme events

- Can be used to study changes in conditional distributions as an end goal or as an intermediate step (e.g., quantile mapping methods for bias correction, coming next)
- As is typical, lots of room for innovation in both modeling and computation to help address climate-specific questions
  - flexibility vs. interpretability of models
  - incorporating climate knowledge into models
  - borrowing strength spatially

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Bias Correction Methods

Goals of "Bias Correction" Methods

Two sources of information about historical and future climate:

- **Observations**
  - Informative about what has really happened, but
  - Limited observational record, relatively small observed changes, and does not speak directly to future changes
- **Climate models**
  - Informative about changes in future scenarios of interest, but
  - Models aren't perfect, have "biases"

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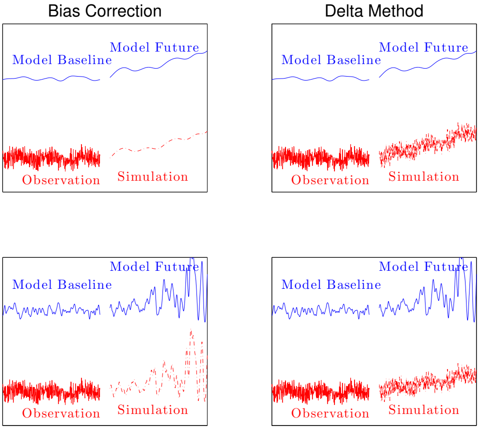
If an **impacts modeler** requires *realistic* future simulations of climate variables, climate model output may be insufficient

- "**Bias correction**" methods combine information from model output with observations to produce hopefully better-calibrated future simulations

Model- vs. Observation-based Methods (Cartoon Illustration)



Simulation = model - bias

Simulation = observation + trend

Simple model-driven methods don't retain higher-order properties of observations

Simple observation-based methods don't account for higher order changes

More sophisticated methods are needed for realistic simulations capturing projected higher-order distributional changes

Quantile Mapping Approaches

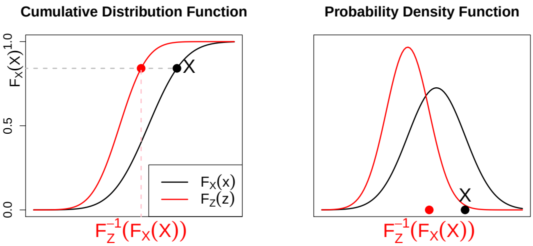
Idea: want to change the "whole distribution," not just the mean.

Quantile Mapping Approaches

Idea: want to change the "whole distribution," not just the mean.

Most popular methods are based on *inverse transform sampling* (called "quantile mapping" in climate literature):

Say X has CDF $F_X(x)$. Then $\hat{Z} = F_Z^{-1}(F_X(X))$ has CDF $F_Z(z)$.



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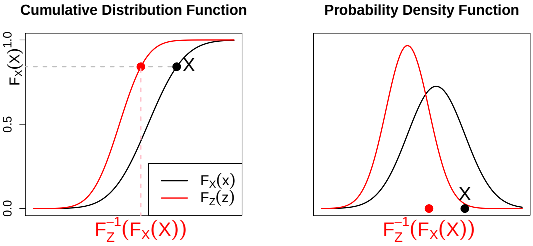

Problem: we don’t know the true future distribution

Model- vs. Observation-based Quantile Mapping

Write

- $Y^{(h)}$ for an observed (*historical*) quantity and
- $\tilde{Y}^{(h)}$ and $\tilde{Y}^{(f)}$ for analogous historical and projected (*future*) quantities from a GCM

Model- vs. Observation-based Quantile Mapping

Write

- $Y^{(h)}$ for an observed (*historical*) quantity and
- $\tilde{Y}^{(h)}$ and $\tilde{Y}^{(f)}$ for analogous historical and projected (*future*) quantities from a GCM

A model-based approach: ideally,

$$\hat{Y}^{(f)} = F^{-1}_{Y^{(f)}}\left(F_{\tilde{Y}^{(f)}}\left(\tilde{Y}^{(f)}\right)\right)$$

1. Assume
   $F^{-1}_{Y^{(f)}} F_{\tilde{Y}^{(f)}} = F^{-1}_{Y^{(h)}} F_{\tilde{Y}^{(h)}}$
2. $\hat{Y}^{(f)} = F^{-1}_{Y^{(h)}}(F_{\tilde{Y}^{(h)}}(\tilde{Y}^{(f)}))$

Model- vs. Observation-based Quantile Mapping

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An observation-based approach: ideally, $\hat{Y}^{(f)} = F^{-1}_{Y^{(f)}}\left(F_{Y^{(h)}}\left(Y^{(h)}\right)\right)$

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2. $\hat{Y}^{(f)} = F^{-1}_{\tilde{Y}^{(f)}}(F_{\tilde{Y}^{(h)}}(Y^{(h)}))$

In practice, lots of important details / modifications

- Multiple variants based on what you're assuming
- How to model and estimate? (Especially since none of these distributions are constant in time.)

Example: Observation-based temperature simulations

Haugen et al. (2019) propose simulating future temperature at time as

$$\hat{T}_t^{(f)} = \underbrace{m_t^{(h)}}_{\text{observed median}} + \underbrace{(\tilde{m}_t^{(f)} - \tilde{m}_t^{(h)})}_{\text{GCM change in median}} + \underbrace{r_t^{(h)}}_{\text{observed scale}} \underbrace{\frac{\tilde{r}_t^{(f)}}{\tilde{r}_t^{(h)}}}_{\text{GCM change in scale}} \underbrace{F^{-1}_{\tilde{Z}_t^{(f)}}\left(F_{\tilde{Z}_t^{(h)}}\right.}_{\text{GCM change in distribution}} \left(\underbrace{\frac{T_t^{(h)} - m_t^{(h)}}{r_t^{(h)}}}_{Z_t^{(h)} = \text{ normalized observations}}\right)\left.\vphantom{F}\right)$$

where the quantile function $F^{-1}_{\tilde{T}}$ is modeled semiparametrically as a function of year, day, and additional covariates using quantile regression and additional details for extremal quantiles.

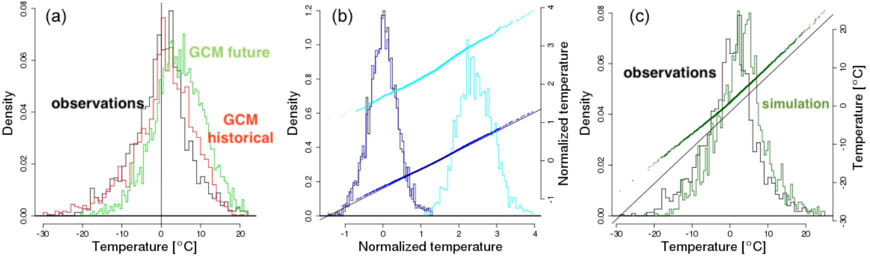


image adapted from Haugen et al. (2019)

Multivariate Methods

Climate change impacts can depend on *multivariate* and *spatiotemporal* relationships.

- Day-to-day vs. interannual temperature variability
- Precipitation events over large or small geographic areas
- Humid vs. dry heat waves

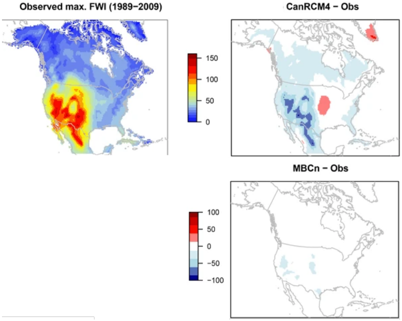
images from
https://warm1069.com/keeping-animals-safe-in-a-heatwave/ and
https://www.theguardian.com/us-news/2021/jun/14/us-heatwave-southwest-utah-california-nevada-arizona

Cannon's MBCn Method

Cannon (2018) proposes an iterative method:

1. Random orthogonal rotation to both climate model and observational target datasets
2. Univariate (model-based) quantile mapping to marginal distributions
3. Rotate datasets back
4. Repeat until convergence

Idea is based on image processing algorithms for color correction



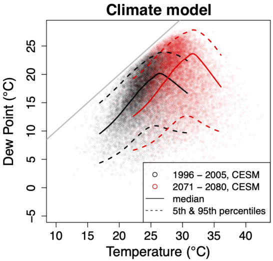
Projecting fire risk from temperature, humidity, wind speed, and precipitation
(image adapted from Cannon (2018))

Observation-based conditional quantile mapping

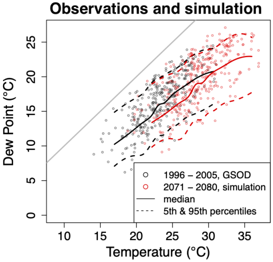
In Poppick and McKinnon (2020), we take a conditional approach (simulating temperature and dew point):

1. Generate an observation-based temperature simulation accounting for changes in mean and *temporal covariance*
   - not discussed, involves Fourier methods for time series (see Poppick et al. (2016))
2. Generate a dew point simulation conditional on the temperature simulation using a quantile mapping approach.

Illustration, Minneapolis JJA Temperature and Dew Point



- CESM1-LE project increase in risk of historically hot and humid events
- But underlying relationship differs from observations



- Proposed simulation produces changes that "look similar" to changes in CESM1-LE
- Smaller increase in risk of historically hot and humid events in simulation compared to CESM1-LE

What Would It Mean for the GCM to Capture "Changes"?

Suppose effect of global mean temperature is the same in the GCM and reality, i.e.,

$$\underbrace{F^{-1}_{\log(\tilde{Y}_{d,y})}(\tau)}_{\text{GCM distribution}} = \tilde{\alpha}_{0,\tau} + \underbrace{\tilde{g}_\tau(d)}_{\text{seasonality}} + \underbrace{\tilde{\alpha}_{1,\tau}\tilde{G}_y}_{\uparrow} + \underbrace{\tilde{h}_\tau(\tilde{T}_{d,y} - \tilde{\mu}_{d,y})}_{\text{effect of local temperature deviation}}$$

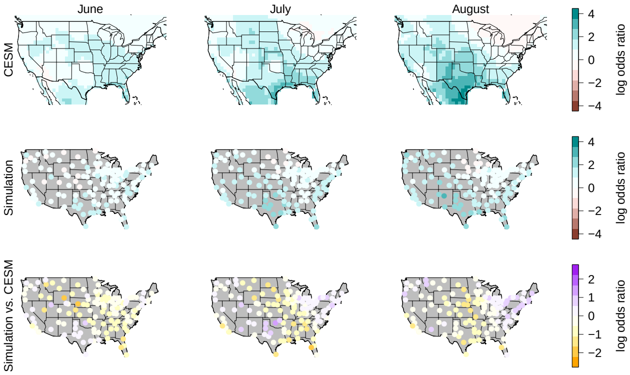
same effect of global warming ↓

$$\underbrace{F^{-1}_{\log(Y_{d,y})}(\tau)}_{\text{real world distribution}} = \alpha_{0,\tau} + g_\tau(d) + \tilde{\alpha}_{1,\tau}\tilde{G}_y + h_\tau(T_{d,y} - \mu_{d,y})$$

where $\tilde{X}$ is a GCM quantity and X is the analogous real-world quantity

Change in risk of historically high humidity (conditional 95th percentile) on historically hot (95th percentile) day

Changes from 2071-2080 vs. 1996-2005



- For historically high temperature, risk of historically high dew point increases
- Increases are smaller in observation-based simulation than in CESM1-LE

Summary

Bias correction methods blend observations and climate models to produce (hopefully) better calibrated future simulations

- Most popular approaches involve "quantile mapping", but differ in
  - what is preserved from observations vs. model output
  - how GCM "biases" or "changes" are encoded
- Important impacts may depend on multivariable and spatiotemporal changes
  - how to correct or adjust complex multivariate distribution?
- Bias corrected simulations also depend on underlying observations and GCM output
- The machine learning/statistics community can help develop and implement new methods, and better understand existing methods

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