Optimal Sparse Decision Trees

Xiyang Hu

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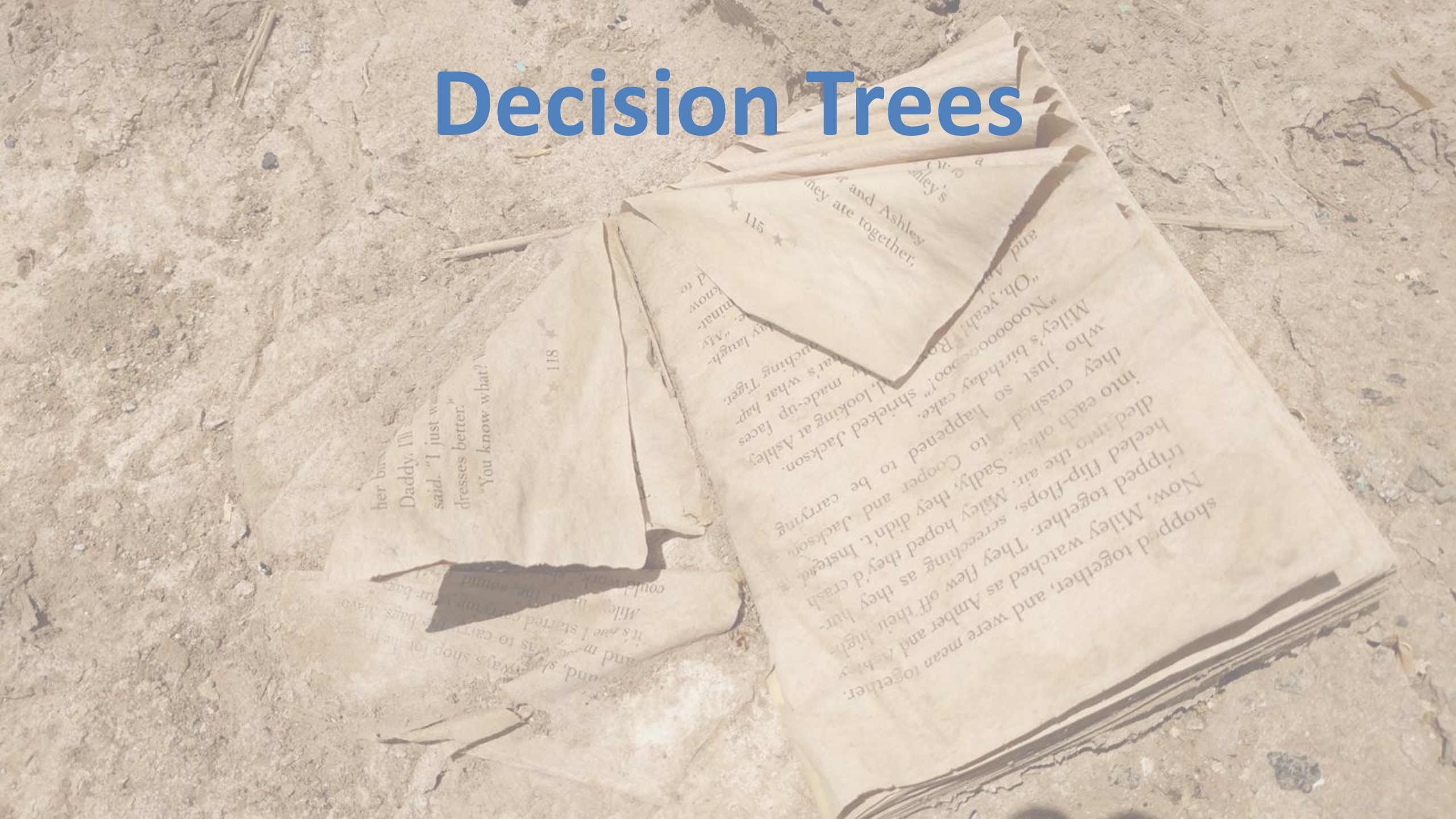
Cynthia Rudin

Duke University

Margo Seltzer

University of British Columbia

Decision Trees



Decision Trees

C4.5

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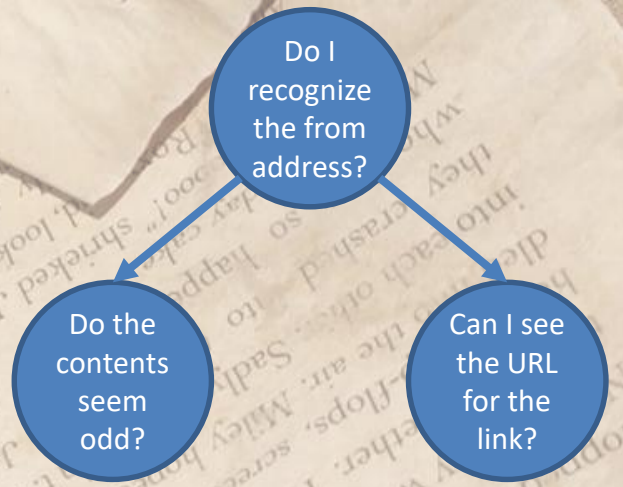
Should I click on the link in this email?



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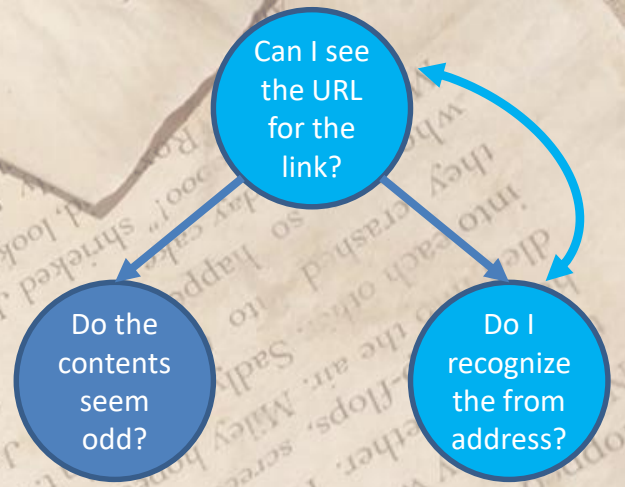
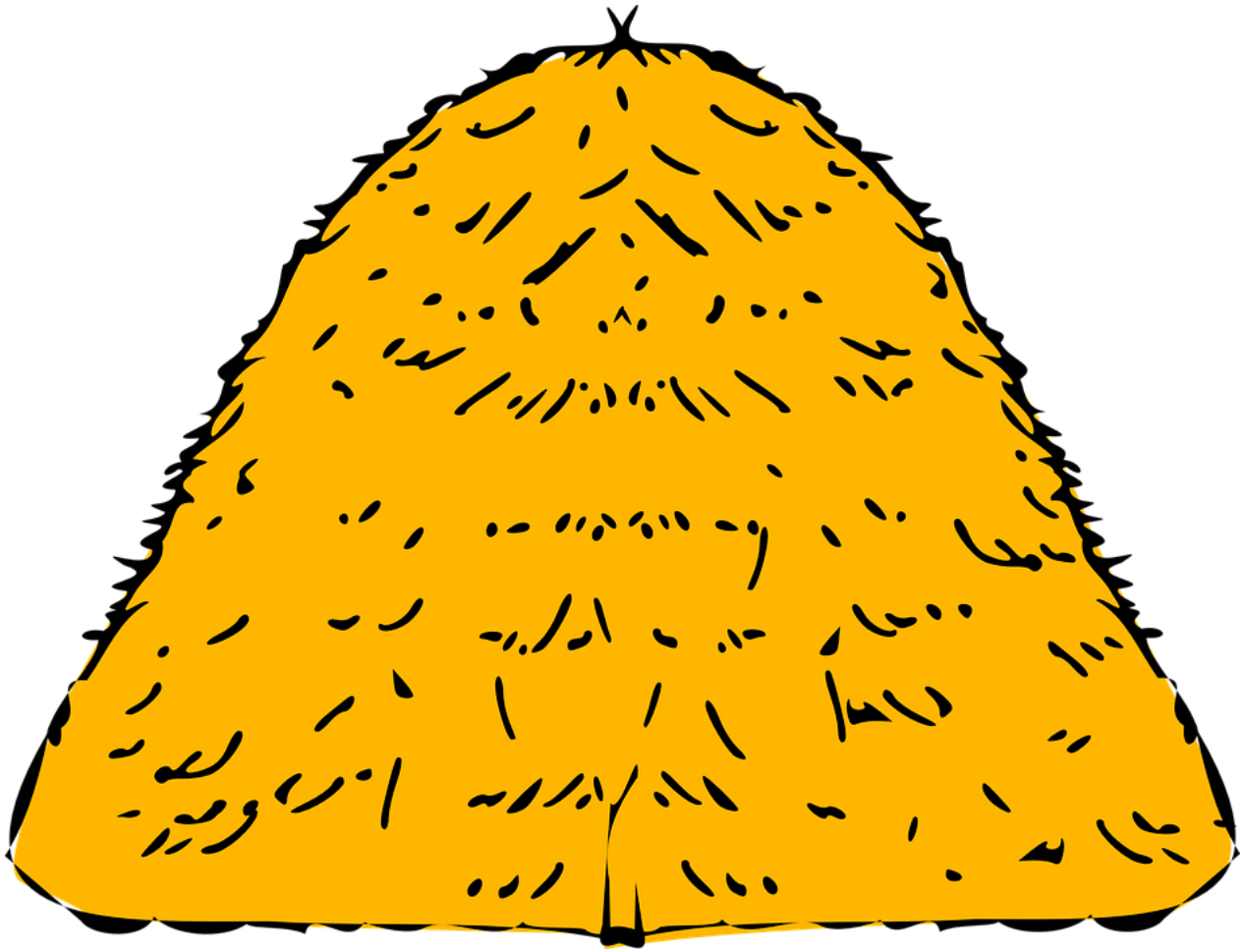


Why not just find the Best Tree?


Why not just find the Best Tree?
O(10^28)

Could we Effectively Search that Space?



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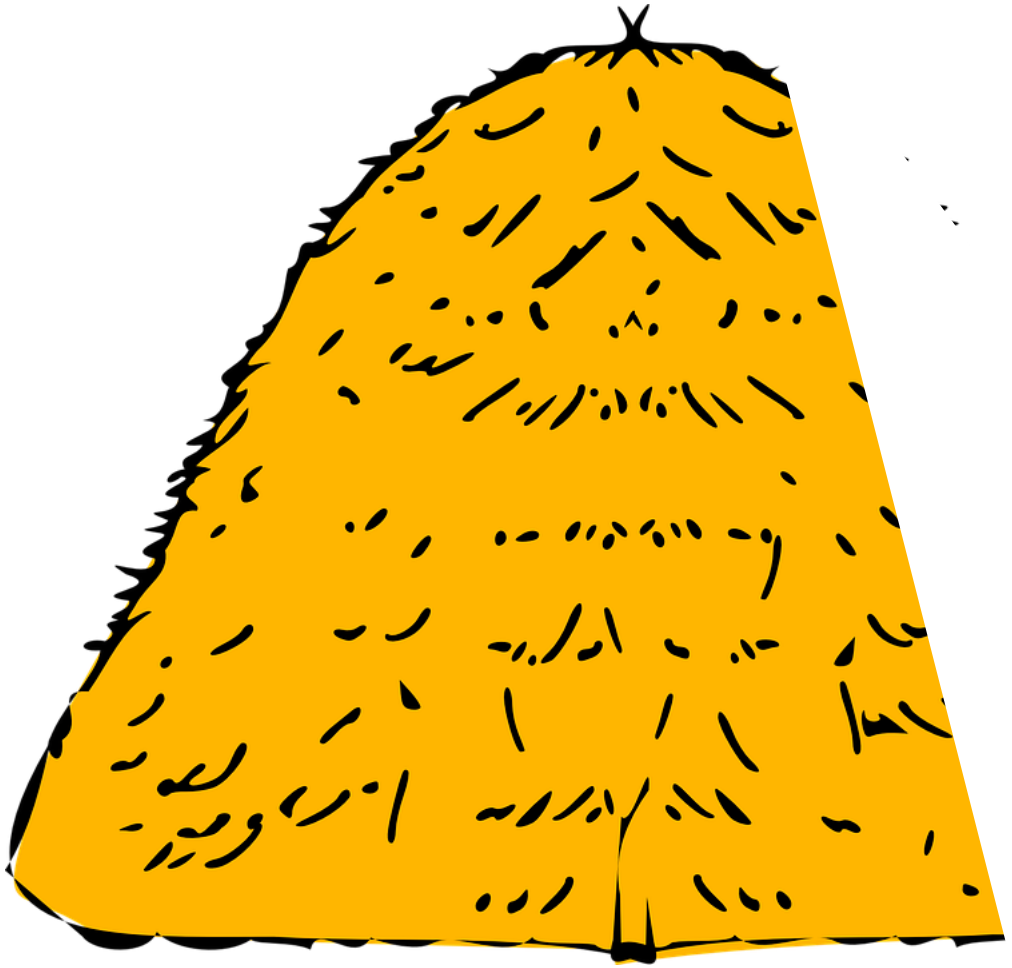

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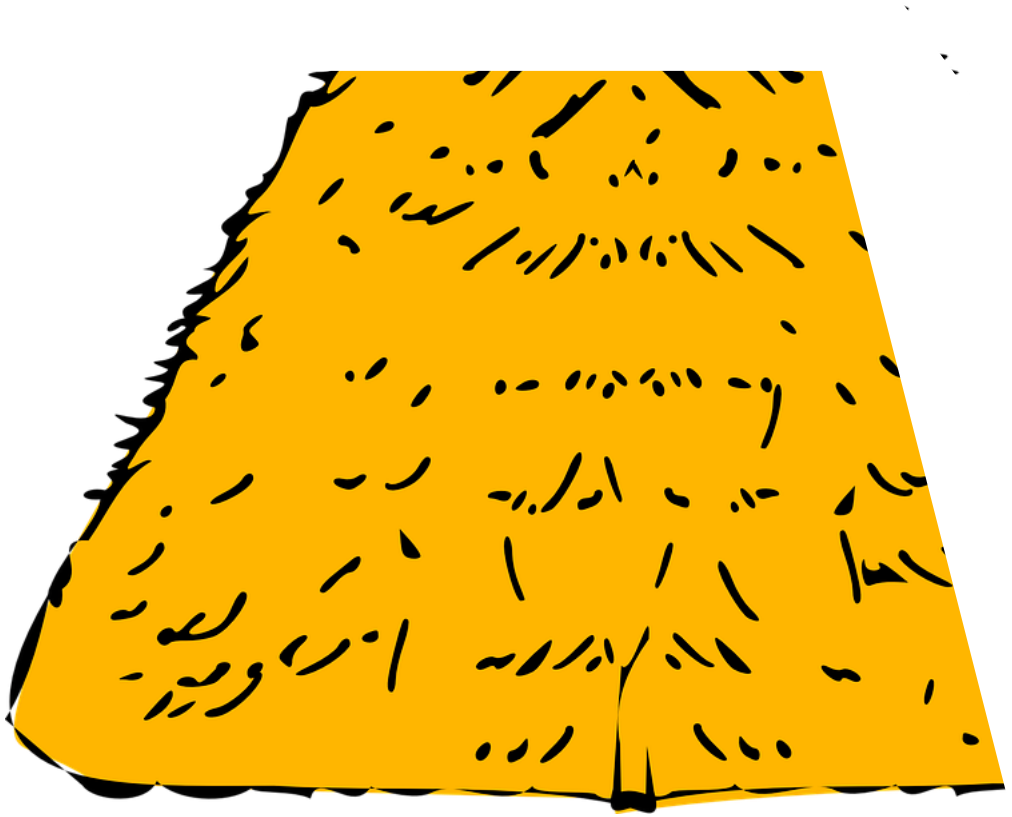

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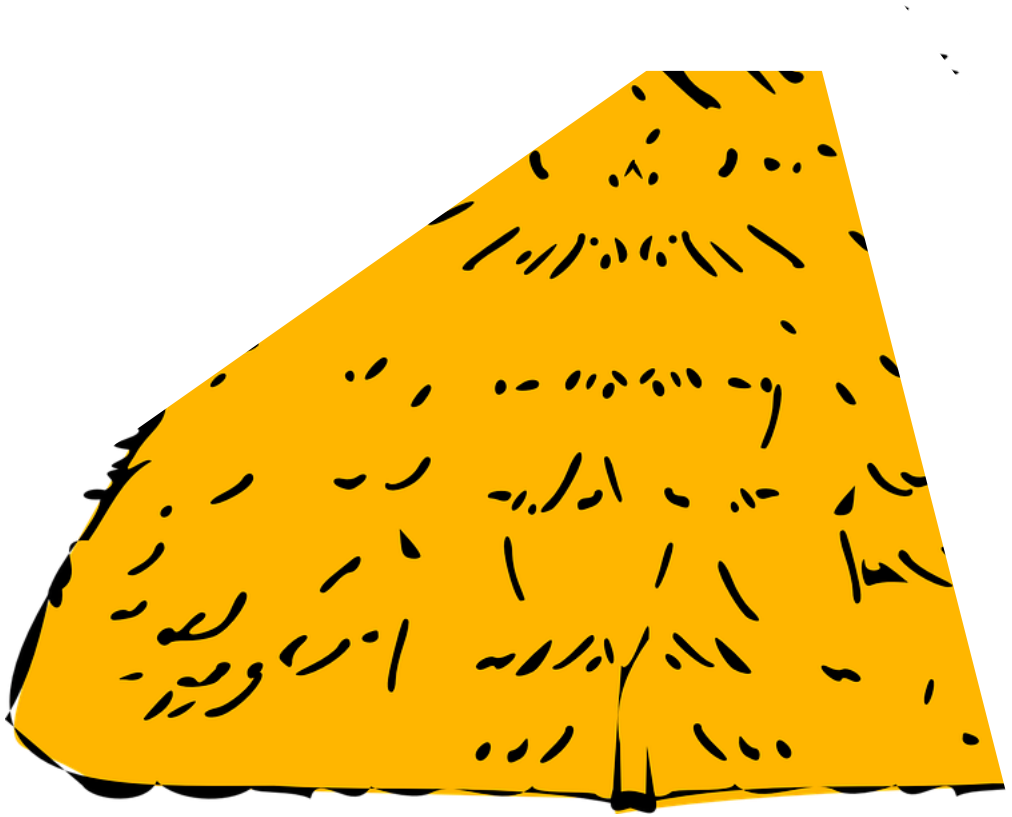

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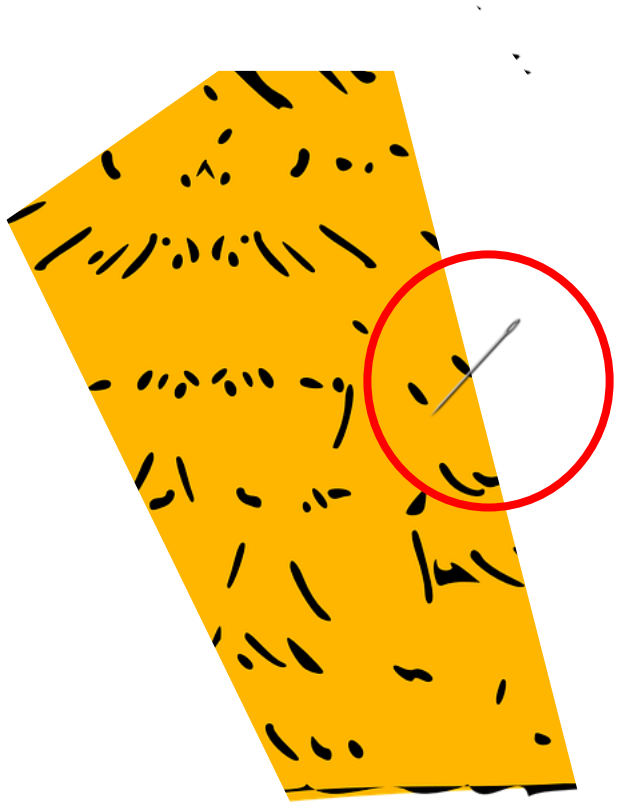

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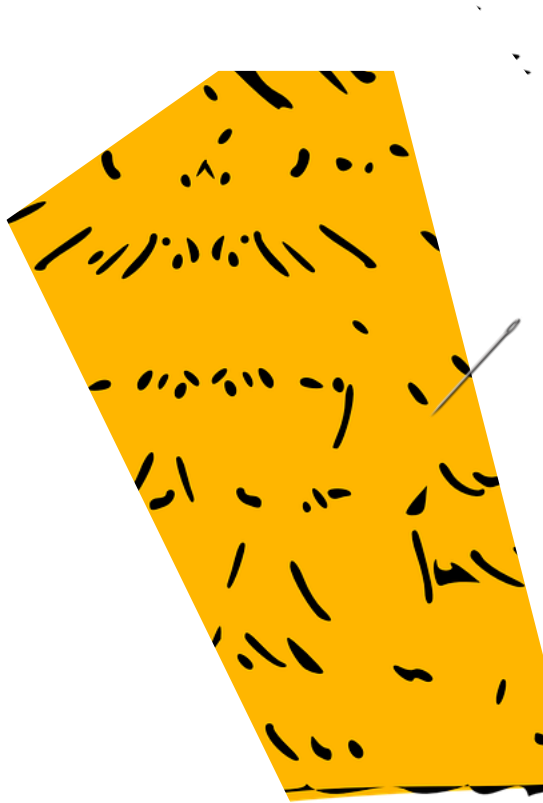

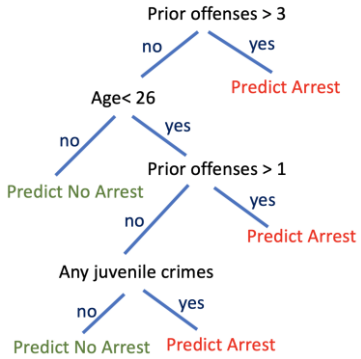

The Optimization Problem

$$\hat{L}(\text{tree},\{(x_i,y_i)\}_i) = \frac{1}{n}\sum_{i=1}^{n} 1_{[\text{tree}(x_i)^1\ y_i]} + C(\#\,\text{leaves in tree})$$

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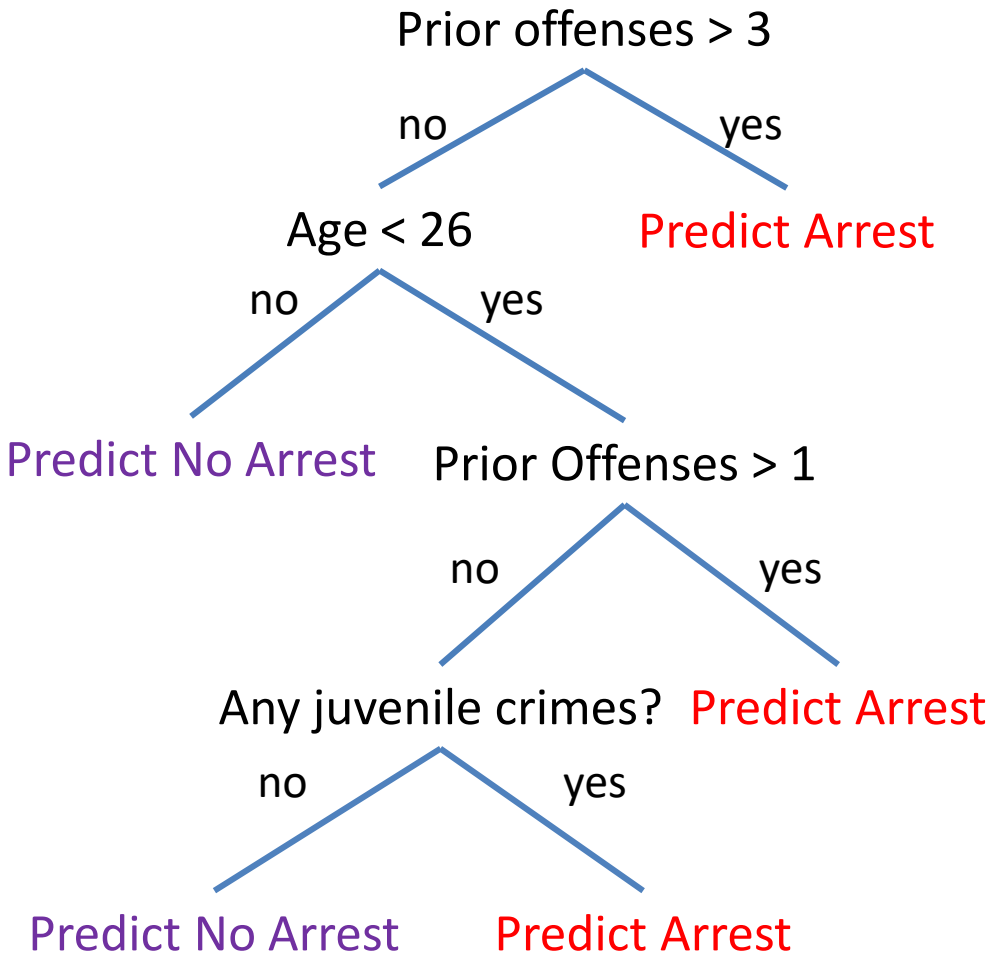
The Optimization Problem

$$\hat{L}(\text{tree}, \{(x_i, y_i)\}_i) = \underbrace{\frac{1}{n}\sum_{i=1}^{n} 1_{[\text{tree}(x_i) \neq y_i]}}_{\text{Misclassification error}} + \underbrace{C(\#\,\text{leaves in tree})}_{\text{Sparsity}}$$

Optimal Sparse Decision Tree

(Broward County Recidivism Data)



Optimal Sparse Decision Trees

Branch and Bound

+

Good Scheduling Order

+ **=**

Strong Bounds

+

Incremental Computation

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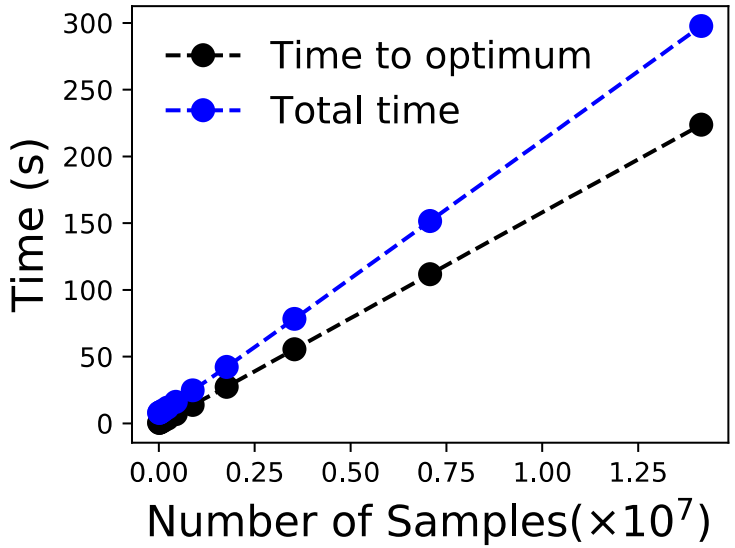

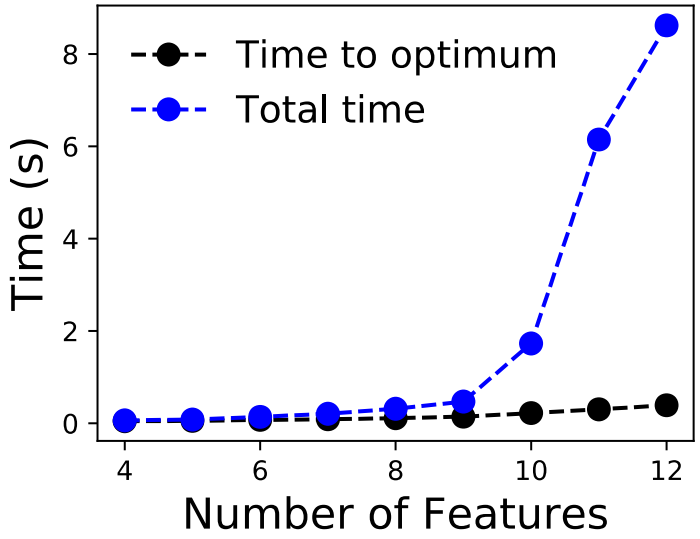

FAST

Optimal Sparse Decision Trees

Branch and Bound

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Good Scheduling Order

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Strong Bounds

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Incremental Computation

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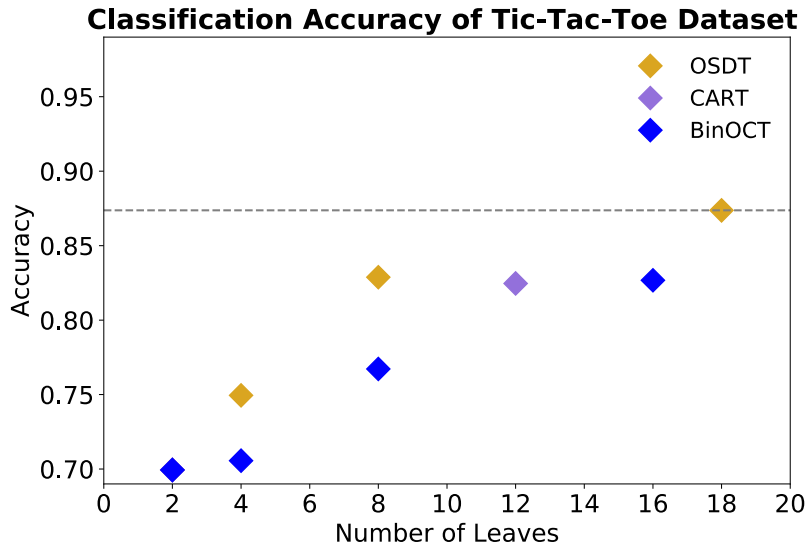
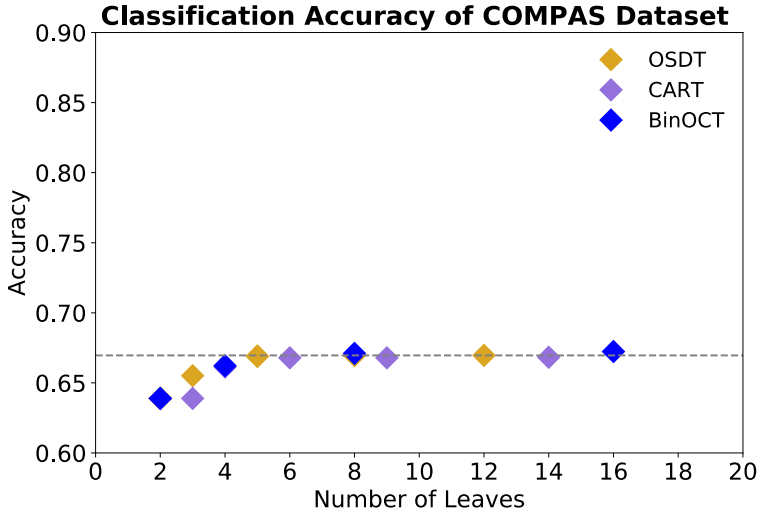


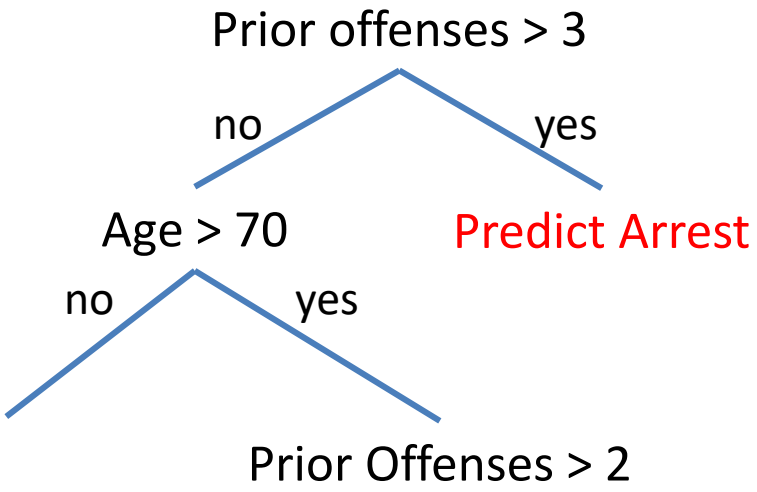


Accurate

Bounding the Search Space

Lower Bound on Node Support

Theorem: For an optimal tree, the support of each node must be above $2C$.



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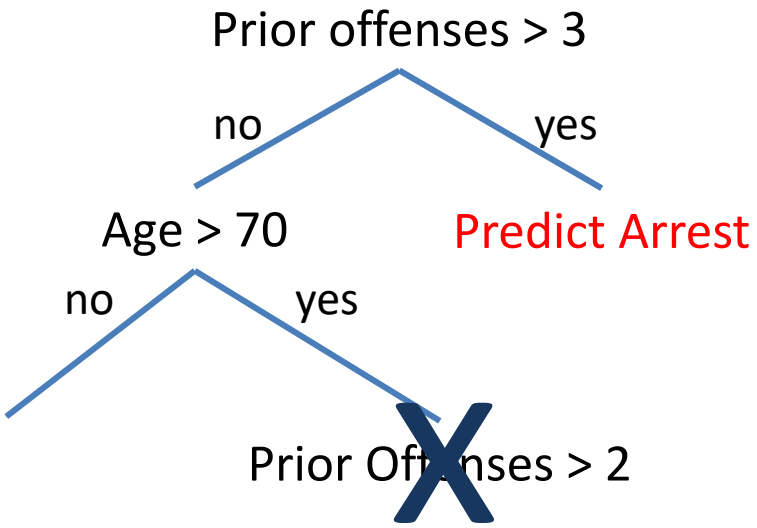

Node support insufficient to produce optimal solution

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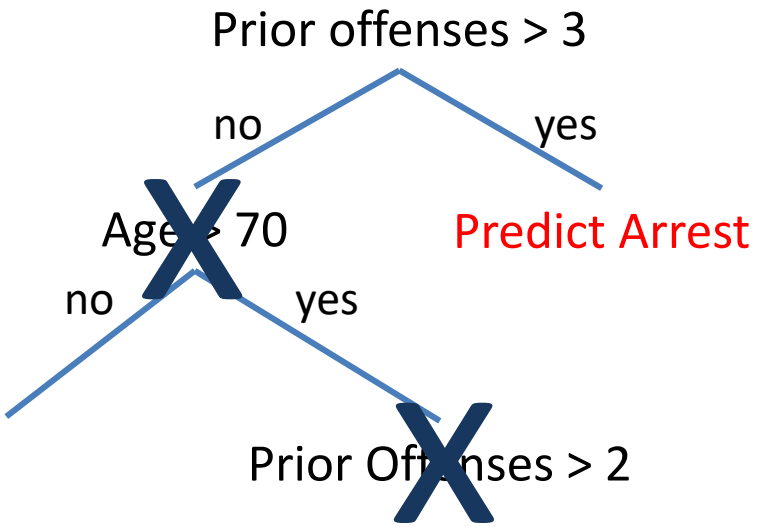

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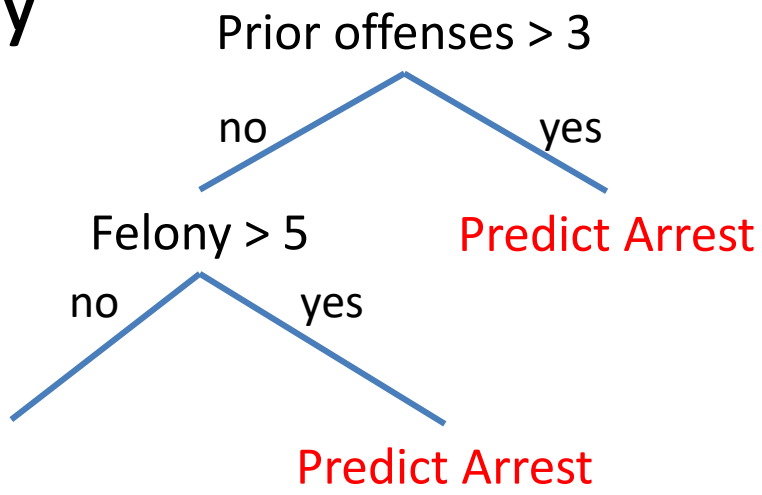
Lower Bound on Classification Accuracy

Theorem: Each leaf of an optimal tree correctly classifies at least fraction *C* of the data



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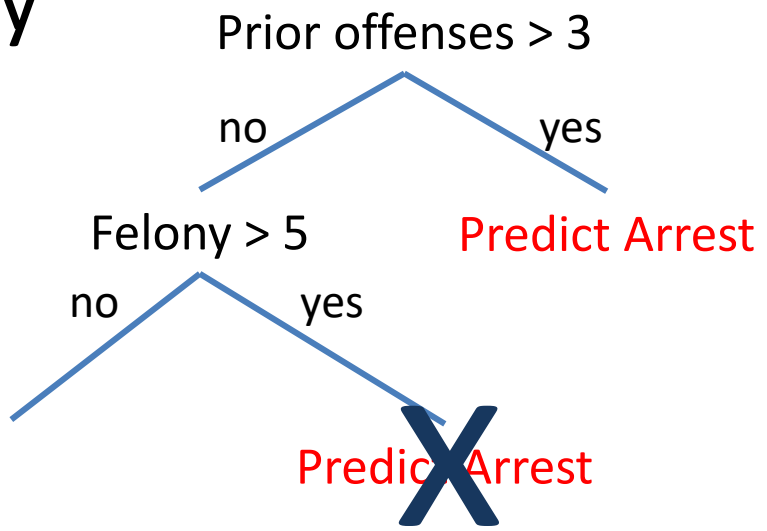

Doesn't classify at least Cn points correctly.

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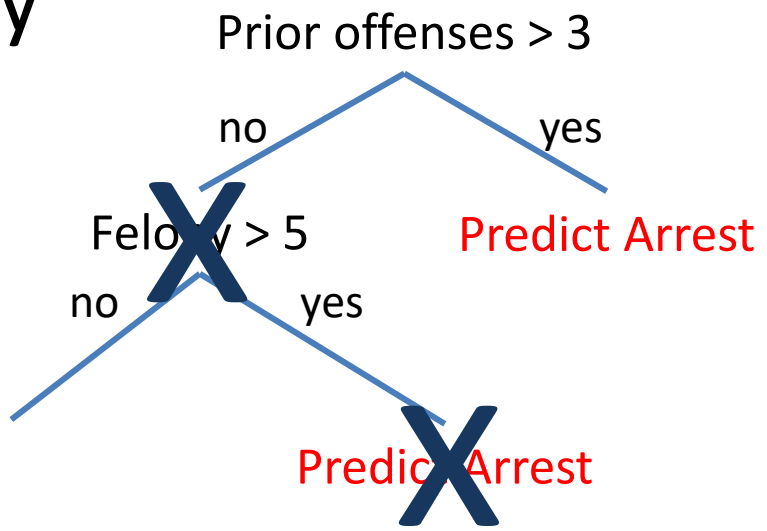

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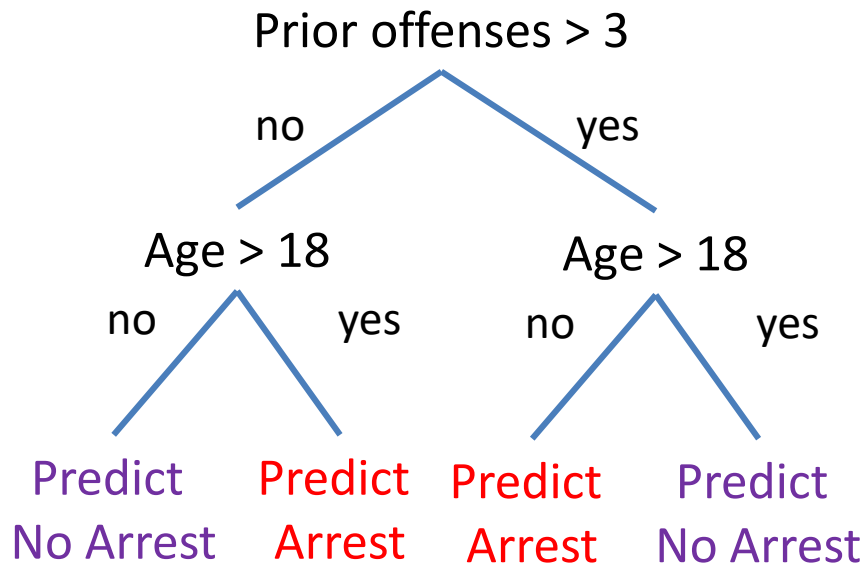
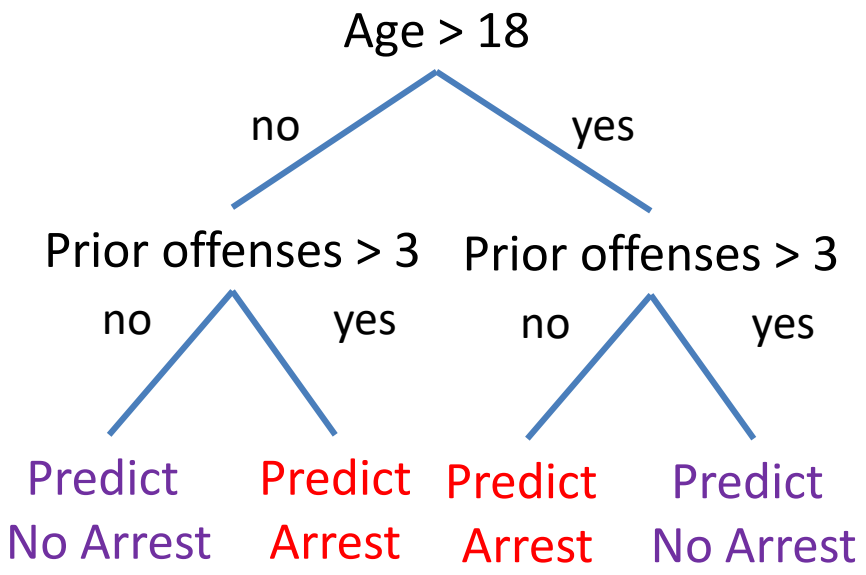
Bounding the Search Space

Permutation Bound

Theorem: If two trees have the same leaves, up to a permutation, all their child trees will be the same, so one of them can be pruned.





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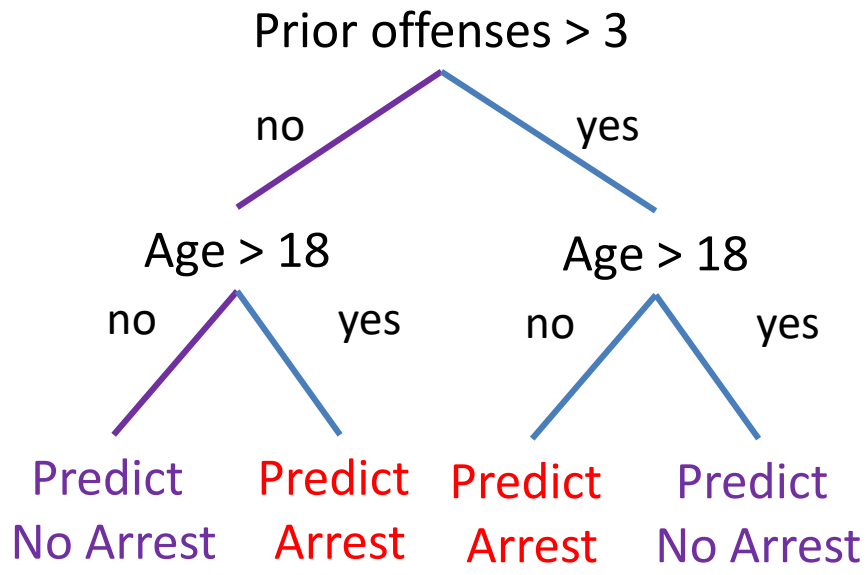

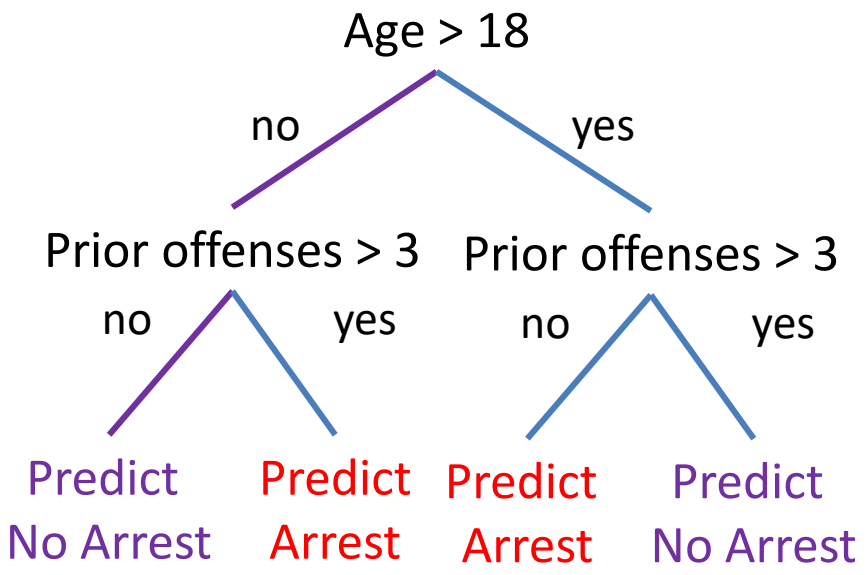

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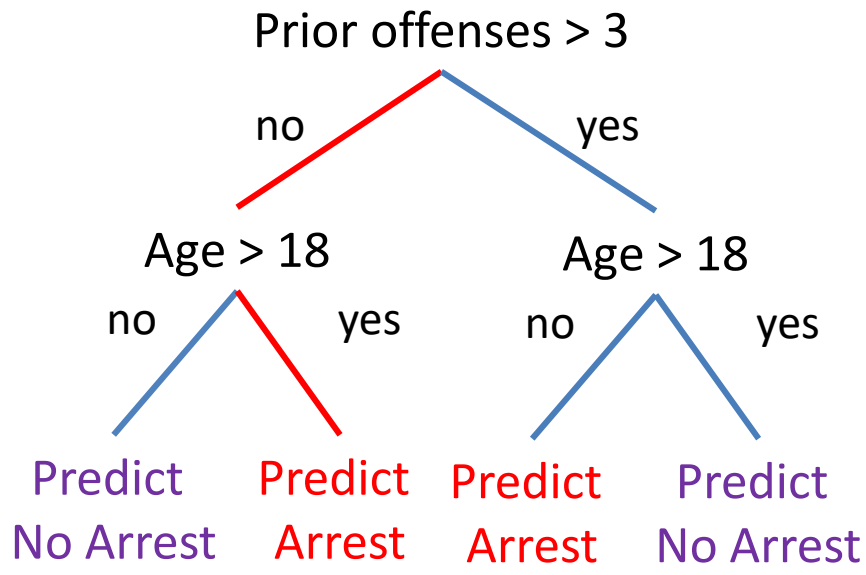

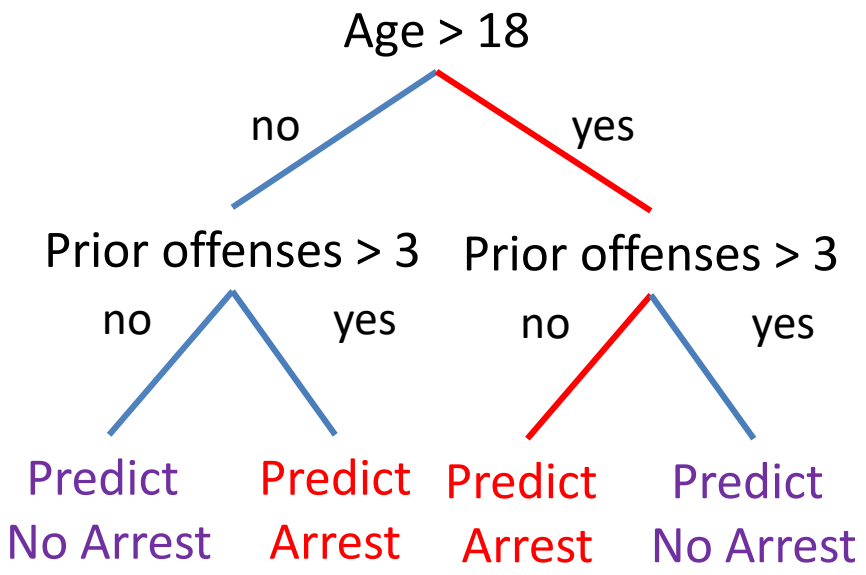

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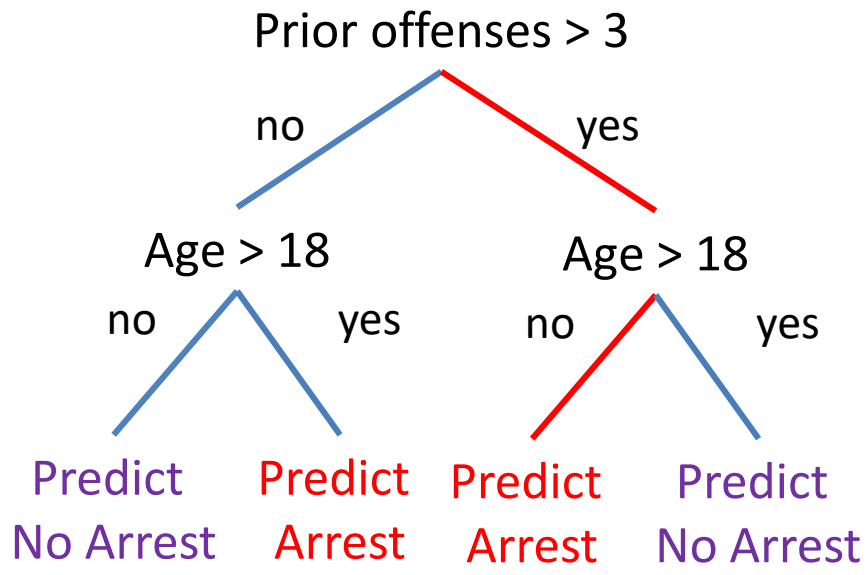

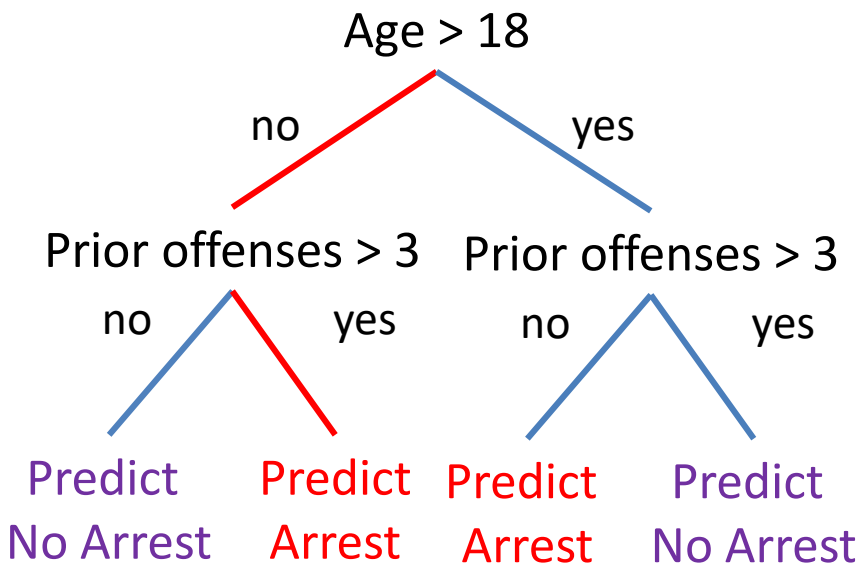

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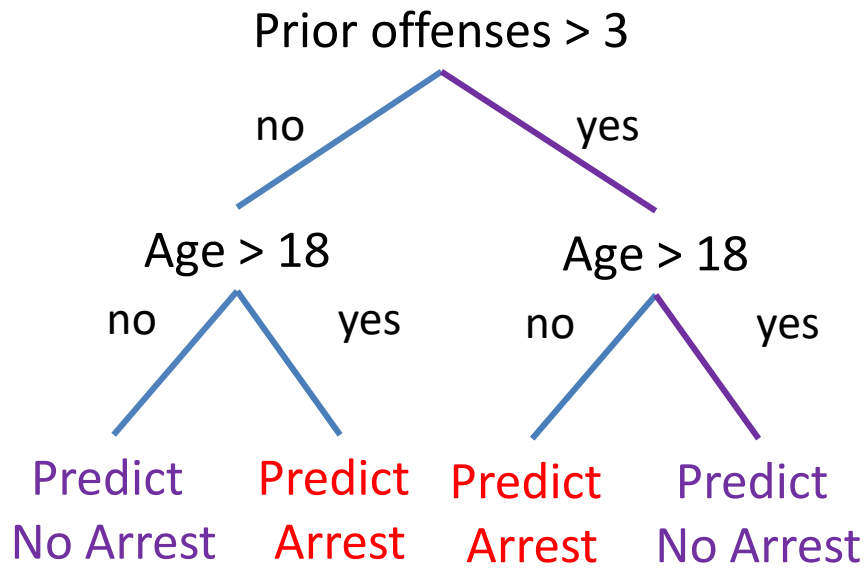
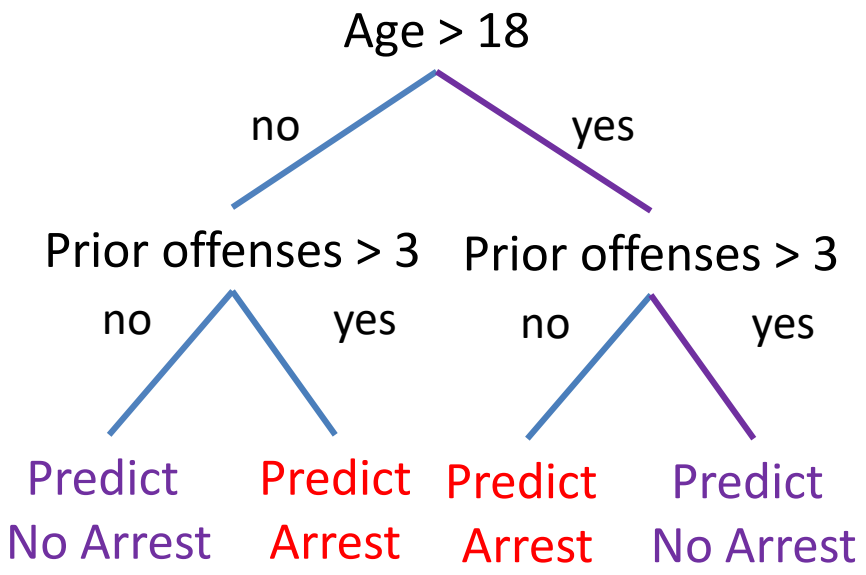




Bounding the Search Space

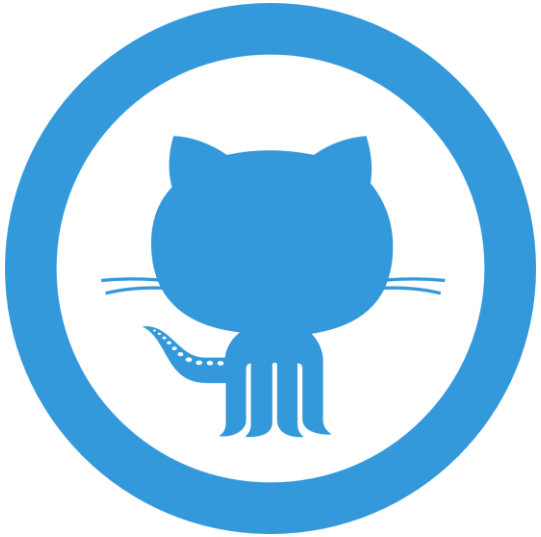
- Other bounds enable even more pruning
  - Equivalent points bound: Samples with the same features, but different predictions will produce misclassifications regardless of model.
  - Bound on the number of leaves: Regularization value bounds the number of leaves.

Optimal Sparse Decision Trees

Accurate

Open Source

FAST



Interpretable

https://github.com/xiyanghu/OSDT