Poster #1189

Numerically Accurate Hyperbolic Embeddings Using Tiling-Based Models

Tao Yu & Christopher De Sa
Department of Computer Science
Cornell University

Euclidean embedding:

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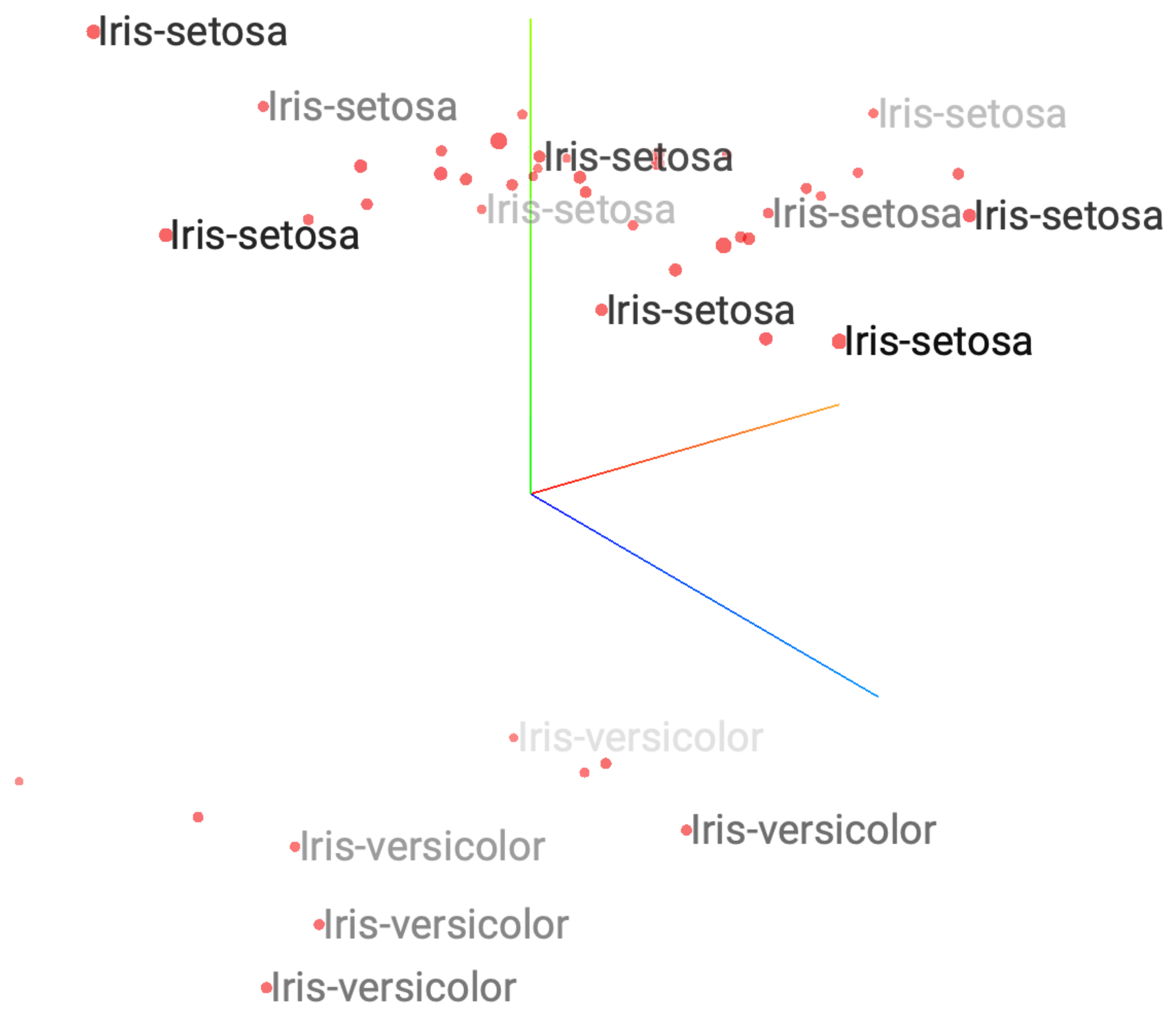

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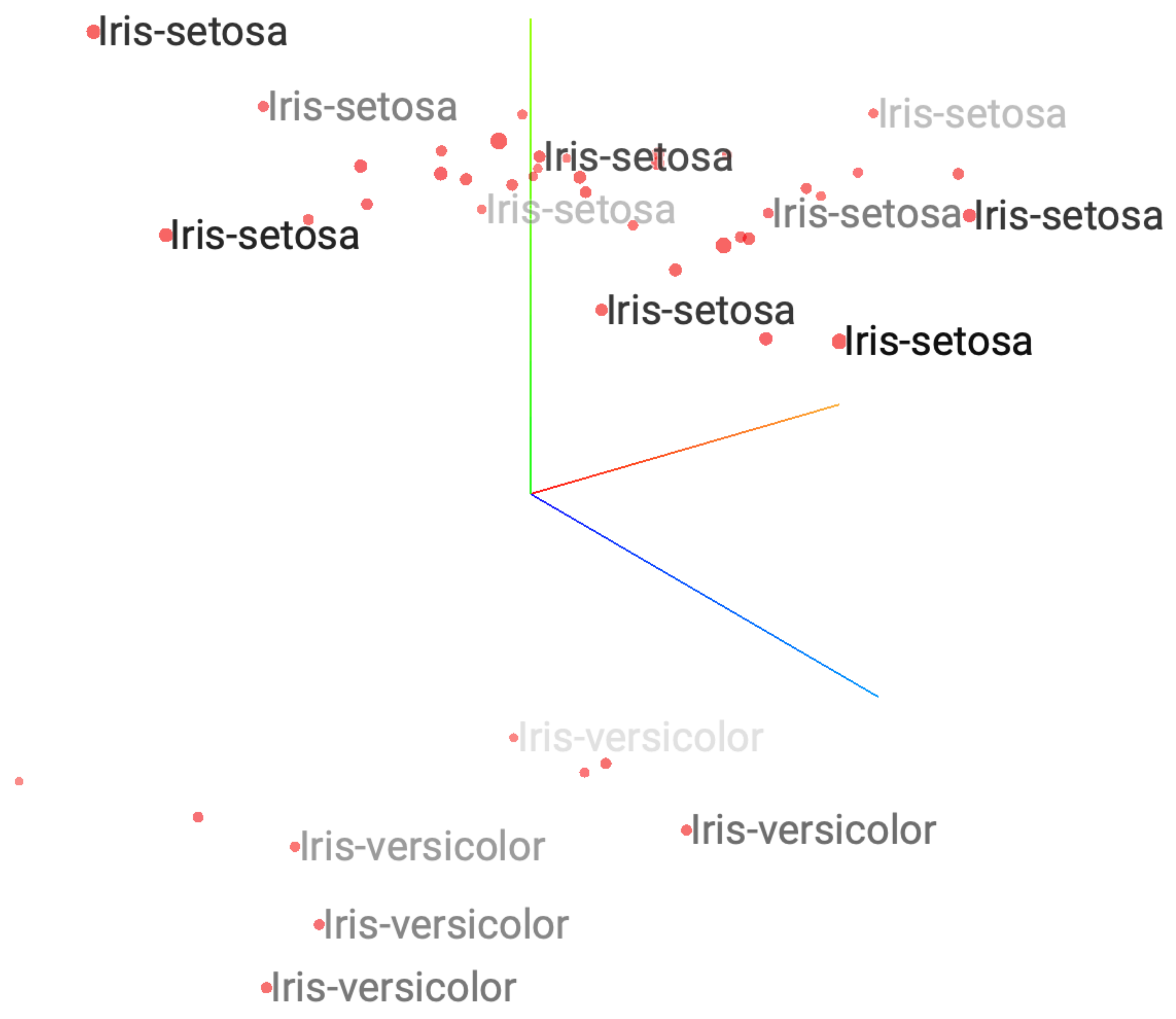

Hyperbolic embedding:

Poincaré Embeddings for Learning Hierarchical Representations

Neural Embeddings of Graphs in Hyperbolic Space

Learning Continuous Hierarchies in the Lorentz Model of Hyperbolic Geometry

Representation Tradeoffs for Hyperbolic Embeddings

LEARNING MIXED-CURVATURE REPRESENTATIONS IN PRODUCTS OF MODEL SPACES

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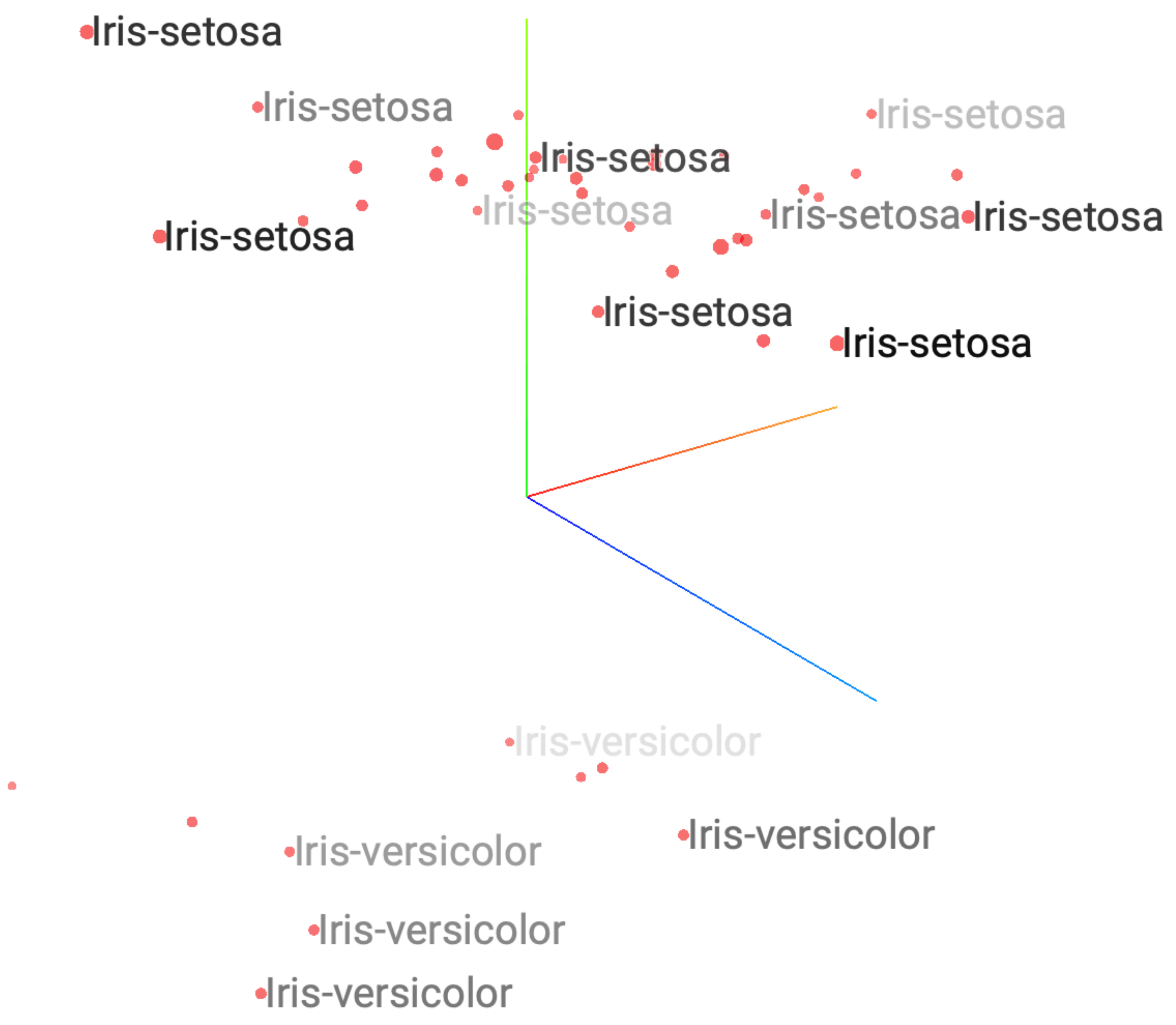

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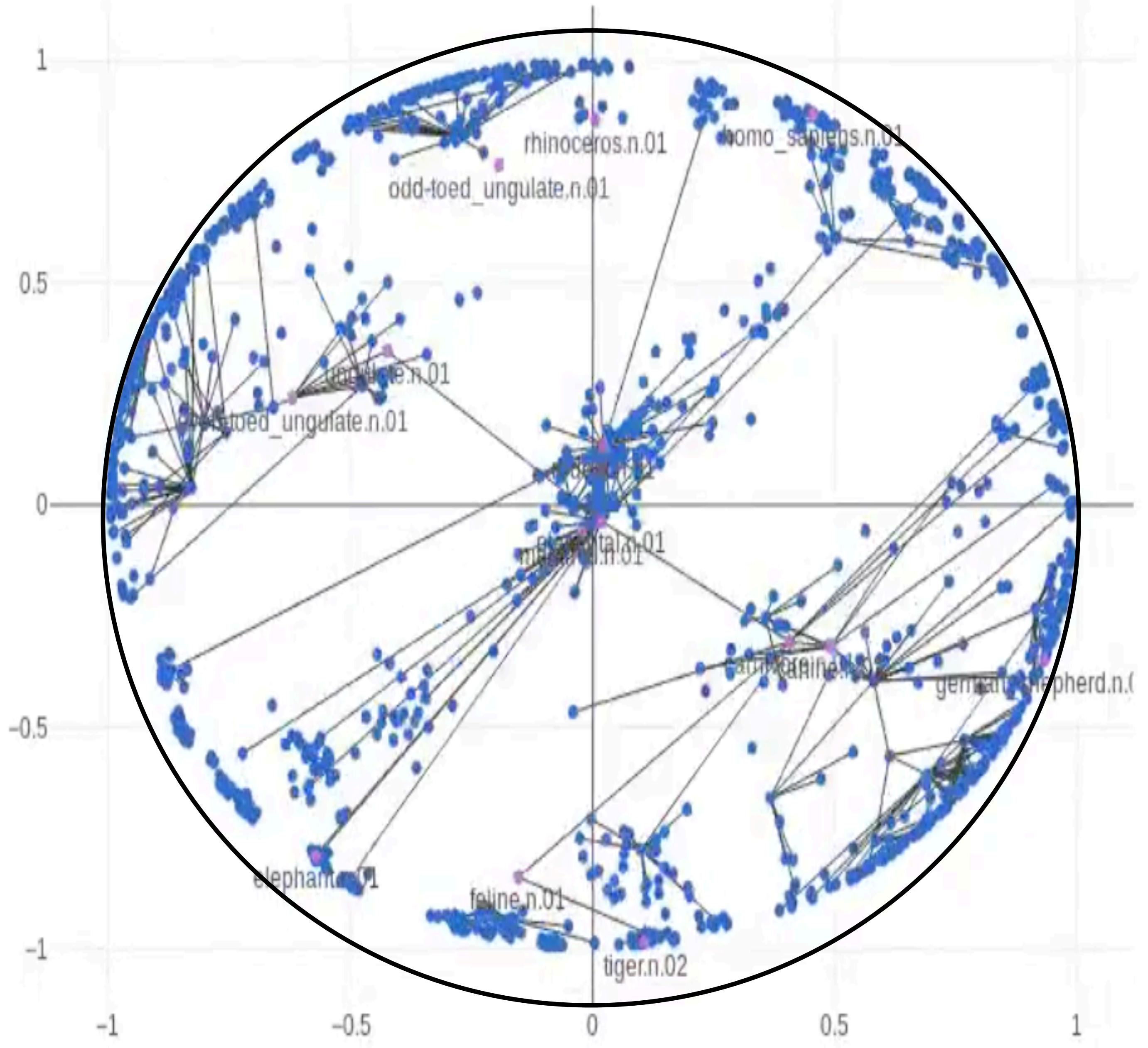

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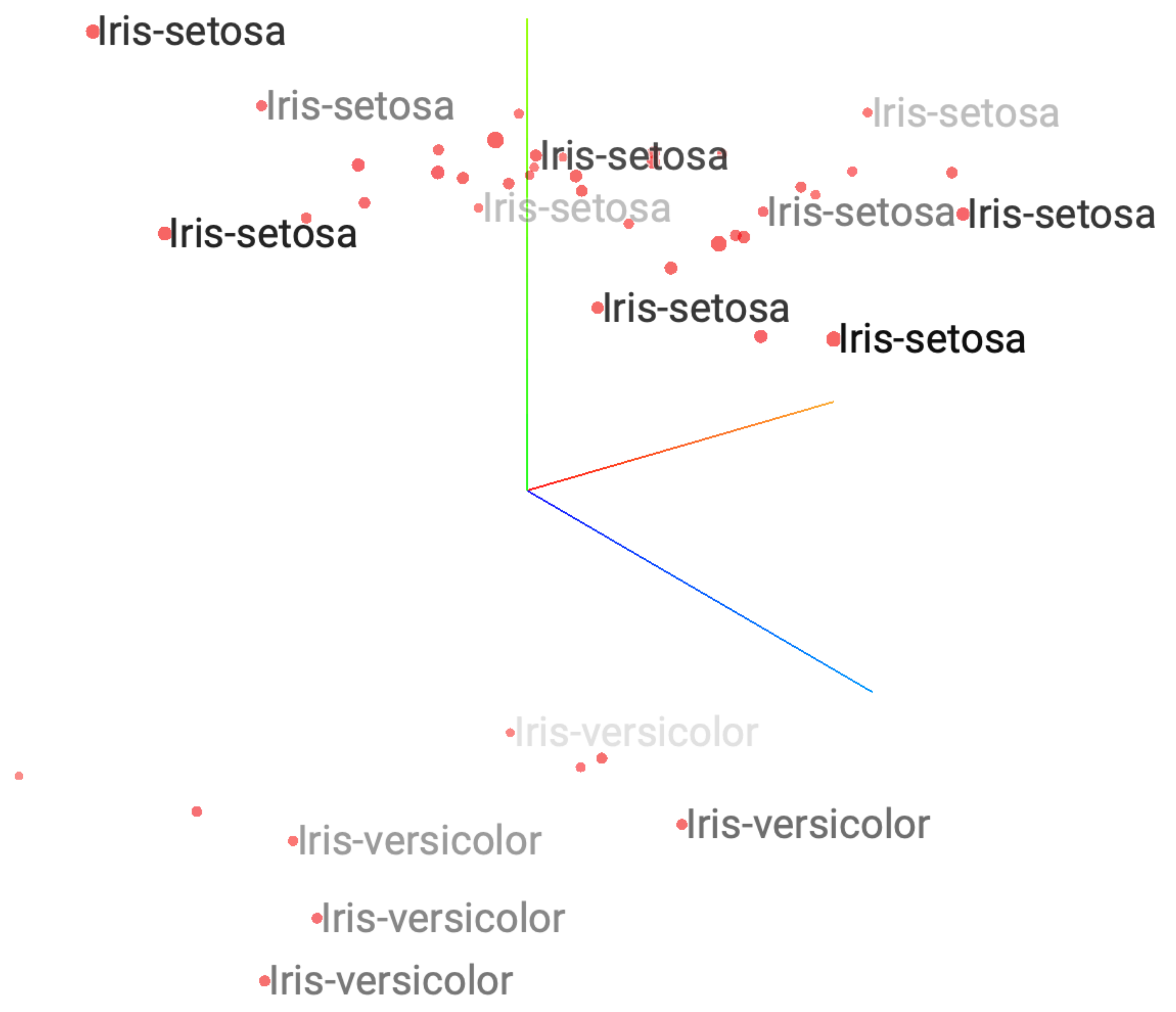
Euclidean embedding: Hyperbolic embedding:



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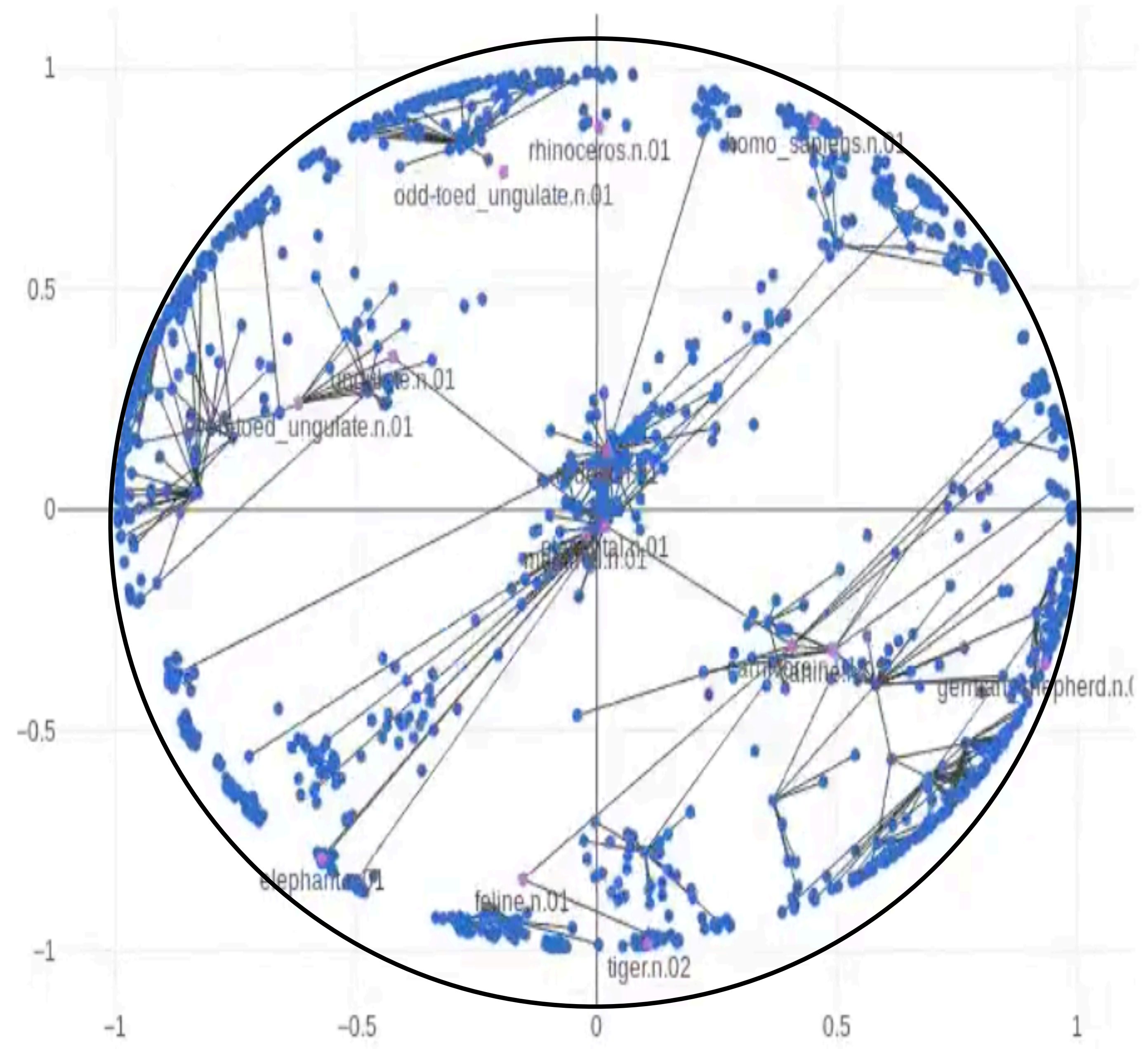

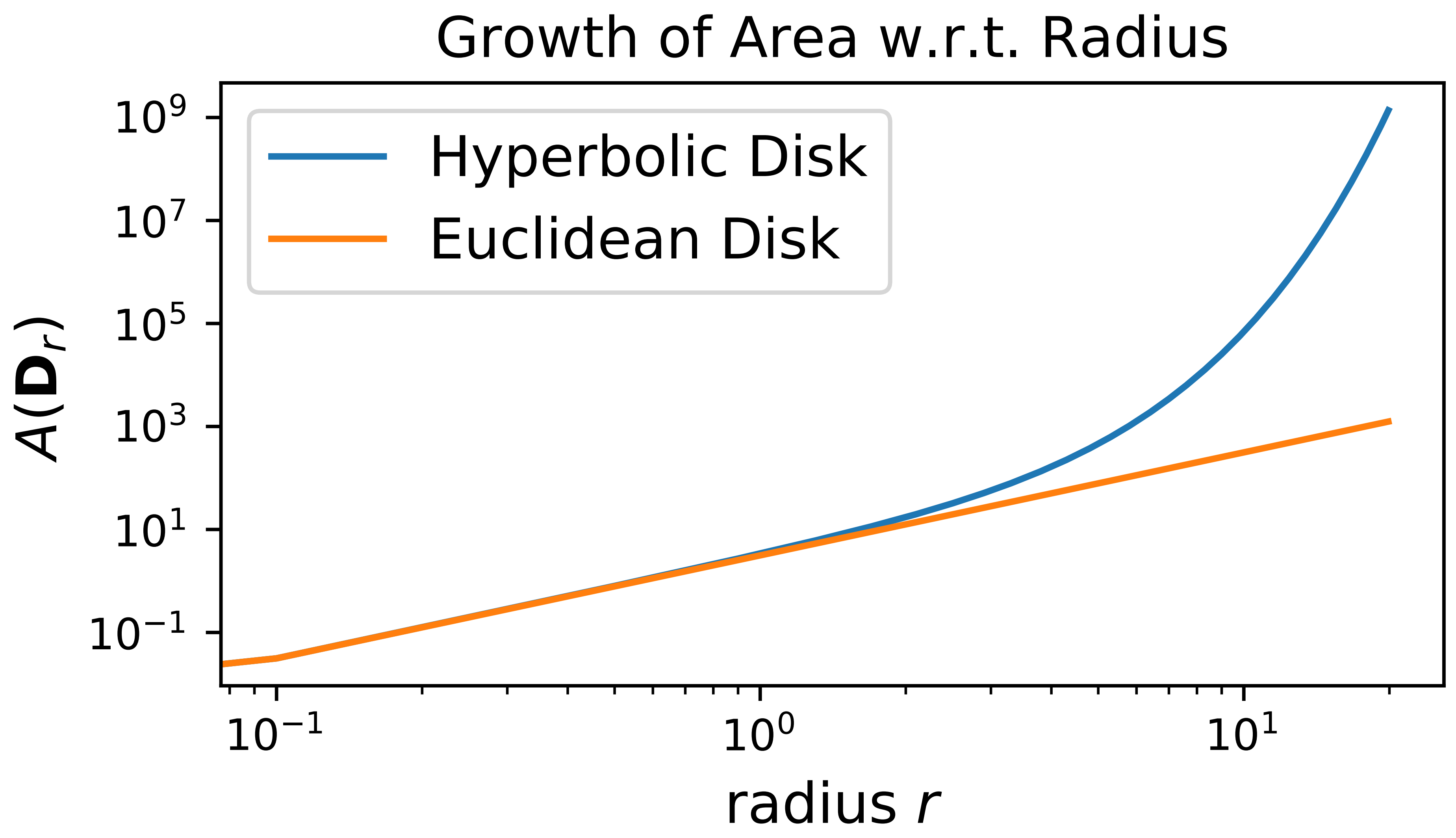

Area of a disk in the hyperbolic plane increases exponentially w.r.t. the radius (polynomially in Euclidean plane).

The NaN problem:

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Hyperbolic embeddings are limited by numerical issues when the space is represented by floating-points, standard models using floating-point arithmetic have unbounded error as points get far from the origin.

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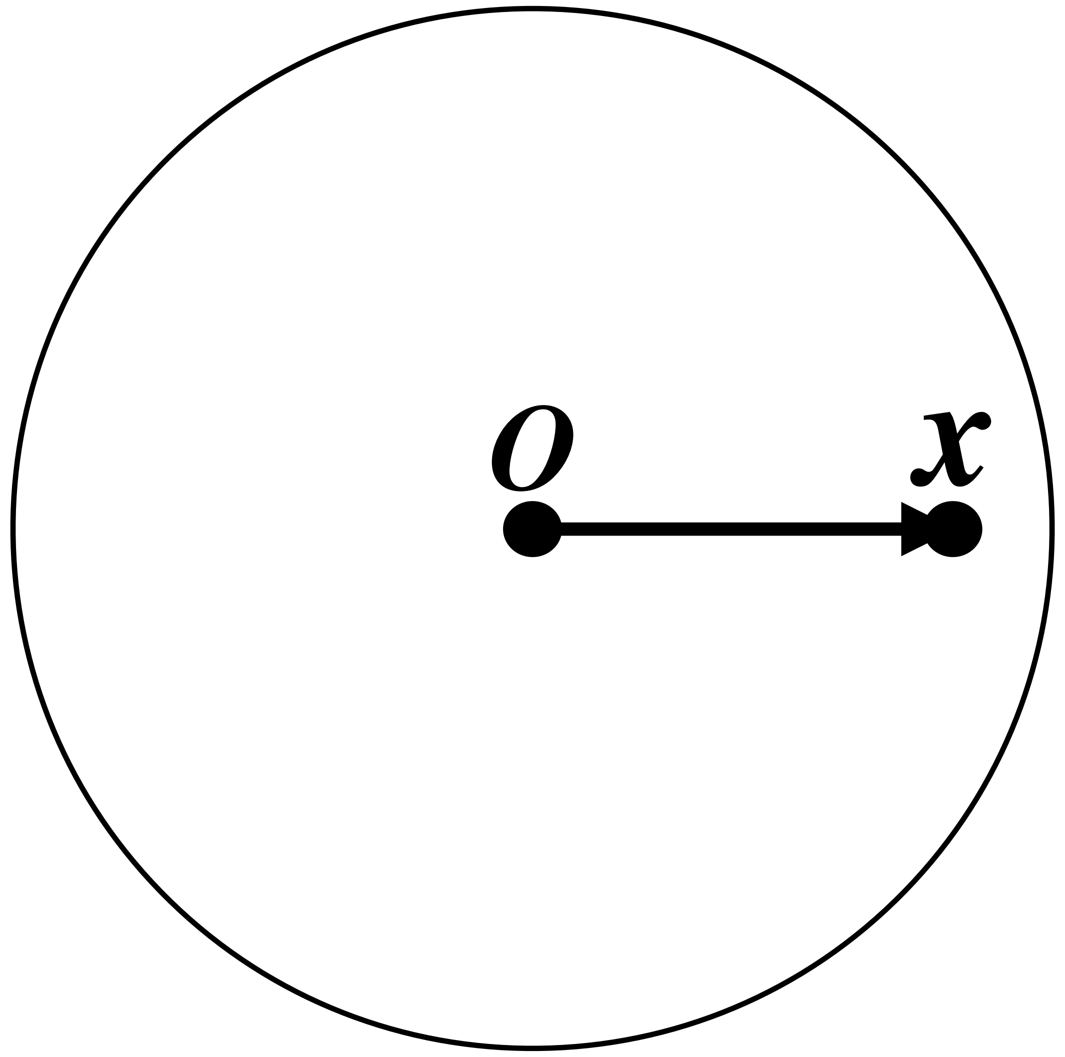

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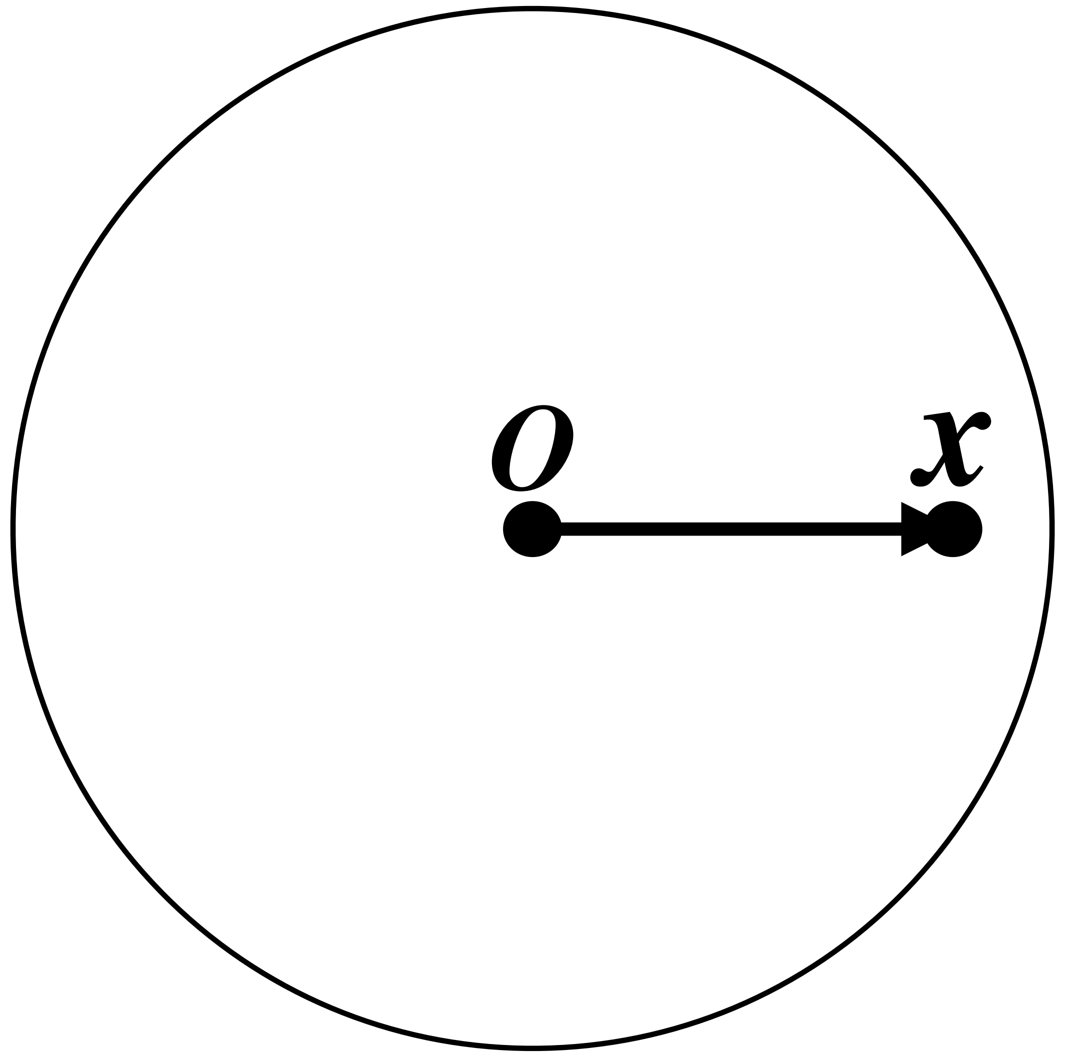

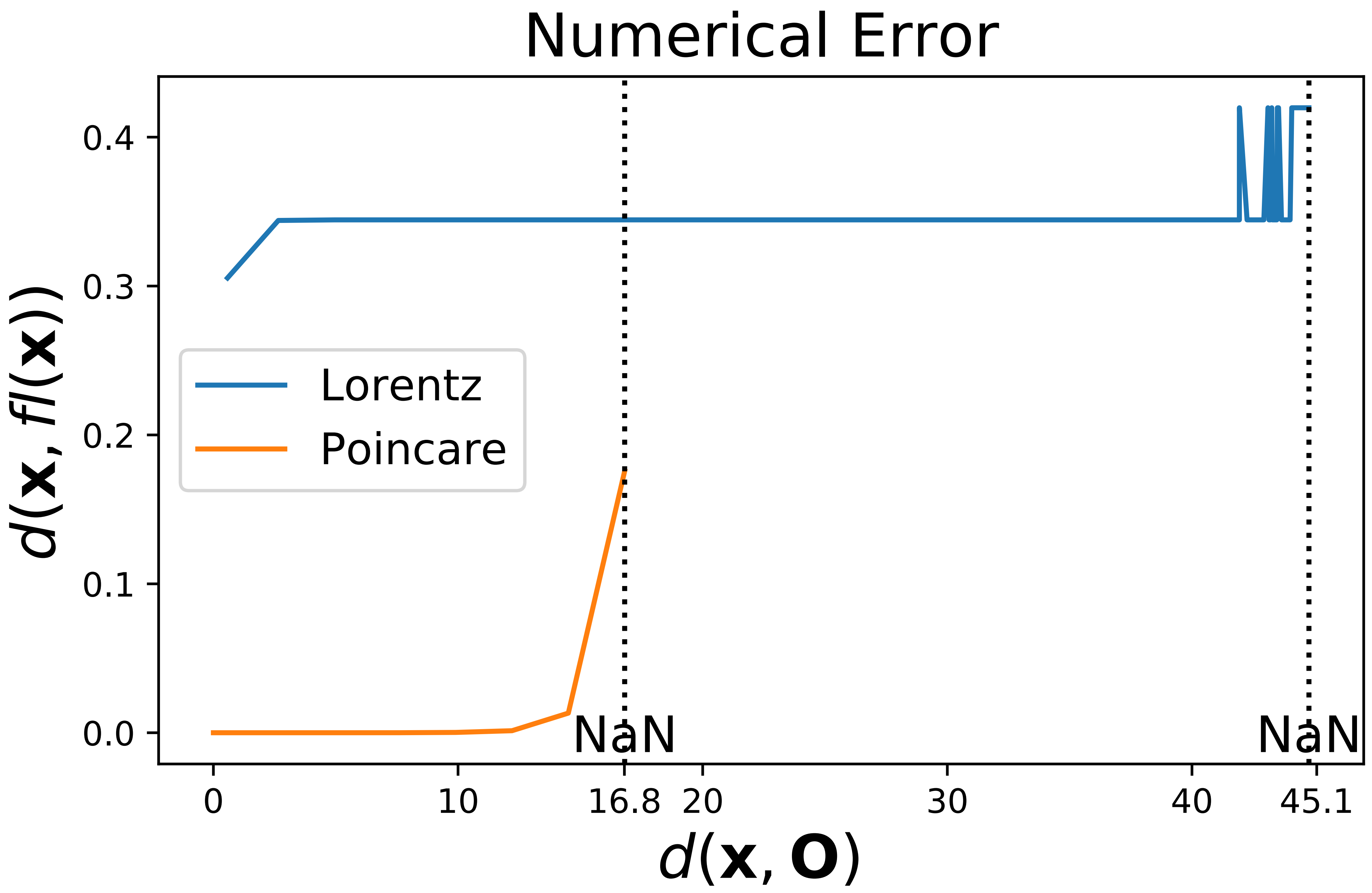

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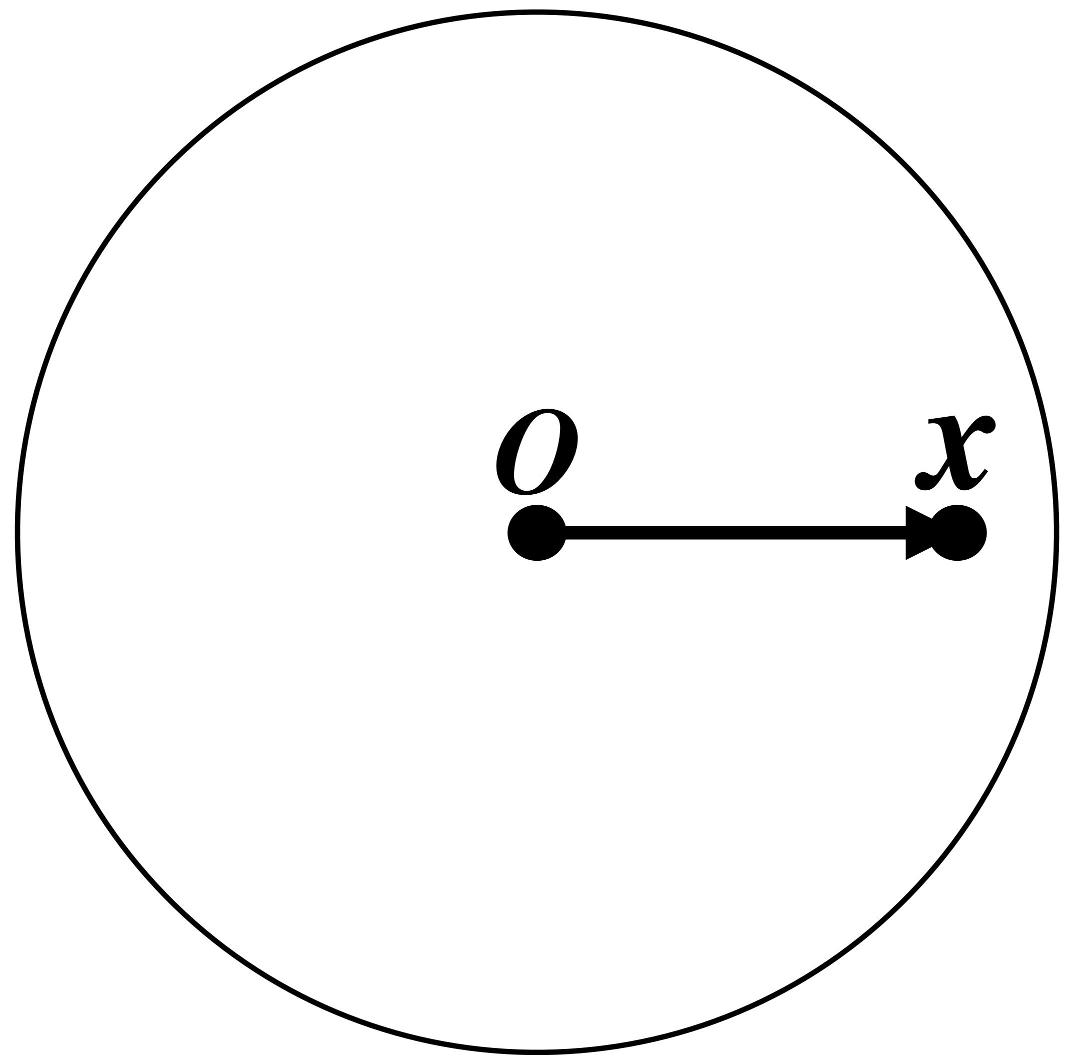

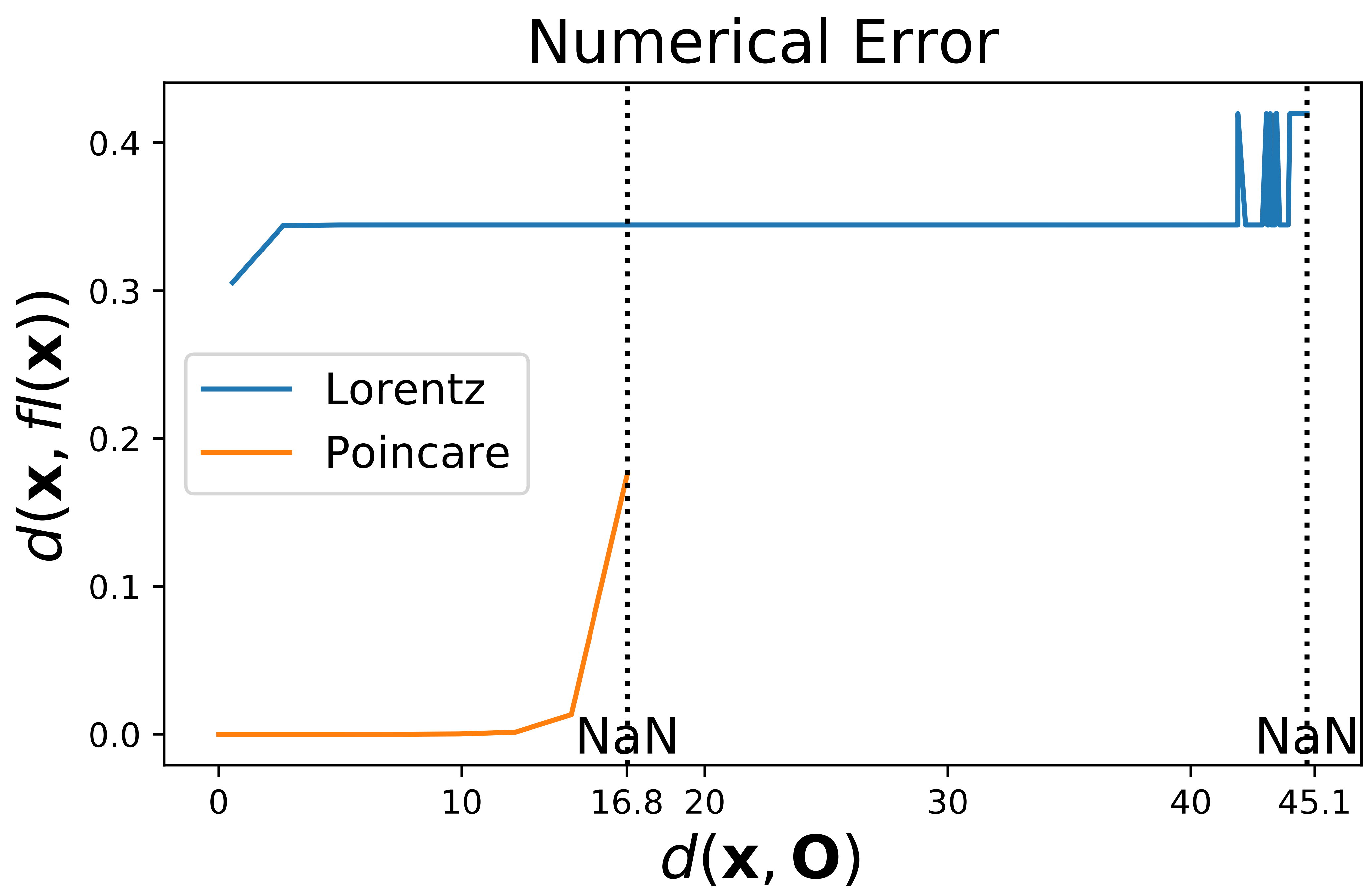

Proved: For standard models of hyperbolic space using floating-point, there exists points where the numerical error is $\Omega(\epsilon_{machine} \exp(d(\boldsymbol{x}, \boldsymbol{O})))$.

Can we be accurate everywhere?

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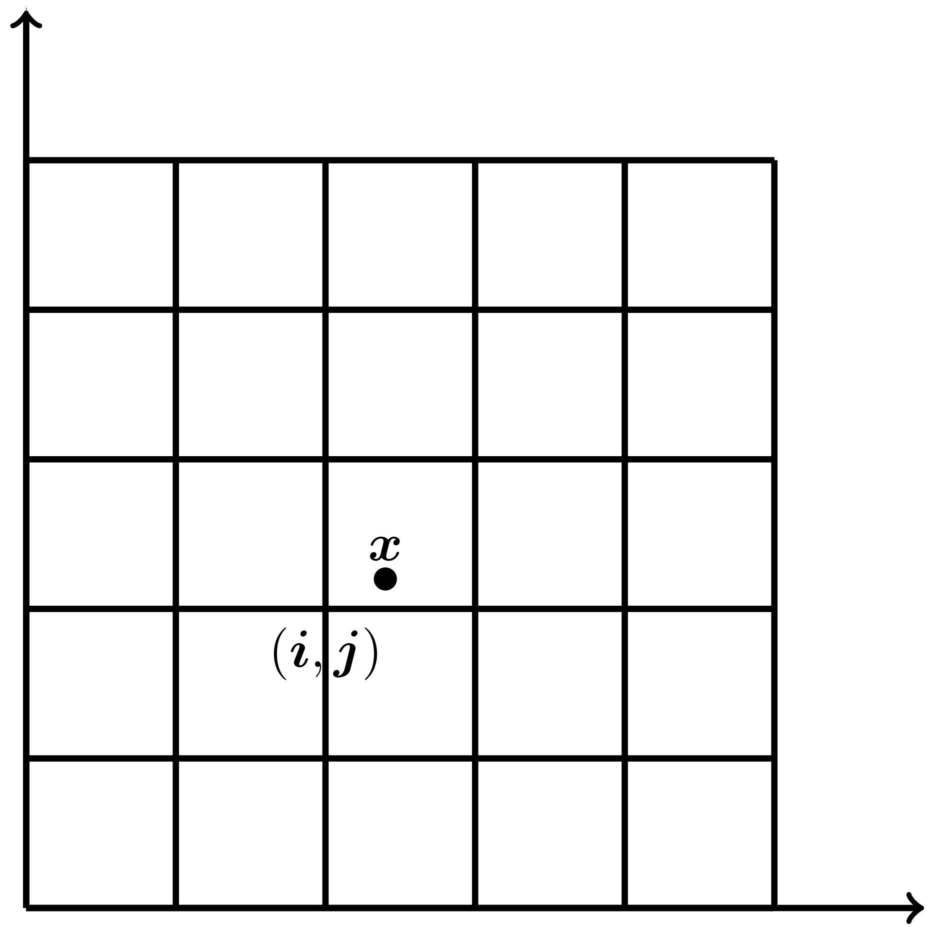
A solution in the Euclidean plane with constant error: using the integer-lattice square tiling, represent a point $\boldsymbol{x}$ in the plane with

(1)Coordinates $(\boldsymbol{i}, \boldsymbol{j})$ of the square where $\boldsymbol{x}$ is located as integer;
(2)Offsets of $\boldsymbol{x}$ within that square as floating-points.



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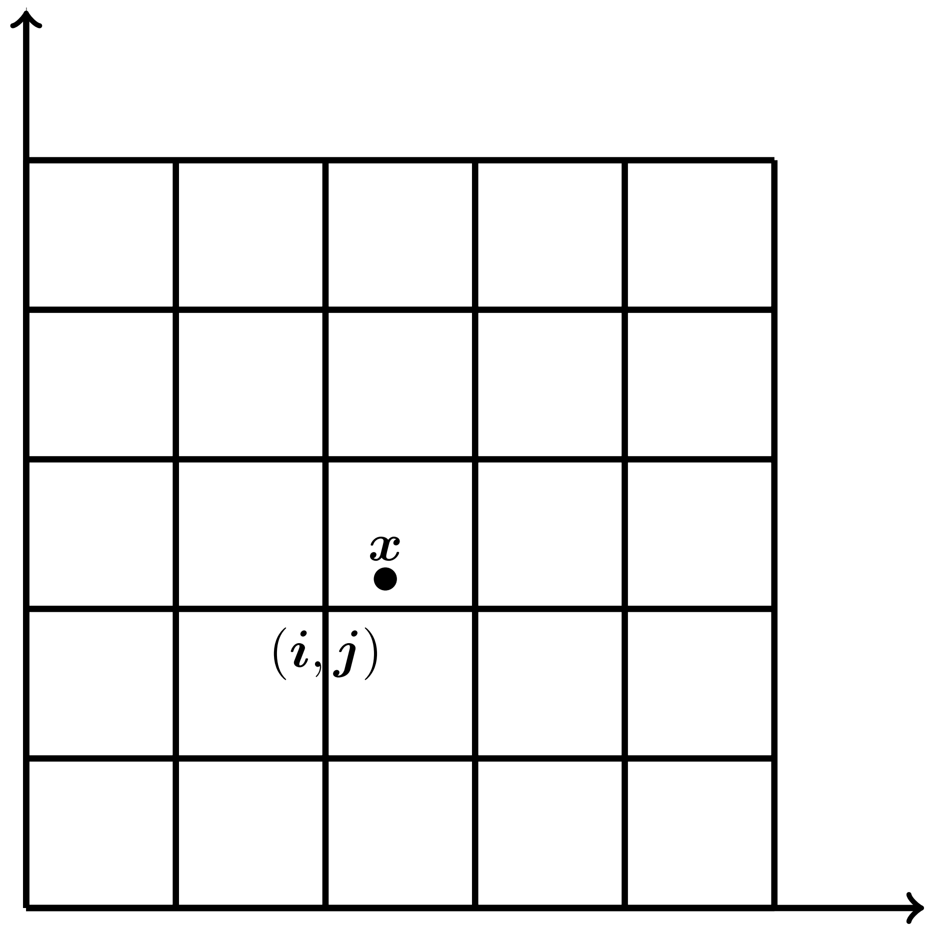
Proved: numerical error will be bounded everywhere and proportional to $O(\epsilon_{machine})$.



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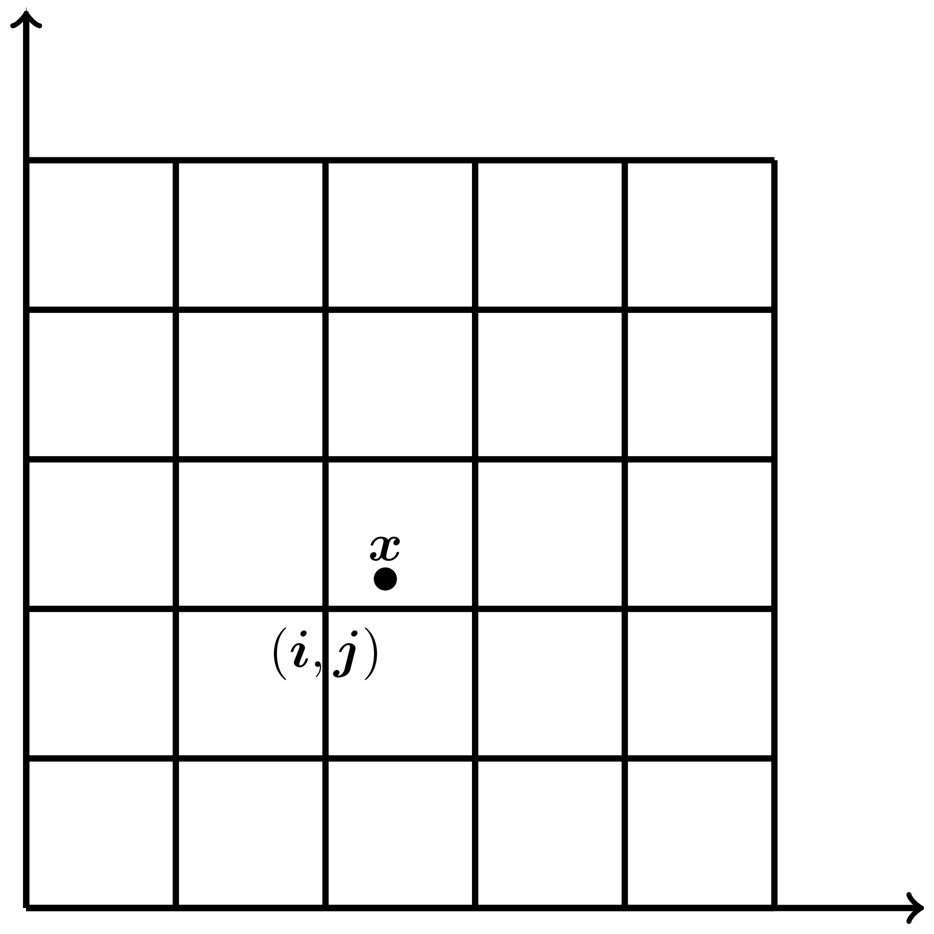
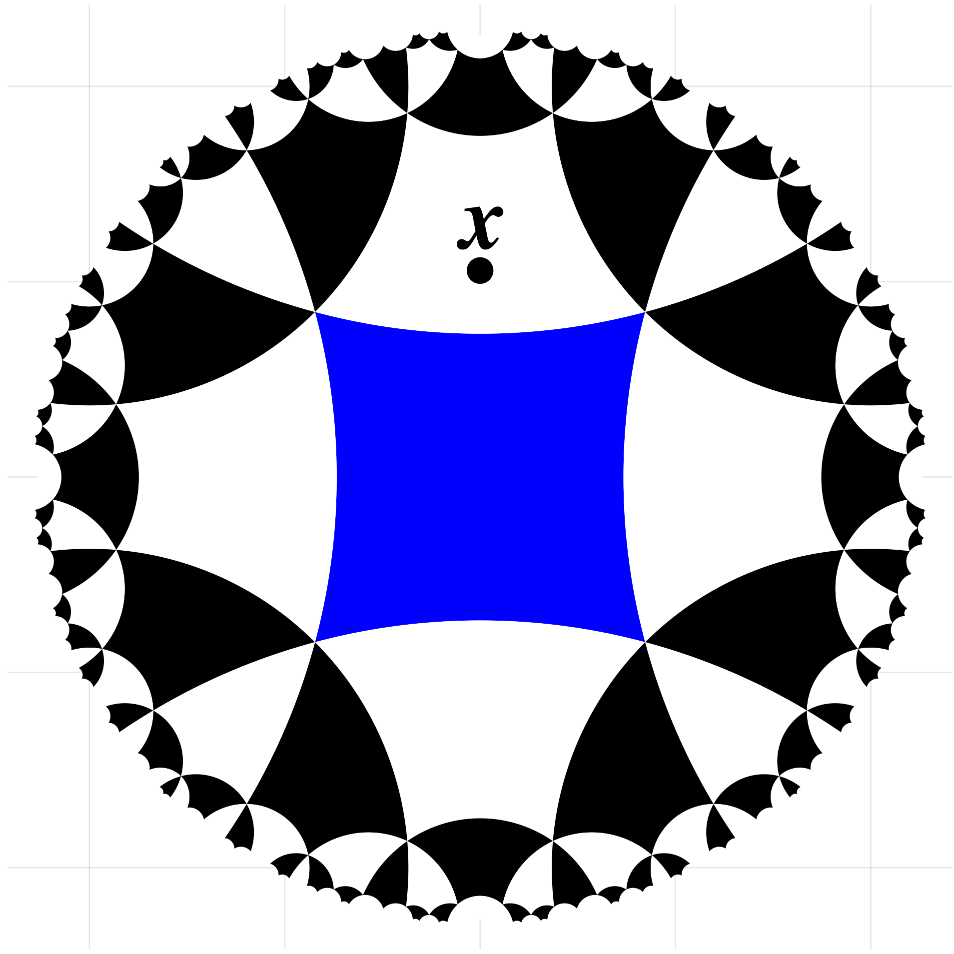


Proved: numerical error will be bounded everywhere and proportional to $O(\epsilon_{machine})$.

Do the same thing in the hyperbolic space: construct a tiling and represent $\boldsymbol{x}$ with:

(1)the tile where $\boldsymbol{x}$ is located;
(2)Offsets of $\boldsymbol{x}$ within that tile as floating-points.



Group-Based Tiling:

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How to identify a tile in the tiling of the hyperbolic plane?

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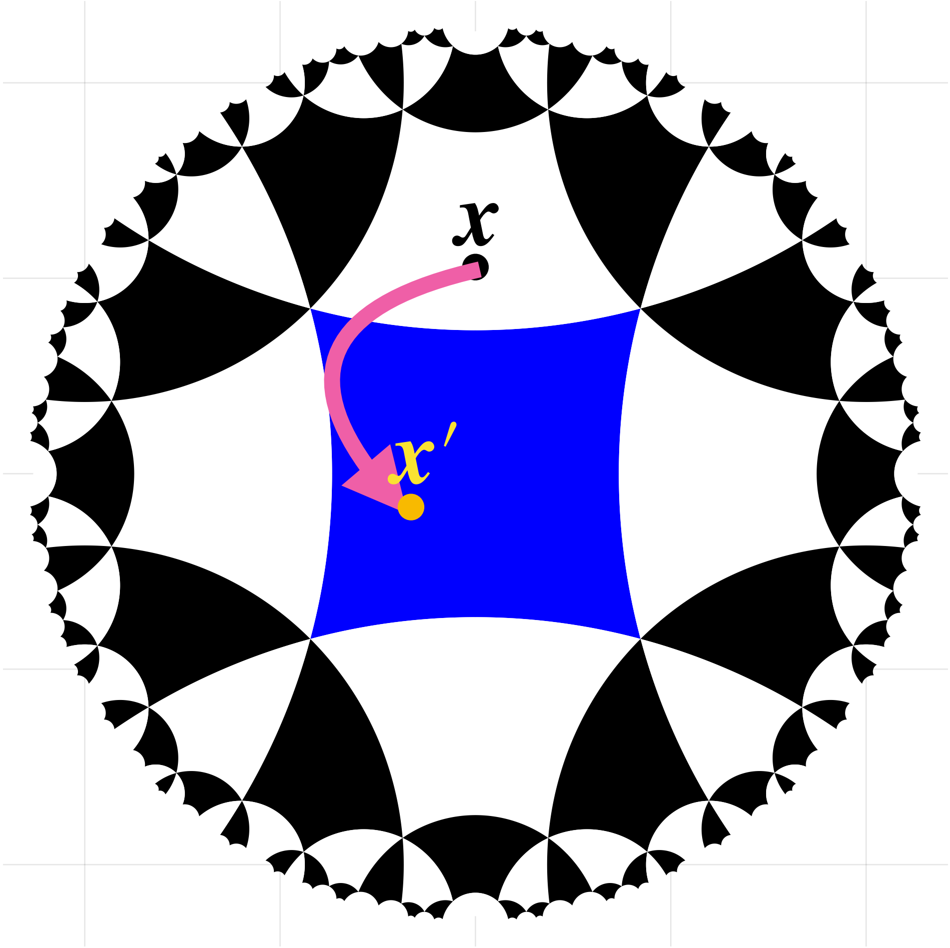

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Isometries!

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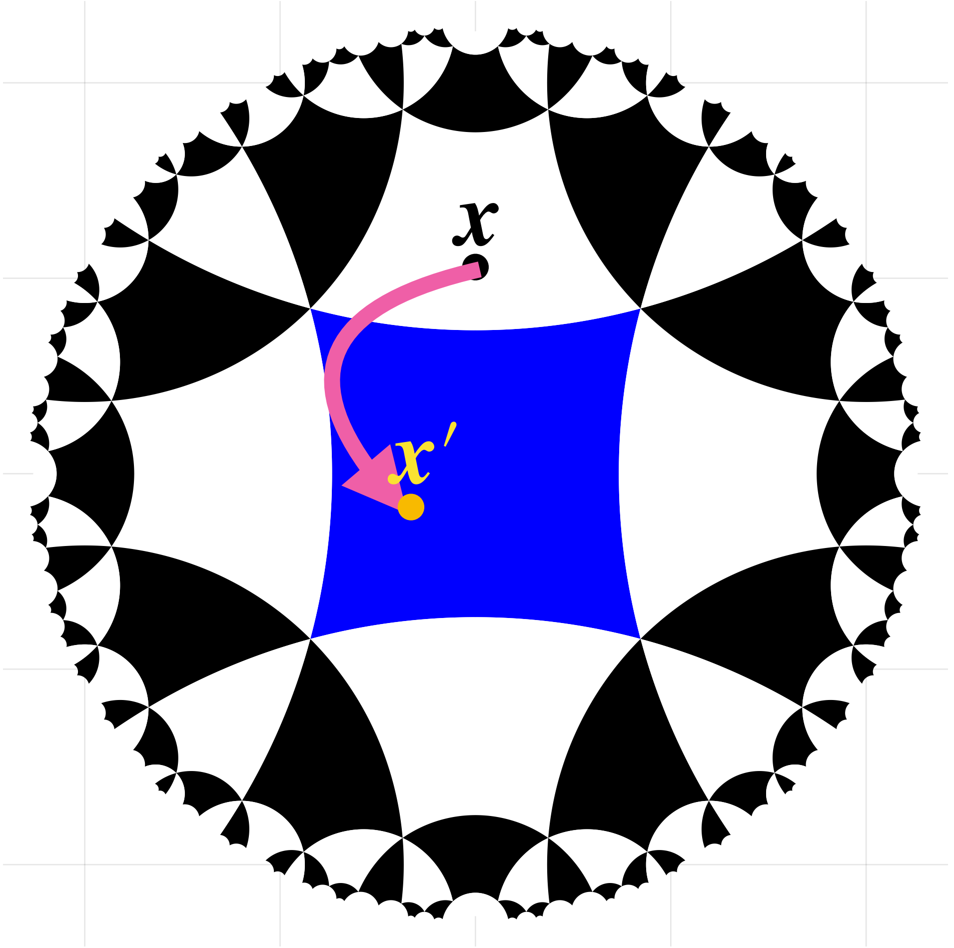

How to identify a tile in the tiling of the hyperbolic plane?

Isometries!

Construct a subgroup G of the set of isometries and represent $\boldsymbol{x}$ with

$$\mathcal{T}_l^n = \{(\boldsymbol{g}, \boldsymbol{x}') \in G \times F : \boldsymbol{x}'^T g_l \boldsymbol{x}' = -1\}.$$

Particularly, elements of G can be represented with integers, F is a bounded region.

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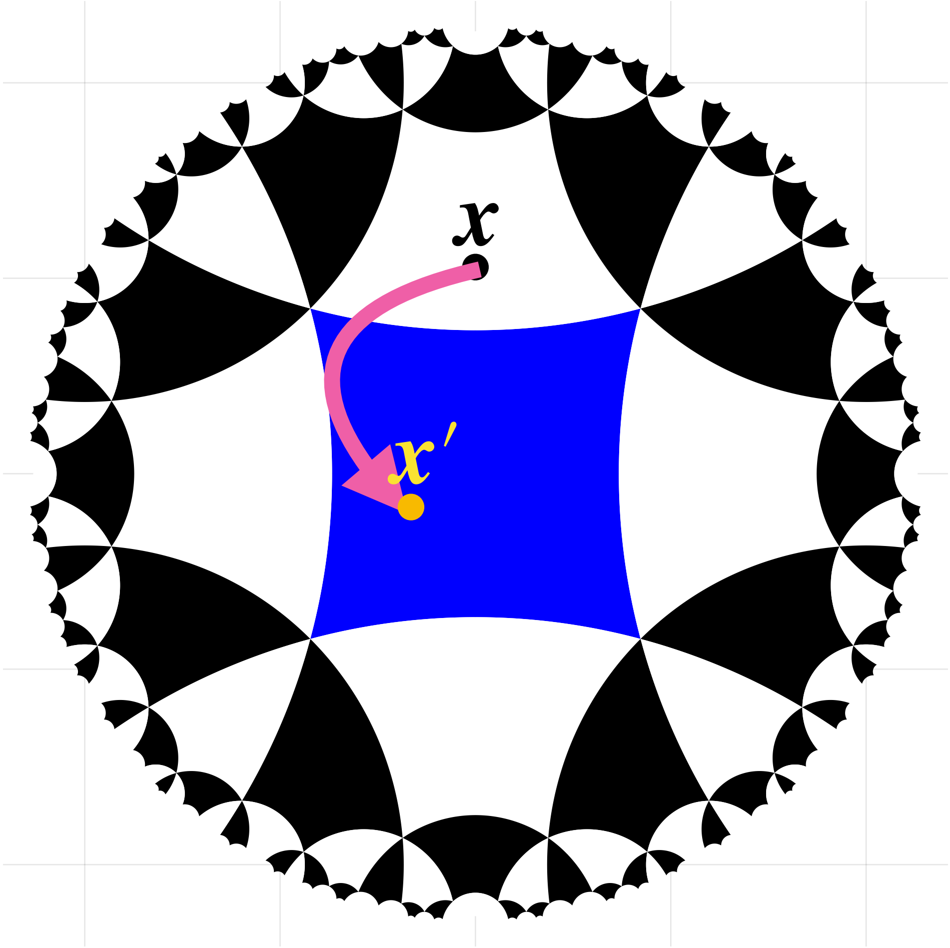

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Construct (non-group-based) tilings in high dimensional hyperbolic space and represent points with more integers.

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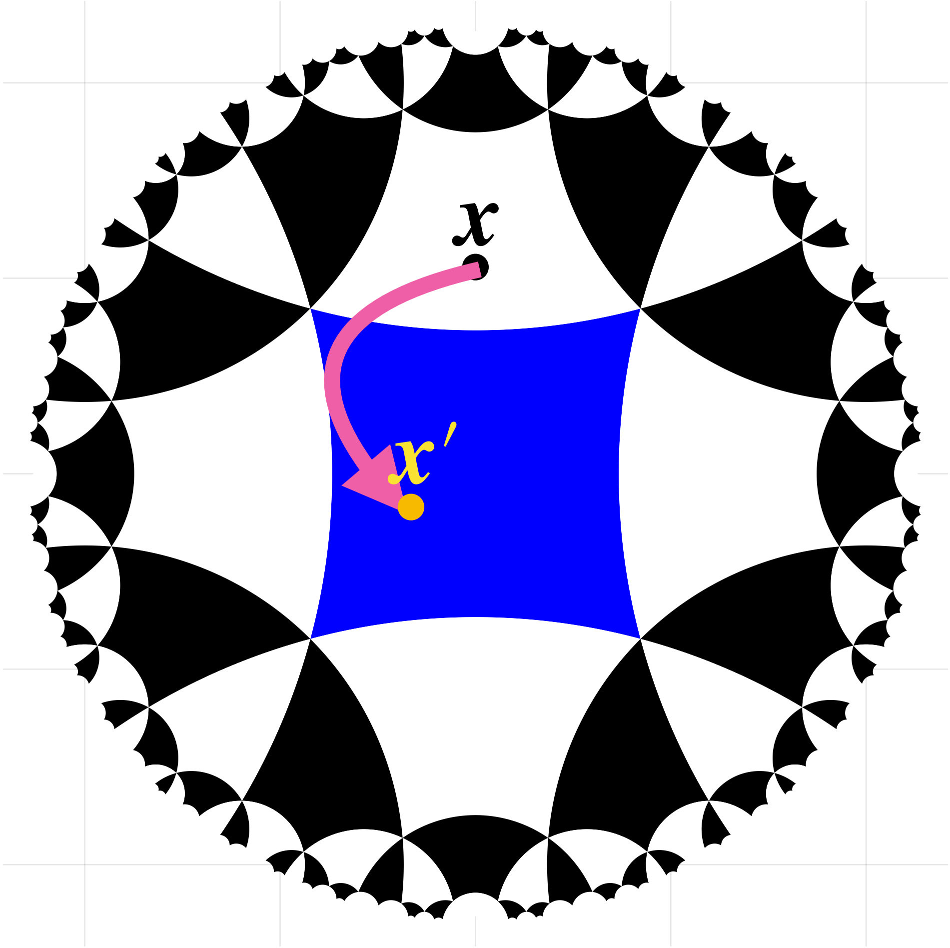

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Particularly, elements of G can be represented with integers, F is a bounded region.

Construct (non-group-based) tilings in high dimensional hyperbolic space and represent points with more integers.

Guarantees: numerical error is $O(\epsilon_{machine})$ everywhere in the space. (Representation, distance, gradients …)

Applications: Compression

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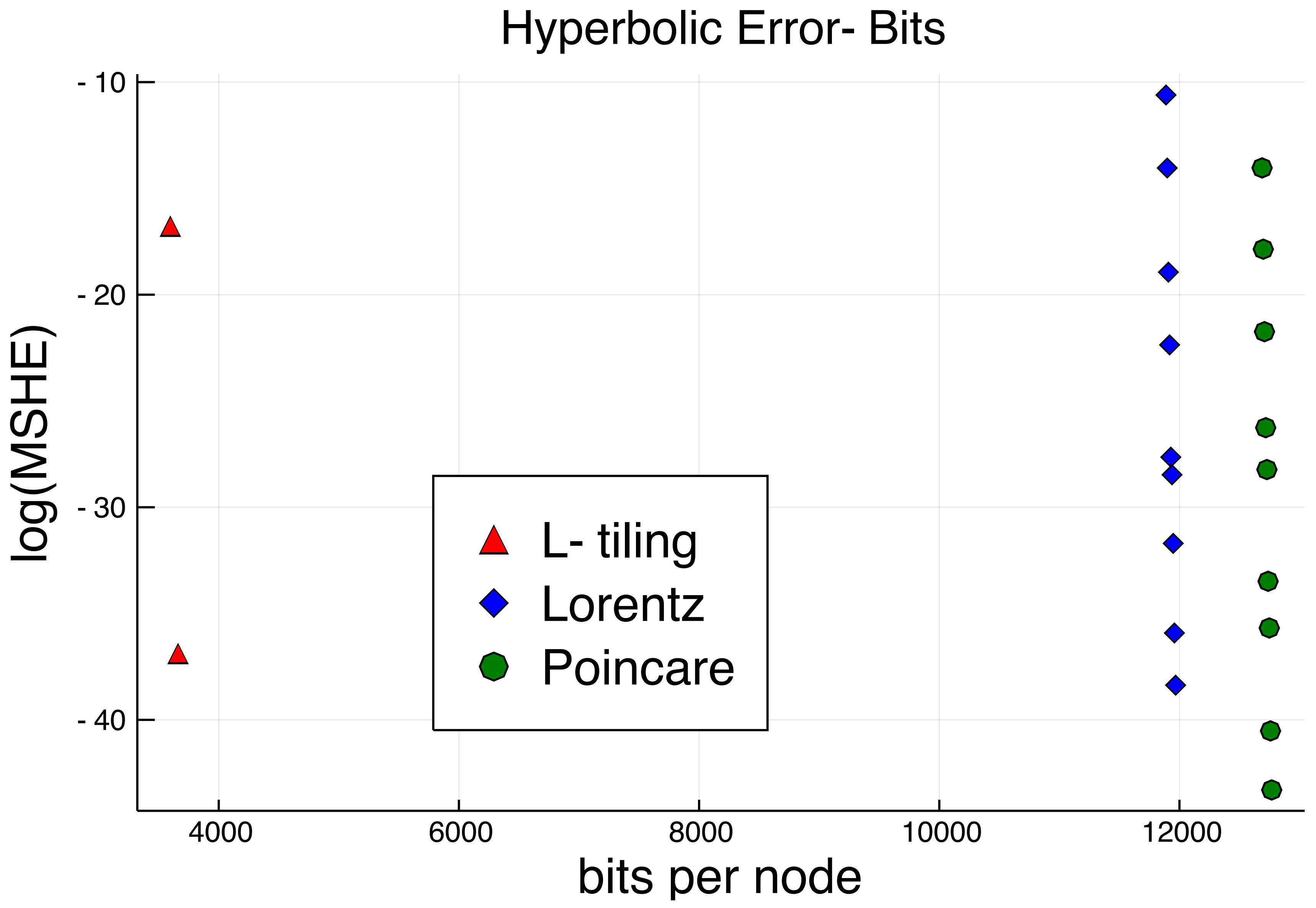
Represent and compress hyperbolic embeddings in tiling-based models to that in the standard models on the WordNet dataset.



Models	size (MB)	bzip (MB)
Poincaré	372	119
Poincaré	287	81
Lorentz	396	171
L-Tiling	37.35	7.13

Under the same MSHE, L-tiling model: 372 MB —> 7.13 MB (2% of 372 MB).

Applications: Learning

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Compute efficiently using integers in tiling-based models and learn high-precision embeddings without using BigFloats.

Dimension	Models	MAP	MR
2	Poincaré	0.124±0.001	68.75±0.26
	Lorentz	0.382±0.004	17.80±0.55
	tiling	**0.413**±0.007	**15.26**±0.57
5	Poincaré	0.848±0.001	4.16±0.04
	Lorentz	0.865±0.005	**3.70**±0.12
	tiling	**0.869**±0.001	**3.70**±0.06
10	Poincaré	0.876±0.001	3.47±0.02
	Lorentz	0.865±0.004	3.36±0.04
	tiling	**0.888**±0.004	**3.22**±0.02

On the largest WordNet-Nouns dataset, Tiling-based model outperforms all baseline models.

Conclusion:

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1. Hyperbolic space is promising, but the NaN problem greatly affects its power and practical use.

2. Tiling-based models solve the NaN problem with theoretical guarantee, i.e., fixed and provably bounded numerical error.

3. Tiling-based models empirically achieve substantial compression of embeddings with minimal loss, and perform well on embedding tasks compared to other models.

Thank You!

Poster #1189, East Exhibition Hall B+C #33, 5-7 pm