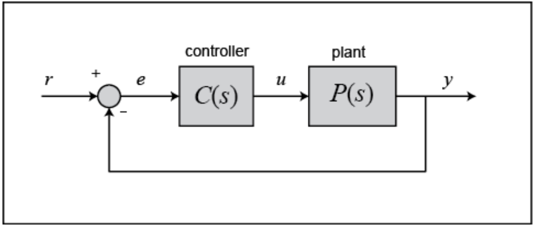
# Beyond Online Balanced Descent: An Optimal Algorithm for Smoothed Online Convex Optimization

Gautam Goel

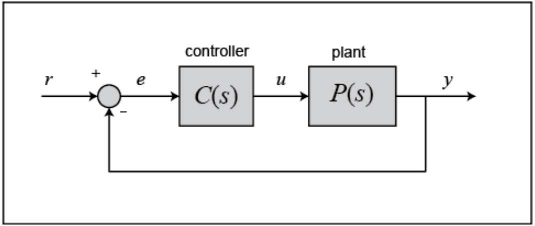
Based on joint work with Yiheng Lin, Haoyuan Sun, and Adam Wierman


Portfolio Optimization




Adaptive Control


Portfolio Optimization




Adaptive Control

**This talk: how do we design online learning algorithms that adapt to dynamic environments while accounting for switching costs?**

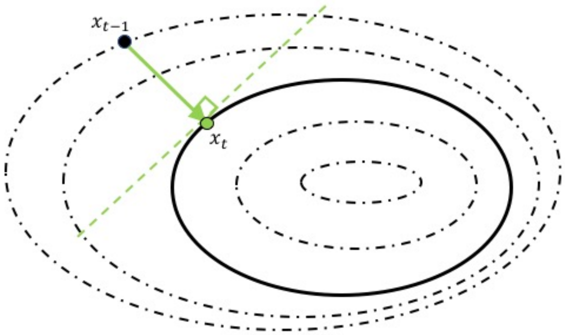
## Online Convex Optimization (OCO) with **one-step lookahead** and **switching costs**

An online learner plays a series of rounds against an adaptive adversary. In the $t$-th round:

1. The adversary chooses an $m$-strongly-convex cost function $f_t : \mathbb{R}^d \to \mathbb{R}_{\geq 0}$.
2. **After** observing $f_t$, the learner picks a point $x_t \in \mathbb{R}^d$.
3. The online learner pays the **hitting cost** $f_t(x_t)$ as well as a **switching cost** $\frac{1}{2}\|x_t - x_{t-1}\|_2^2$ which penalizes the learner for changing its decisions between rounds.

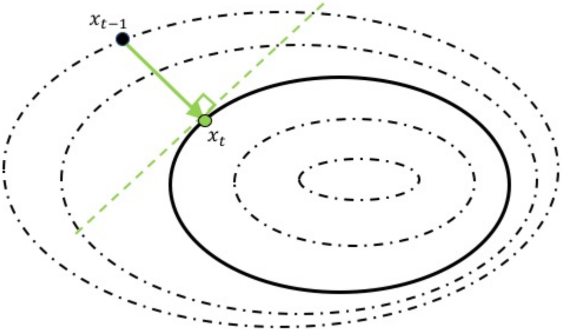
$$\texttt{Competitive Ratio} = \sup_{f_1,\ldots f_T} \frac{\sum_{t=1}^{T} f_t(x_t) + \frac{1}{2}\|x_t - x_{t-1}\|^2}{\underbrace{\min_{x_1,\ldots x_T} \sum_{t=1}^{T} f_t(x_t) + \frac{1}{2}\|x_t - x_{t-1}\|^2}_{\textbf{Dynamic}\text{ optimal solution}}}.$$

# Online Balanced Descent (OBD)




**Key idea #1:** Project onto level sets (otherwise you incur extra switching cost!).

# Online Balanced Descent (OBD)




**Key idea #1:** Project onto level sets (otherwise you incur extra switching cost!).

**Key idea #2:** Pick level set so that switching cost ≈ hitting cost.

**Theorem (Goel, Lin, Sun, Wierman '19)**

*Suppose the hitting cost functions are m-strongly convex with respect to the $\ell_2$ norm and the switching cost is given by $c(x_t, x_{t-1}) = \frac{1}{2}\|x_t - x_{t-1}\|_2^2$. Any online algorithm must have a competitive ratio at least $\frac{1}{2}\left(1 + \sqrt{1 + \frac{4}{m}}\right)$. A modified version of OBD, called Regularized-OBD (R-OBD)* **exactly** *achieves the optimal $\frac{1}{2}\left(1 + \sqrt{1 + \frac{4}{m}}\right)$ competitive ratio.*

# Thanks for listening! See poster #50 at 5pm today.


Gautam Goel


Yiheng Lin


Haoyuan Sun


Adam Wierman

**Connections to statistics and control:** An Online algorithm for Smoothed Regression and LQR Control [Goel and Wierman, AISTATS'19]

**Non-convex cost functions:** Online Optimization with Predictions and Non-convex Losses [Lin, Goel, and Wierman arXiv 1911.03827]