Exact Recovery of Multichannel Sparse Blind Deconvolution via Gradient Descent

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Multichannel Sparse Blind Deconvolution

Given multiple measurement

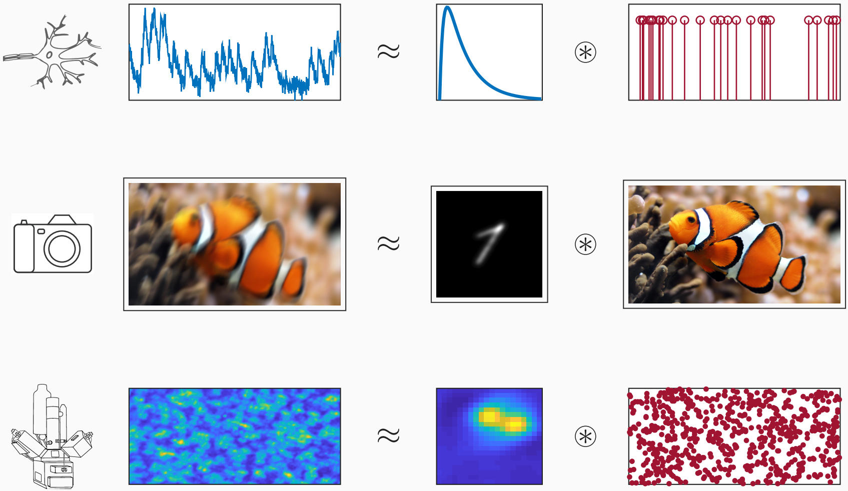
$$\boxed{\boldsymbol{y}_i = \boldsymbol{a} \circledast \boldsymbol{x}_i, \quad (1 \leq i \leq p),}$$

can we recover both $\boldsymbol{a}$ and $\{\boldsymbol{x}_i\}_{i=1}^{p}$ simultaneously?

- We assume $\boldsymbol{y}_i, \ \boldsymbol{a}, \ \boldsymbol{x}_i \in \mathbb{R}^n$.
- **Invertible** kernel $\boldsymbol{a}$.
- **Sparse** signal $\boldsymbol{x}_i$

$$\boldsymbol{x}_i \sim_{i.i.d.} \text{Bernoulli} - \text{Gaussian}(\theta)$$

Motivating Applications



Symmetry Leads to Nonconvex Problems

- **Scaling Symmetry**: $\boldsymbol{y}_i = \boldsymbol{a} \circledast \boldsymbol{x}_i = \alpha \boldsymbol{a} \circledast \alpha^{-1} \boldsymbol{x}_i$
  - easy to handle, $\|\boldsymbol{a}\| = 1$;

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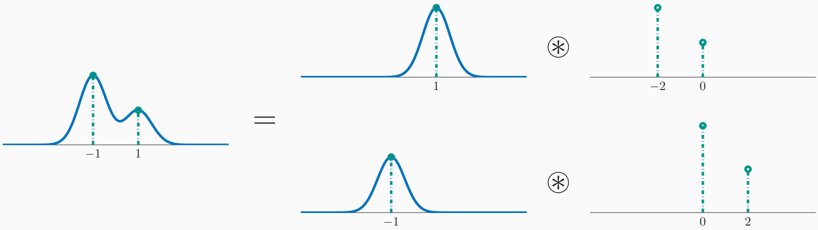

$$\boldsymbol{y}_i = \alpha\, s_\ell[\boldsymbol{a}] \circledast (1/\alpha)\, s_{-\ell}[\boldsymbol{x}_i]$$

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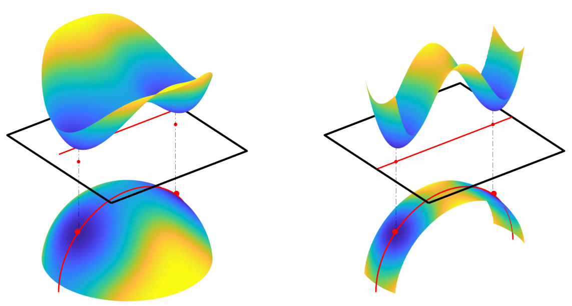
  - easy to handle, $\|\boldsymbol{a}\| = 1$;
- **Shift Symmetry** creates equivalent solutions:

$$\left(\boldsymbol{a}, \{\boldsymbol{x}_i\}_{i=1}^{p}\right) \quad = \quad \left(\mathrm{s}_\ell\left[\boldsymbol{a}\right], \{\mathrm{s}_{-\ell}\left[\boldsymbol{x}_i\right]\}_{i=1}^{p}\right)$$



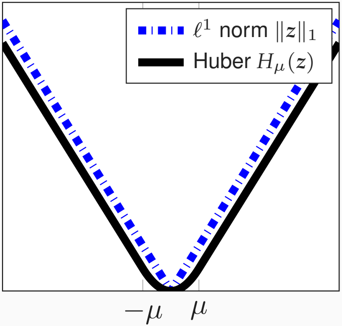
Nonconvex Formulation

Finding a shift of the filter $\boldsymbol{a}$ by solving

$$\boxed{\min_{\boldsymbol{q}} \ \frac{1}{np}\sum_{i=1}^{p} H_\mu\left(\boldsymbol{C}_{\boldsymbol{y}_i}\boldsymbol{P}\boldsymbol{q}\right), \quad \text{s.t.} \quad \boldsymbol{q} \in \mathbb{S}^{n-1}.}$$

Huber loss: 1st-order smooth & sparsity promoting

$$H_\mu(z) := \begin{cases} |z| & |z| \geq \mu \\ \frac{z^2}{2\mu} + \frac{\mu}{2} & |z| < \mu \end{cases}$$



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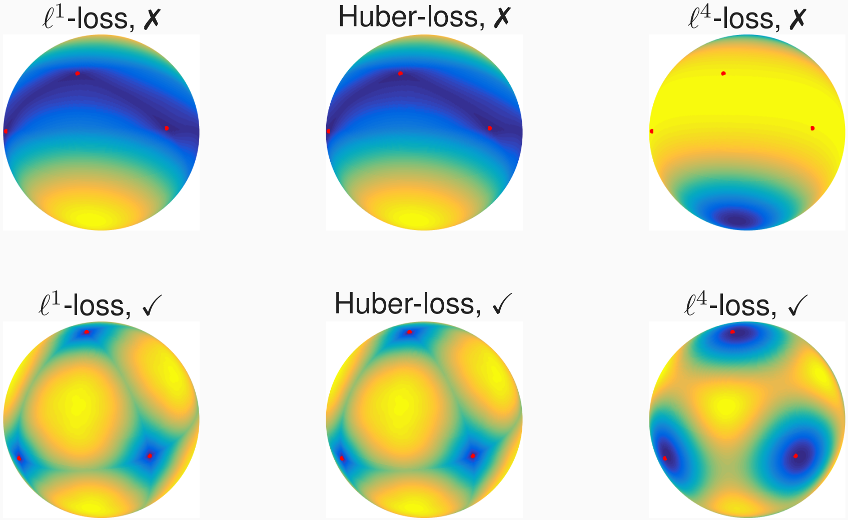
- Preconditioning leads to better landscape

$$\boldsymbol{P} = \left(\frac{1}{\theta np}\sum_{i=1}^{p} \boldsymbol{C}_{\boldsymbol{y}_i}^{\top}\boldsymbol{C}_{\boldsymbol{y}_i}\right)^{-1/2} \approx \left(\boldsymbol{C}_{\boldsymbol{a}}^{\top}\boldsymbol{C}_{\boldsymbol{a}}\right)^{-1/2},$$

Landscape with/without Preconditioning



Main Result

With random init., gradient descent solves sparse blind deconvolution in a linear rate.

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- Study the geometry properties of optimization landscape.
  - regularity condition, implicit regularization, sharpness.
- Benign geometry enables efficient optimization.

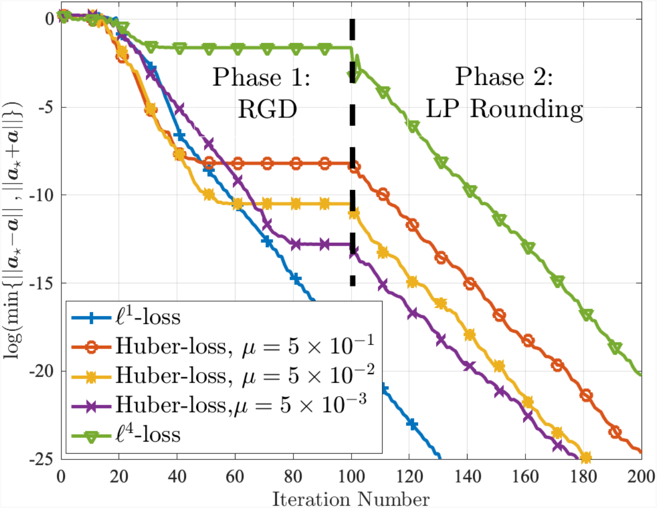
Comparison with Literature

Significant improvements in sample and time complexity.

Methods	Wang et al.[1]	Li et al.[2]	**Ours**
Assumptions	$\boldsymbol{a}$ spiky & invertible, $\boldsymbol{x}_i \sim_{i.i.d.} \mathcal{BG}(\theta)$	$\boldsymbol{a}$ invertible, $\boldsymbol{x}_i \sim_{i.i.d.} \mathcal{BR}(\theta)$	$\boldsymbol{a}$ invertible, $\boldsymbol{x}_i \sim_{i.i.d.} \mathcal{BG}(\theta)$
Formulation	$\min_{\lVert \boldsymbol{q} \rVert_\infty = 1} \lVert \boldsymbol{C_q Y} \rVert_1$	$\max_{\boldsymbol{q} \in \mathbb{S}^{n-1}} \lVert \boldsymbol{C_q P Y} \rVert_4^4$	$\min_{\boldsymbol{q} \in \mathbb{S}^{n-1}} H_\mu \left(\boldsymbol{C_q P Y} \right)$
Algorithm	interior point	*noisy* RGD	*vanilla* RGD
Recovery Condition	$\theta \in \mathcal{O}(1/\sqrt{n})$, $p \geq \widetilde{\Omega}(n)$	$\theta \in \mathcal{O}(1)$, $p \geq \widetilde{\Omega}(\max\left\{n, \kappa^8\right\} \frac{n^8}{\varepsilon^8})$	$\theta \in \mathcal{O}(1)$, $p \geq \widetilde{\Omega}(\max\left\{n, \frac{\kappa^8}{\mu^2}\right\} n^4)$
Time Complexity	$\widetilde{\mathcal{O}}(p^4 n^5 \log(1/\varepsilon))$	$\widetilde{\mathcal{O}}(pn^{13}/\varepsilon^8)$	$\widetilde{\mathcal{O}}(pn^5 + pn \log\left(1/\varepsilon\right))$

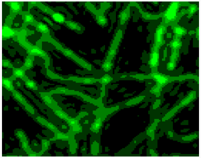
2. Wang et al., blind deconvolution from multiple sparse inputs, 2016.
3. Li et al., Multichannel sparse blind deconvolution on the sphere, 2018.

Experiment I: Convergence Comparison

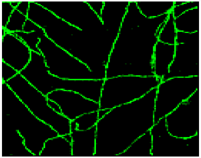


Experiment II: Super-resolution Microscopy

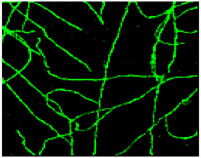
Observation



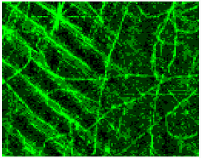
Ground truth



Huber-loss



ℓ^4-loss[3]



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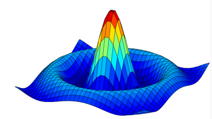

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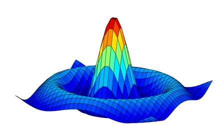

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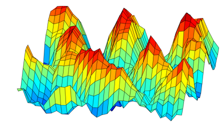

Take home message

With random init., gradient descent solves sparse blind deconvolution in a linear rate.

Poster: Hall B + C #207

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Xiao Li
(CUHK, EE)

Zhihui Zhu
(JHU, MINDS)

THANK YOU!

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